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Evenement Event

Heyting Day 2025: Models of intuitionism and computability
Symposium in honour of Jaap van Oosten

In December of 2024 Jaap van Oosten retired from the University of Utrecht. At the Heyting Day 2025, which was held on 14 March 2025 at the KNAW and organised by the authors together with Albert Visser, we celebrated his many contribution to research and teaching. The meeting was entitled "Models of intuitionism and computability" and consisted of three talks by international experts and the Heyting Lecture held by Jaap van Oosten. The meeting was attended by about 100 people and the recordings of the talks can be found on the website of the KNAW[11] and YouTube[12]. In this article we give an overview of Jaap's life, his scientific work and his other contributions to academic life.

Biography

Jaap was born on 19 December, 1957, in Klundert, the Netherlands. In 1976 he started his studies at the University of Amsterdam; not in mathematics, but in Dutch Language and Literature. He switched to mathematics in 1981 and finished his master in 1986. After that, he did his PhD under the supervision of Anne Troelstra from 1987–1991, also at the University of Amsterdam.

After his PhD, he did postdocs in Parma, Århus, Amsterdam and Utrecht, before obtaining a permanent position in Utrecht in 2000. During his time as a postdoc, he married Tine Blankenstein in 1994.

Throughout his scientific career, Jaap wrote many papers on categorical aspects of realizability, bringing together computability and logic.

This culminated in his book "Realizability: an Introduction to its Categorical Side", published in 2008 [10]. It has become *the* standard reference for the treatment of realizability from a categorical perspective.

In recognition of his scientific work, Jaap was an invited plenary speaker at the Logic Colloquium (three times) and at the CT conference in 2014, the most important scientific venues for Mathematical Logic and Category Theory in the world.

Scientific work

To understand the context of Jaap's scientific work, we strongly recommend reading his historical essay on realizability, published in 2002 [9]. This paper recognises three milestones in the development of the subject of realizability:

- Kleene's founding paper from 1945 called "On the interpretation of intuitionistic number theory" [4].
- Troelstra's *magnum opus* called "Metamathematical Investigation of Intuitionistic Arithmetic and Analysis" from 1973 [7].
- Hyland's landmark paper called "The effective topos" from 1982 [2].

Kleene

It was Kleene who invented realizability in response to Brouwer's work on intuitionism, a particular philosophy of mathematics. In Brouwer's philosophy every valid proof of an existence statement should be underpinned by a *construction* of the thing that is claimed to exist. What Brouwer meant by a construction is not easy to say, but Kleene decided to read this as an *algorithm* (a computer program). This made use of the, then very new, area of computability theory, which gave a mathematically precise definition of an algorithm and a computable function, developed to a large extent by Kleene himself. Nowadays we are

very familiar with this concept and we can say: a function is computable if it can be computed by an (idealised) computer (with unbounded memory).

Kleene's realizability can be understood as a mathematical answer to the question: if you give a constructive proof, what is the implicit algorithm that is hidden in the proof? One of the things realizability allows you to prove is the following: if you prove $\forall x \in \mathbb{N} \exists y \in \mathbb{N} \varphi(x,y)$ and your proof is constructive, then you can obtain an algorithm that finds a suitable y as a function of x.

Troelstra

After Kleene's work, it became clear that the way he extracted an algorithm from a constructive proof was not the only one and many more "realizability interpretations" were found. All of these were collected and systematically explained in Troelstra's *magnum opus* from 1973. As Jaap emphasises in his book, this systematisation involved a lot of original work, so Troelstra's book should be seen as much more than an exposition of known material.

Hyland

In Troelstra's book realizability is always a *proof-theoretic interpretation*, a syntactic transformation of logical formulas preserving their constructive provability. However, Martin Hyland's work on the *effective topos*, and his work on *tripos theory* together with Johnstone and Pitts [3], started a different *semantic* tradition. This semantic tradition is crucial for understanding Jaap's work.

In this tradition we understand realizability as a *mathematical structure* which is interesting in its own right, irrespective of own's philosophical convictions. The question, of course, is what kind of structure realizability should be, and the answer given by Hyland is that it is a *topos*, or more precisely: an *elementary topos*. Elementary toposes were defined by Lawvere and Tierney in 1969–1970 and axiomatise a categorical object that acts as a world of sets and functions. The main class of examples at the time were the so-called Grothendieck toposes, as invented by Alexandre Grothendieck for the purposes of algebraic geometry.

