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In Memoriam Wim Veldman (1947-2024)

# An inspiring intuitionistic mathematician

**With great sadness it falls upon us, former students, to commemorate the life and mathematical work of Wim Veldman: beloved teacher, supervisor and researcher at Radboud University Nijmegen.**

Wim Veldman passed away in November 2024, only months after his 2023 cancer treatment was discovered to have been unsuccessful. In these last months he was closely supported by his wife Beatrix and his children Daniël and Liesbeth, who came back home to face this difficult time together as a family.

Many people came to visit him while his

health still allowed it. On hearing the bad news one of his students took the first plane from Japan to the Netherlands, wishing to provide solace and support as well as to express what Wim meant to her, and had meant in her education.

## Wim the teacher

Wim was extraordinarily well-liked and loved, having played for many students a major part in the course of their study of mathematics in Nijmegen. But also by his close colleagues at the faculty, and many mathematicians around the world who came to know him as an uncommon thinker in the field of constructive mathematics and foundations. Most of these research colleagues probably remained unaware of Wim's exceptional qualities as a teacher. A striking characteristic of Wim was the broadness of his talents, and the depth of his knowledge and abilities therein.

Drawing upon these, Wim's teaching attracted many students to his field of logic, foundations, constructive mathematics and his favourite intuitionistic mathematics. His lectures came with beautifully handwritten lecture notes, which contained a detailed exposition as well as an historical background and motivation of the subject. For students

he was very approachable, and a pillar of support way beyond his teaching.

One then understands that, upon Wim's 'retirement' in 2012, a truly large lecture hall was packed with faculty and students from 30+ years of teaching, who came to hear his 'farewell lecture' (after which of course he continued to teach for 12 more years). At the end of said lecture he received a minute-long standing ovation, not quite a daily occurrence in the world of mathematics.

Wim himself studied mathematics under Johan de longh, who in turn studied with Brouwer and was later to found the mathematics department in Nijmegen, in the early 1960's [1]. In those days strict hierarchies existed between students and faculty, and even between the full professors and the other faculty. On receiving his MSc, Wim joined the faculty and witnessed first-hand the democratization of the university in the 1970's, of which De longh by the way did not approve.

De longh was an inspiring teacher, who held to high standards regarding clarity, precision and didactics. After De longh's retirement, Wim was part of a small group which endeavoured to expand these standards to large parts of the bachelor programme. Professor Arnoud van Rooij recalls decades of daily lunch walks in which they discussed each other's lecture notes and shared their cares about students, both as mathematicians-in-spe and as young people finding their way in life.



Wim Veldman

### Wim the researcher

Every advantage has its disadvantage, according to a well-known football coach. The downside of De Iongh's high standards was that he was seldom satisfied with a piece of mathematical writing, saying that if its subject was of true interest, then it surely merited an even better exposition. For this reason De Iongh never published anything.

Thankfully Wim inherited only the better half of this mentality, and published many articles which clearly exhibit being well thought through in content and presentation. Still, Wim's PhD thesis *Investigations in intuitionistic hierarchy theory* [2], took almost ten years to finish, owing much no doubt to its having to pass both Wim's and de Iongh's scrutiny. The subject matter of his thesis retained Wim's interest throughout his career and from time to time he would revisit his earlier results with new publications reflecting his progressing insight.

As a young researcher Wim drew attention by proving a completeness theorem for intuitionistic predicate logic, constructively [3]. Not only is the proof ingenious, it also was in marked contrast to arguments from Georg Kreisel that such completeness implied non-constructive axioms. Wim showed that this implication was brought on by the classical proof-theoretic framework in which Kreisel had sought to interpret the metaconcepts involved. Under a semantical interpretation

based on Kripke schemes, completeness can be proven even constructively (with a caveat, see below).

The researcher Wim combined modesty with a certain stubbornness. Like De Iongh he was well-read in the foundations and philosophy of mathematics, and enjoyed reading the original publications of Cantor, Brouwer, Frege, Russell, Gödel, Poincaré, Wittgenstein, Kleene,... in four modern languages, not to mention the *Elements of Euclid* which he read in ancient Greek. Convinced of the beauty and correctness of Brouwer's insights he did not disavow intuitionistic mathematics, as many constructivists did and still do, in order to gain recognition, funding and other benefits from being part of an established research area.

His publications all concern intuitionistic mathematics (INT), and against the tide he would keep reminding others that clarity of foundations deserved more time than seemingly allowed by the prevailing 'publish or perish' winds. It was surprising to many to find in him also a very competent classical mathematician, and occasionally some had the misfortune to see their results lose their shine from just a few of Wim's comments. He did not hesitate to question his own results either, for instance his completeness theorem of which he later expressed doubts that the Kripke model involved was truly acceptable according to Brouwer's views.

A very nice reference to Wim and his contributions in the field of logic can be found in Joan Moschovakis's lemma *Intuitionistic Logic* in the Stanford Encyclopedia of Philosophy (SEP) [4]. In her acknowledgements, Wim is given the special place that indeed only he occupied. The authors of this in memoriam, both students of Wim, are grateful to professor Moschovakis for this recognition. We would like to repeat her advice to read Wim's 2021 publication *Intuitionism, an inspiration?* [5] which she characterizes as the best approach to intuitionistic mathematics for anyone interested.

