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Column Better than blackboard

How students learn

In this column ‘Better than blackboard’ Nelly Litvak writes about teaching mathematics at university. She will address problems that many university teachers face.

Recently I saw a quote from the famous technology executive and author Sheryl Sandberg: “We cannot change what we are not aware of, and once we are aware, we cannot help but change.” This is a spot-on explanation why I cannot teach the same way as I did ten years ago, and why I am so passionate about the change in education.

I’ve been reading a lot of books and articles about how people learn in general, and learn mathematics in particular. It all started with the book *Mathematical Mindsets* [1] by Jo Boaler that landed on my hands by sheer chance in 2017. When I’ve got an opportunity, I made a point to meet Boaler at Stanford, where she works, because her work meant so much to me. After that more books followed, perhaps most remarkable was *How We Learn* [3] by Stanislas Dehaene, and there were many others.

By now my self-acquired knowledge about how people learn, formed a virtual wall between me and my lectures ten years ago. I cannot stand in front of the class in full awareness how little learning is happening in students’ brains right now. I cannot unknow what I know about the learning process. And, as Sheryl Sandberg rightfully noticed, I cannot help but change.

I mentioned bits and pieces about how people learn, in my previous columns, but I believe the topic deserves a column in its own right. I don’t expect that every reader will immediately share my drive for

change. Maybe you will find some points controversial, even simply wrong. But I hope at least you will see why we cannot ignore this knowledge in how we teach.

The four factors of effective learning

All aspects of effective learning, of course, cannot be covered in one page, as I will try to do. There are tons of books about it. Yet, I will try to provide a concise summary of the main points. My short formulations might not be sufficiently nuanced, but they work for me, and, in my experience, for my colleagues as well.

From all my readings, I have parsed the following four factors that greatly influence students’ learning.

1. Engagement. People cannot learn by passively listening or watching. These modes of learning are good enough to gain information, but not to learn as in ‘master the subject’. Learning happens when people are actively involved, when they do something. This is why the innovative university Minerva [4] has a non-negotiable requirement for design of every class: 100% of the students must be actively involved at least 75% of the time. When they choose activities, their driving question is: “What are the other students doing?” And “listening” is not a good answer. Each student must be actively working on something at least 75% of the time.

2. Inconsequential error feedback. We all gladly say that “we learn from mistakes”. For learning, it is very important that students are encouraged to make mistakes. This is obvious, but this is not the whole story. Turns out, it is equally important that students feel safe to make mistakes. The students should be able to come up with a wrong solution and get feedback, with no consequences. This was the idea that struck me most in Jo Boaler’s book [1]: the errors must be celebrated because when students make errors it means they are learning! Celebrating errors is not common in our classrooms. As for feedback, I am often annoyed when an exam is called ‘a feedback moment’. Because an exam is exactly the opposite of the ‘inconse-



Engagement

quential error feedback'. In the exam, students receive a grade; the number may tell them that they are not good enough but doesn't tell how to improve. And for sure this feedback is far from inconsequential: students are punished for every mistake by points reduction. 'Punished' is not my formulation, this is what students say. This sounds like anything but a safe space for errors. And this is not a good news for learning.

3. *Spacing.* We learn best when we do it consistently in small portions at regular intervals. This is a well-known fact, but for me the most convincing argument came from [3]. It is about the working of our brain, and very roughly it goes as follows. When we learn something during the day, the new knowledge is stored in our operational memory. Then during sleep this knowledge is transferred to the long-term memory. There is only so much information that our operational memory can hold during the day, and only so much that our brain can transform to the long-term memory during the night. The information that is not held or transformed, is basically lost. And this is why it is not possible to learn a lot in a very short time. In fact, [3] suggests optimal spacing intervals. The best strategy is to learn in small portions during several weeks, and then come back to this knowledge in increasing intervals of a month, two months, six months, one year, et cetera.

4. *Sleep.* It is not under our control how much students sleep at night, but the importance of sleep for learning is huge and fascinating [3,4]. During sleep, information is transformed from short- to long-term memory, and new neuronal connections appear to tie the newly learned information to the existing knowledge. This is why sometimes you wake up with a new idea, or you can suddenly play a difficult piano passage that you practiced yesterday with no success. The process of forming new neuronal connections takes several days. So, for reliable learning, good sleep is needed every day! Maybe, as an organization, we can take this into account. For instance, we can create a safe and friendly environment so that our students don't lose their sleep to stress. And maybe we shouldn't start classes at 8:45?



Students learn from mistakes when they feel safe to make them

Illustration: Mara Chelărescu

We do all these already, don't we?