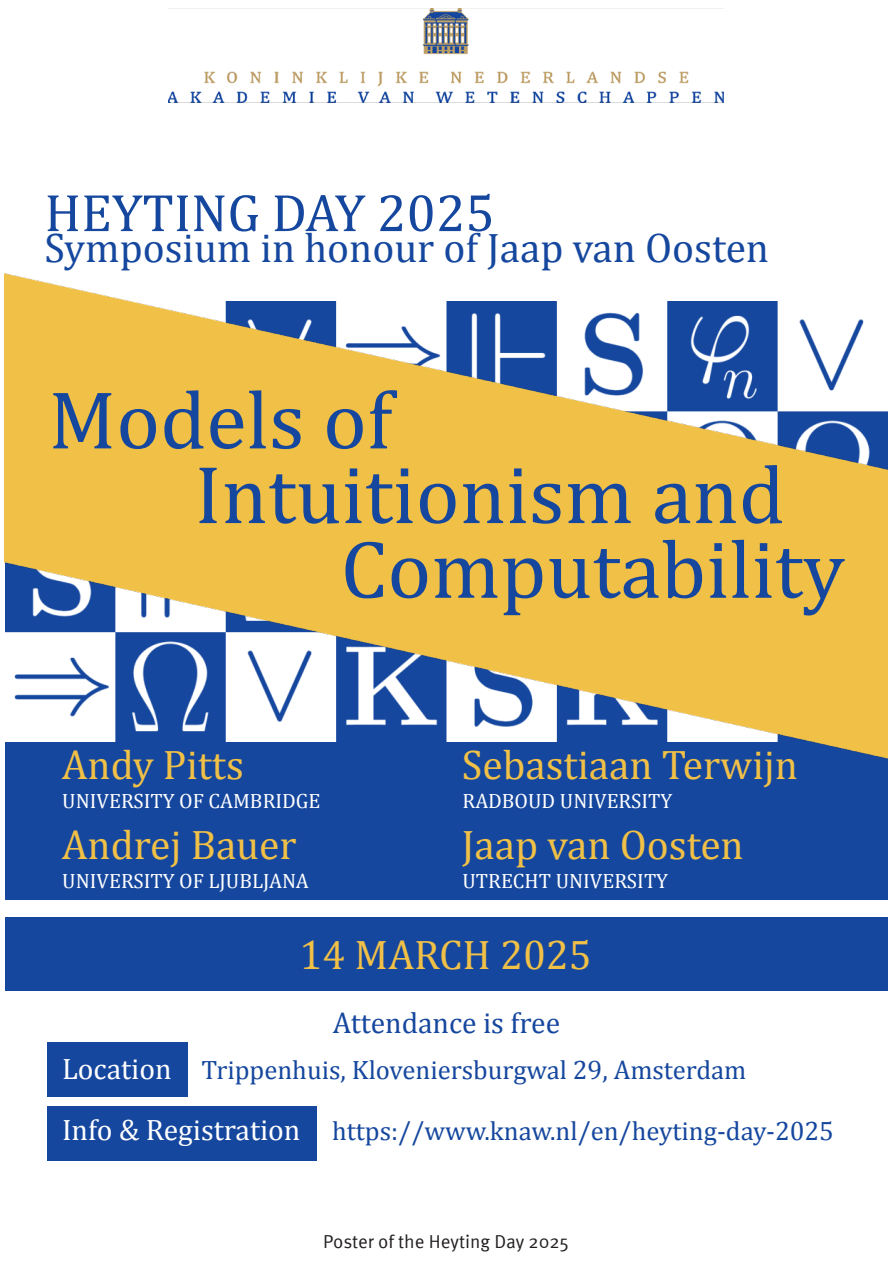
The triposes defined by Hyland, Johnstone and Pitts allowed one to construct a new class of elementary toposes, and Hyland's effective topos is the prime example. This is a world which is ruled by realizability in that in this world everything is computable by nature: it is truly a computable universe.

Jaap's work

It turns out that many of the other realizability interpretations that can be found in Troelstra's book can also be understood as being the logical laws in suitable computable universes. Trying to formulate these proof-theoretic interpretations in a more semantic way, using the notions of topos and tripos in particular, has proven to be an incredibly fruitful endeavour. Pitts started doing this in his PhD thesis [6] and also Grayson [1] did important work.