Wim's intuitionistic proof of Kruskal's theorem [6] is widely used today in theoretical Computer Science. It is but an example that Brouwer's intuitionism forms a sizeable and growing part of modern mathematics. It was at times difficult for Wim to know the validity and modern-day relevance of INT in contrast to a certain general perception that INT is quaint and belongs to the past. As his PhD-student the first author already had many discussions with Wim on INT itself, and on the general misperception still persisting from Brouwer's feud with Hilbert. After my graduating in intuitionistic topology, we kept in touch both personally and mathematically. It was comforting to Wim to know that he had passed on his torch to a next generation, and over the years he kept an active interest in the few publications that I managed to produce. Likewise I tried to read his articles, which was not always easy since after his so-called retirement, Wim worked harder than ever.

One endeavour needs to be mentioned here: Wim recently formalized what is now known as intuitionistic reverse mathematics, in a formal setting called BIM (Brouwer's Intuitionistic Mathematics) [7] building on Kleene's seminal work in *The Foundations of Intuitionistic Mathematics* [8] (FIM, 1965). Kleene eventually proved the equiconsistency of classical and intuitionistic analysis, and while doing so also developed the notion of realizability which has become central in different areas of proof theory and computer science. Time will tell in what way BIM will be used by future scholars, the authors are confident, however, that the foundational clarity of BIM along with the stability of INT (its axioms as described in FIM have remained unchanged) will remain attractive.



De Iongh at Wim's PhD defence.



Wim defending his PhD thesis, 1981.

### Wim the supervisor

Four students wrote their PhD thesis under the (co-)supervision of Wim Veldman [9]: Tonny Hurkens (*Borel Determinacy without the Axiom of Choice*, 1993), Franka Waaldijk (*Modern Intuitionistic Topology*, 1996), Giuseppina Ronzitti (*On the Cardinality of a Spread*, 2002 – defended at the University of Genoa) and Bas Spitters (*Constructive and Intuitionistic Integration Theory and Functional Analysis*, 2003).

It would take many boxes to store all the Master's theses that Wim supervised over the years. Especially in this capacity he inspired students to hold to their own path, and to trust in their capacity for quality, whilst pursuing their further career outside of mathematics.

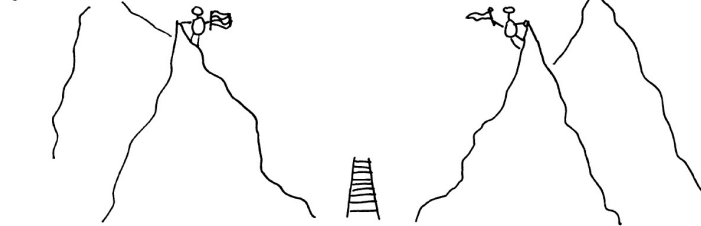
He was very well-read, but rather than displaying his broad scholarship, he would be sincerely curious to hear from students whose broader interests he knew of (and shared), how they would connect those to their mathematical work. In the case of the second author, Wim's subtle philosophical questions (of her claim in a casual piece she wrote on Alan Turing's biography, and in his assessment of her Master's thesis), while somewhat daunting at the time, have inspired her throughout her work, in academia and beyond, to never be afraid to ask such questions — even (or especially) if you're not sure they can be answered.

### Wim the person

At the same time, Wim compensated the depth of his inquiries with a very light, disarming kind of humor. It emanated in his classes but also in his official ("serious") lectures. While explaining abstract theory, he would always create a narrative that made it personable, through fictional characters (such as Alice in Wonderland) or historical figures (Euclid, Wittgenstein

As another consequence of the hyperarithmetical hierarchy theorem, we have, that, for each  $\tau \in \text{HIF}$ ,  $\Pi_\tau^0$  is not closed under the operation of countable union, and  $\Sigma_\tau^0$  is not closed under the operation of countable intersection.

9.11 This is where we stand now:



Perhaps because of breathing deeply the thin air of higher mathematics, we are feeling slightly euphoric...

An illustration of Wim's handwriting and humor from his PhD thesis.

and Ramsey). It made the audience feel to be on a joint adventure in thought and time, meeting interesting characters, rather than dissociating into abstraction. He clearly enjoyed these adventures himself, often almost giggling at the blackboard, at the imagination of the story playing out. In his last months, when reading was becoming increasingly strenuous, Wim could still find joy in reading children's stories and poems with similar qualities of addressing big life questions without using big assuming words.

Wim was also a dedicated musician and violinist. In fact, it was through music that he also deeply connected with others, both within and outside the mathematical community. For the Open Podium on the occasion of the first lustrum of the student association Desda in 1991, Wim conducted an orchestra of students and staff members — with a similar humility and precision towards the music, but also with his sincere joy in establishing a joint experience of beauty.

Wim Veldman kept developing and challenging his thoughts for as long as he could. Many have been inspired by him to keep doing so as well in our own lives, aspiring to the same combination of curiosity, humility (don't be 'vermetel' and build on claims whose truth you cannot construct), and care: for intellectual precision as well as for other people.

It has been hard for Wim, and for many who knew him, to accept the finiteness of his physical presence. However, his mind will transcend in the life and work of those he touched with his.

### Acknowledgments

We thank Herman Geuvers, Tonny Hurkens and Arnoud van Rooij for our conversations about Wim which substantially contributed to this piece. Tonny also provided the illustration from Wim's PhD thesis, and Arnoud provided the pictures of Wim's PhD defence.

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