Strictly speaking, the classical 'lecture-tutorial-exam' course design doesn't contradict the four factors of effective learning that I described above. Students can stay engaged in the lectures, for instance, by writing notes and asking questions. Students can practice and make mistakes, and tutorials are there for a perfectly safe inconsequential feedback. Students can follow the course plan and learn gradually week by week. And then they can sleep all they want with piece in their heart.

They can do it. Except they don't. They massively skip the lectures, listen passively, and cannot stay focused. The attendance of tutorials takes a drastic nosedive after the first one-two weeks. It would be fine if students used this time to study independently. But they usually don't. Most of the learning happens right before the

test. And without spacing, there is no deep understanding. The knowledge is scattered and grade-driven. At the end, even if the student passed the test, we cannot count on their knowledge in follow-up courses.

I often hear from colleagues, "So what? We offer the opportunity and it's up to the students to take it." I used to share this opinion long ago, and now I don't. I try to help my students to learn effectively, and I hope I will convince you that I have a point.

Attempts to implement effective learning

In my courses, I explicitly include engagement, inconsequential error feedback, and spacing in the course design. Not always successfully, but I consciously try.

For engagement, I use flipped classroom. I ask the students to watch videos or read a book before the class, and this leaves the time in the class for interaction.

I already wrote about some of my methods, see my previous columns [5, 6, 7]. Usually I use online quizzes or let students work on problems in groups. Lately I noticed that students get tired if the same activity (say, an online quiz) is used again and again during the same class, so now I am experimenting with alternating quizzes with solving problems in small groups. In Minerva [4] they have several templates of in-class activities and they may use several different activities in one class. Ideally, I would like to do so, too.

Inconsequential error feedback is probably the most challenging part because it is infeasible to give written or face-to-face feedback individually to each student in any medium-to-large course. Usually I do this by letting students work in groups and then the teachers walk around and give feedback on the group work. Another method is to let students work on a problem and then show the solution and let them correct their own solutions together. I also use digital tests where we add, in advance, comments to the correct and wrong answers; the students can see the comments after submitting the test. Very recently we tried the system where the final grade was composed of many small weekly tests (mostly digital), and students could retake any test many times. This is not exactly ‘inconsequential’ feedback, but due to many attempts students were not afraid to fail, and this is exactly what inconsequential feedback is about.

For the spacing, most effective system that I used so far were weekly tests that contribute to the grade. Almost all students attempted the tests each week. Successfully or not, they were working on our course each week, and this is exactly what the idea of spacing is. But I also use mandatory submissions. There is no grade, but without submission the student cannot take part in the exam. I try not to impose unnecessary dead-lines, sometimes my dead-line is the exam date. And yes, some students submit everything on the last day. Yet, this is rather exception than the rule. It is quite clear to any reasonable student that it is impossible to solve eight quizzes and eight problems in two days. Usually, if there is no dead-line, the students submit their work gradually but with delay of two/three weeks.

I cannot comment about the sleep. I just hope that students don't lose sleep because of my courses.

“Don’t you pamper students too much?”

This is the comment that I hear most of the time: “Don’t you pamper your students too much?” This is usually followed by: “The students are adults, therefore they should take responsibility for their own learning.” Okey, I hear you. And as a mathematician, I suggest to look logically at this statement.

First, the assumption is that the students are adults. What does that mean? Let’s try to come up with a definition. By law, we define adult as a person who is at least 18 years old. This does imply, for instance, that the person has the right to drive, to vote, to get married, and is responsible for their action in the court of law. However, does this immediately imply that this person can study effectively?

Suppose, going by my readings, effective studies means: being engaged, looking for feedback, studying at regular intervals, and going to bed on time. This sounds like awful lot of will power, motivation and discipline. Nobody suddenly wakes up with all these qualities on their 18th birthday. These abilities have to form gradually. Being an 18+ in itself doesn’t imply that the person is able to study effectively.

Okey, let’s say that the person is adult when their body is full-grown. Then vast majority of students don’t fall under this definition. Human brain grows till age of about 25. In that sense, our students are not yet completely grown ups.

Alright, let’s say the person is adult when they are responsible, disciplined, understand the value of feedback and don’t procrastinate. If this describes your students, good for you! In fact, good for

you, if this describes your PhD students, or co-workers. By that definition, many people never grow up! Otherwise, why do we have all these time management courses for perfectly grown up professionals? Why all these books about procrastination? Why all these New Year resolutions broken by the end of January? Think about yourself, do you never procrastinate and always look for error feedback?

