Olga Lukina
Mathematical Institute
Leiden University
o.lukina@math.leidenuniv.nl

Event Workshop Lorentz Center, Leiden, 13–17 May 2024

Dynamics on zero-dimensional spaces: new connections

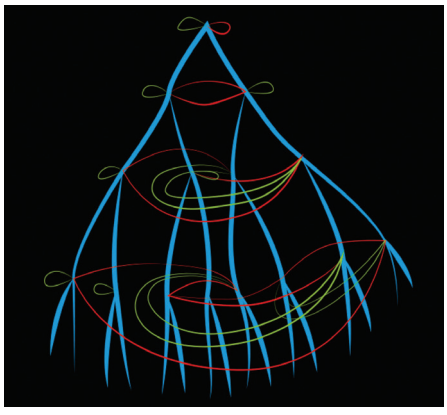
In May 2024, we gathered 55 researchers from around the world, representing four different subfields of mathematics, for a week-long workshop at the Lorentz Center in Leiden. The unifying principle was that in each subfield, group actions on Cantor sets arise as an object of study, or as a technical tool. The goal of the meeting was to facilitate the knowledge transfer between disciplines, and new collaborations. My co-organizers were María Isabel Cortez (Pontificia Universidad Católica de Chile, Chile) and Samuel Petite (Université de Picardie Jules Verne, France), and the goal of the workshop was stated in its title: *Dynamics on zero-dimensional spaces: new connections*.

To illustrate how Cantor sets arise in different disciplines, let us have a look at the picture in the workshop's poster, where we see a rooted binary tree superimposed with a collection of graphs with green and red edges.

The tree in the picture grows from top to bottom, with each vertex branching into two. The set of all infinite paths in the tree is uncountable, and is a Cantor space with a suitable topology. The tree is equipped with an action of the infinite dihedral group. The action of two generators of the group is represented in the picture by green and red edges respectively. These edges join vertices at the same level of the tree, i.e. at the same distance from the root. For a given vertex, the group elements which fix this vertex form a subgroup of finite index of the infinite dihedral group, called the stabilizer. The Schreier coset graph associated to the acting group and the stabilizer is precisely the pattern formed by the green and red edges at a finite level. Taking the limit of pointed Schreier graphs with respect to the Gromov–Hausdorff metric, one obtains infinite graphs of orbits of the action on the Cantor set and can study the dynamics of the system. This is a typical picture of a group action in topological dynamics and in geometric group theory.

In arithmetic dynamics a rooted tree has solutions of polynomial equations as vertices, and the action at each level is given by the action of the Galois group of the corresponding finite field extension. The inverse limit of the finite Galois groups is a profinite group (an arboreal representation), whose properties one is interested in. Next, collapsing vertices at a each level into a single vertex, and the edges joining them according to a certain rule, one obtains another infinite graph, called a Bratteli diagram. Using a Bratteli diagram, one defines certain types of operator algebras, and their classification is related to the classification of dynamical systems up to orbit equivalence.

One of the famous groups acting on a binary rooted tree is the Grigorchuk group. This group was the first known example of a group with intermediate (neither polynomial nor exponential) word growth, answering a question by John Milnor, whether groups with this property exist. Initially this group was defined as a group of infinite interval exchange transformations of a unit interval, but currently the most well-known representation of this group is as a group acting on a rooted tree.



Bringing together people working in different areas is often like jumping in at the deep end of the pool and trying to swim: will the participants interact, will they find each other's problems and results interesting, will the workshop be a success? In our workshop, everything went great: our wonderful audience created a friendly and vibrant atmosphere, in which ideas thrived. There were lively discussions during the lectures, continuing into the breaks. A few new interdisciplinary collaborations have started: at the intersection of topological dynamics and descriptive set theory, topological and arithmetic dynamics, operator algebras theory and topological dynamics.

We would like to express our thanks to the Lorentz Center for their help in organizing the workshop. The facilities were excellent: the lecture room was well equipped for both beamer and blackboard talks, and there were plenty of refreshments available throughout the day. The participants were given shared offices, which they could use to discuss new ideas with each other, or just check their email in peace. When we found out that one of our senior participants had a birthday and decided to celebrate by serving cake from Jacob's Patisserie in Leiden in one of the breaks, the Lorentz Center was up to the challenge: they had plates and spoons for 50 people, making this a memorable occasion.

Thanks are also due to the organizations which contributed funding to this workshop and helped making it a success: the Lorentz Center, STAR, NDNS+ and GQT mathematical clusters, Foundation Compositio Mathematica, ANR (French National Research Agency), Université de Picardie Jules Verne.