Jaap's PhD thesis [8] should be understood against this background. It contributes to the semantic understanding of various notions of realizability, in the footsteps of the works just mentioned, and focusing on Lifschitz realizability in particular. Jaap's PhD thesis contains many more things, some of which he would further develop in later work. All of this goes under the rather understated title of *Exercises in Realizability*.

After his PhD thesis, he analysed, among other things, relative, modified and extensional realizability, always using topos theory as his general framework for



Courtesy of Jacob Neumann

Poster of the Heyting Day 2025

trying to understand the phenomena he observed. He also contributed to the understanding of the morphisms of these realizability toposes and the ways realizability toposes can be obtained from computational models known as pcas and their various generalisations. Much of this work was done together with his MSc and PhD students.

Teaching

Jaap was also passionate about teaching and raising the next generation of Dutch logicians.

We already mentioned his book *Realizability: an Introduction to its Categorical Side*. The book starts by quoting Martin Hyland saying that the world is not ready for a book on realizability; nevertheless, Jaap decided to write one and explicitly says the main motivation for writing this book was to help MSc and PhD students who wanted to find their way into this subject, and as such it has been enormously helpful to many students.

He has also been crucial in keeping mathematical logic alive in the Mathematics Department in Utrecht. The course on "Grondslagen van de Wiskunde" (Foundations of Mathematics) has been the first introduction to logic for a whole generation of math students in Utrecht. The lecture notes for this course have become Jaap's second book *Sets, Models and Proofs* [5] (published in 2018, and written together with Ieke Moerdijk).

Indeed, if you look at the names of Jaap's PhD, MSc or BSc students, it becomes a "who is who" for Dutch mathematical logic. His PhD students were Pieter Hofstra, Wouter Stekelenburg and Jetze Zoethout; and we would also like to mention Bram Arens and Claire Gentil, who both started, but did not finish for personal reasons.

Among the many master students Jaap supervised (certainly more than 40), we would like to mention Eric Faber, Sori Lee, Niels Voorneveld and Tingxiang Zou in particular, as their work led to joint publications.

One special quality of Jaap is that he wanted students in logic to become acquainted with the full breadth of mathematical logic. In particular, he would often run student seminars on topics like the topology of rewrite systems or o-minimality, which were quite far from his own area of expertise.

Both authors have very fond memories of Jaap's lecture notes on *Recursion Theory and Nonstandard models of Peano Arithmetic*. Also his lecture notes on *Category Theory & Topos Theory* have become a reference for many generations of students. Characteristic of these notes is that they are always very to the point, stimulating, but also expect the student to do a serious amount of work.

In short, Jaap has made considerable contributions to mathematical logic in the Netherlands and the Dutch logic community is indebted to him for his many efforts on its behalf.

References and notes

1 R. J. Grayson. *Modified realisability toposes*. Handwritten notes from Münster University, 1981.

2 J. M. E. Hyland. The effective topos. In A. S. Troelstra and D. van Dalen, editors, *The L. E. J. Brouwer Centenary Symposium*, volume 110 of Studies in Logic and the Foundations of Mathematics, pages 165–216. Elsevier, 1982.

3 J. M. E. Hyland, P. T. Johnstone, and A. M. Pitts. Tripos theory. *Mathematical Proceedings of the Cambridge Philosophical Society*, 88(2):205–232, 1980.

4 S. C. Kleene. On the interpretation of intuitionistic number theory. *Journal of Symbolic Logic*, 10(4):109–124, 1945.

5 I. Moerdijk and J. van Oosten. *Sets, Models and Proofs*. Springer Undergraduate Mathematics Series. Springer, 2018.

6 A. M. Pitts. *The Theory of Triposes*. PhD thesis, University of Cambridge, 1981.

7 A. S. Troelstra. *Metamathematical Investigation of Intuitionistic Arithmetic and Analysis*, volume 344 of Lecture Notes in Mathematics. Springer-Verlag, 1973.

8 J. van Oosten. *Exercises in realizability*. PhD thesis, University of Amsterdam, 1991.

9 J. van Oosten. Realizability: a historical essay. *Mathematical Structures in Computer Science*, 12(3), 239–263, 2002.

10 J. van Oosten. *Realizability: An Introduction to its Categorical Side*, volume 152 of Studies in Logic and the Foundations of Mathematics. Elsevier, 2008.

11 https://www.knaw.nl/en/heyting-day-2025

12 https://www.youtube.com/@DeKNAW/videos