Expecting a perfectly disciplined class is not only unrealistic. Recently, I was taken aback by yet another thought, from yet another article: we don’t teach the students self-discipline, but do expect it in our course design, therefore, we strongly benefit those students who had better chances to learn it, for instance, at home. Not all students had such chances. We say, we give them responsibility, but de facto we introduce inequity.

When I say it, colleagues mostly agree, but they ask: “Then, is it my responsibility to teach those skills?” No, not really. I believe that ideally there should be a course, at the beginning of the study, that explains to students how human brain learns, and teaches them successful learning strategies.

However, as teachers, we can do something, too. I believe, I don’t pamper my students. Engagement, spacing, and error feedback are necessary for effective learning. Therefore, as a teacher, I believe it is my responsibility to design interactive classes, to give students specific tasks with specific timeline, and give them feedback on these tasks. It is the students’ responsibility to execute the tasks to the best of their ability, do this on time, and use feedback to improve. In my view, this is an adequate amount of responsibility.

Difference bachelor and master students?

The next comment I often get, is that such close guidance is perhaps necessary for first year students, maybe bachelor students, but master students should be able to direct their own learning. My answer to this is that engagement, inconsequential error feedback, and spacing, are necessary for learning of anything, by any human, including the master students. However, indeed, implications on course design are different because master students generally are more experienced and motivated, and we can count on their cooperation. Also, the classes are often smaller, we



Illustration: Mara Chelărescu

Spacing

might know each student by name. This makes a huge difference because in a small group every student naturally feels seen and heard.

Therefore, when we teach a specialized MSc course, we have much more freedom in choosing learning forms. For instance: a classical blackboard explanation of a proof with follow-up discussion may work for a small motivated audience; in a small class, we can involve everyone in a live conversation, we don't need an online quiz for that; we may give students more freedom in choosing assignments or formulating their own; we may even let them choose their own deadlines.

The bottom line is that the teacher must organize engagement, inconsequential error feedback, and spacing in any course, regardless the age, background and motivation of participants. The specifics of the audience are important only in how to include this in a course design.

Is math special?

This is another comment that I often get: "Math is not just the same as other skills, math is different, right?" My answer is, somewhat yes, but not really. Math is perhaps more cumulative than other subjects, next bit of knowledge builds strongly on everything that came before. Also, math is a purely intellectual skill, it generally doesn't involve any equipment.

Yet, the process of grasping a mathematical topic, connecting it to other topics, and mastering the art of mathematical argument, is the learning process that occurs in a human brain and requires the same elements: engagement, inconsequential error feedback, and spacing. In fact, I have first read about this in the book of Boaler [1] which is about teaching math.

So, with respect to effective learning, mathematics is not special. I would even say, due to its purely intellectual nature, learning mathematics is the example of learning in its purest form.

Is there an evidence that it works?

In the last twenty years there has been a boom in commercial education for adults, and all these courses are organized this way: with engagement, feedback and spacing, adjusted to the audience.

We at university shouldn't look down at commercial education. My mother runs an online school on communication, so



Sleep

Illustration: Mara Chelărescu

I know first-hand that it is a very difficult business. They must deliver effective learning, because their client pays money to learn. If the learning process is ineffective, the company simply goes bankrupt. I may only hope that universities take a note of how people teach when they must earn every cent on the quality of learning only.

Classes bleeding empty

From somewhere about 2017, attendance of my classes started noticeably reducing, especially in courses for non-math students. It was a common phenomenon, all teachers experienced that, and it became even worse after the Covid pandemic. From what I know from others and my experience, it is not uncommon to have less than 30% students at the lecture (or it's interactive equivalent), and tutorials bleed almost empty by week 4.

What pained me most was that I saw 30 students at the lecture and 150 at the exam. This made me feel completely useless as a teacher. I felt that the only thing the students wanted from me is a grade, as in "Tell me what I have to write so that you give me a six." I could be pragmatic about it, but I happened to be idealistic and believing in my added value for the students. This is why I strongly feel, this low attendance is not okay.

Many colleagues say: "It is not a problem, it is academic freedom. Every student learns differently and chooses what works for them." No and no.

No, every human without brain disorder, learns in exactly the same way: by

getting information, trying, failing, getting feedback, and trying again. Think about anything that you learned: math, playing instrument, sport, cooking... anything! I bet you didn't learn anything in any other way.

And no, students have a very limited information which class is important because they don't know the subject. I am hired as expert in the subject. As an expert and a teacher, I have a better judgement which activity is useful for students' learning. I know exactly which errors they make, which parts are difficult. Yes, the students may have individual preferences for one work form over another. But any work form is better than skipping the class.

It is simply not true that the students skip the classes because these classes are genuinely not useful. Procrastination, other priorities, personal problems, just forgetting to set the alarm, are much more likely reasons. You know it yourself. Usually the students who skip classes are the ones who need the classes most. This is not the evidence of a good judgement.

Some people say, I am fine if even one student attends. I see your point, but unfortunately this doesn't sound like an economically viable workflow. A teacher in empty classroom that sits 70 students is a massive waste of resources.

And I don't buy the argument of academic freedom. Academic freedom is about choosing your topic of study and research, not about skipping classes. I also don't see the logic, why we are so adamant and sensitive about that freedom. In high school, attendance is mandatory. At work, professionals tend to attend all their scheduled meetings. Why the university is suddenly the life-time low, when one can choose whether or not to attend their scheduled activities?

Let's be honest. We don't believe in good judgement of students which classes to attend. In such conversations I often feel that all these arguments about students being adults and their freedom are mostly excuses to resist the change.

A plea for mandatory attendance

So I have this unpopular opinion: I am for mandatory attendance. Don't get me wrong. I don't suggest to demand mandatory attendance for the courses in their current form. I do believe that for a student

in the back row, a video is a superior substitute to the live lecture.

What I suggest is this: every new course should be designed for effective learning, and there should be mandatory attendance for every class. Is the class useful? Then I don't see the reason why students should skip it. Is the class not so useful? Let's not give it. Is the class useful for some students? Let's make attendance mandatory for this particular group. Is there an adequate online substitute? Let's monitor that students actually worked on it.

Implementing mandatory attendance is easier than one may think. In our recent weekly tests in a course for 300 students, we took example from our colleagues in Computer Science at the TU/e and used attendance slips, see Figure 1. During the test, student assistants collected the slips and marked attendance in a shared Excel file. In another course for Computer Science at the TU/e, my colleagues are experimenting with registering attendance automatically with students' campus IDs. And my favourite is the Wooclap quiz software where I can ask students to login. Then a teacher may require, say, answering at least 75% of quiz questions during the entire course. This is not only mandatory attendance but also mandatory engagement.

Administration is greatly simplified if you require not 100% attendance but, say, 75%. This will prevent the avalanche of e-mails from students who are ill or have other good reasons to skip a class. If they want to skip more than allowed, you may require to contact their study advisor for exception. There will be probably a few cases like this, but not many if you are flexible enough.

[course name], attendance card

Weekly test nr:	
Name:	
Student id:	
Signature:	

Figure 1 Attendance slip. We copy five slips per page in Word and then print and cut them in the university printshop.

Once attendance is marked in Excel, you are basically done. If you are an Excel wizard, you can easily implement any requirement. If you are Excel-incapable like me, then a good student assistant will easily do the job. We were particularly lucky with a TA who not only registered attendance and grades, but also wrote a manual how to do it for the next year.

While writing this column, I saw a tweet from Robert Talbert, one of the authors of [2], about the strong correlation between attendance and results, and the promise to write more about attendance in his upcoming blog. The time is ripe, and I am going to do it.

In Q1 of 2024/25 I will start teaching a new elective course. The course has a very innovative design, mainly by my talented young colleague Mike van Santvoort. When we were filling out the Osiris form, I paused a bit before I checked the box 'mandatory attendance'. It was uncommon for me, and somewhat daring. But I do believe it is right.

Reclaiming my joy of teaching

My main motivation for staying in academia, was teaching. I loved it from day one. I loved giving lectures, and I had many compliments from the students. Until gradually I saw that no matter what I say, I was losing the contact with the students,

and turning into a grading machine. This greatly threatened my joy of teaching. And I know I am not the only one.

Now that I committed to engagement, error feedback, and spacing, I am enthusiastic again.

I cannot say that students love and support my methods one hundred percent. Some do. Some others may complain about my mandatory assignments, beg for an extra point, and ask to revert to the traditional system. But I know I am doing the right thing, even if not all experiments work out as expected. Theoretically, I know this from my readings. But I can see it, too. When I set students to work, I do often notice the spark again, I see them making errors and helping each other, I get wonderful deep questions even from non-math students. I also see enthusiastic reactions from many colleagues, especially young colleagues, and some try new methods, too.

I realize now that my joy of teaching was very much centred around my story, my enthusiasm, my ability to explain clearly. I still like to tell a story, but now my motto is 'It's not important what I say, it's important what students do'. Now I enjoy my attempts, successful or not, to put the students in the centre of attention. And I will keep trying because this is their right place. Seeing them learning effectively is the greatest reward for me.

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