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History

Medieval surveying of wetlands with a shadow quadrant

Last year Teunis Klapwijk wrote his Master Thesis *The Frisian Maasland wilderness and Lotharingian learning in the early 11th century*. This article, based on this thesis, is about the very early influence of Arabic geometric knowledge via Lorraine to the area south of Delft. From the perspective of the history of mathematics, it means a diffusion of knowledge through practice, a hundred years before translations from Arabic to Latin emerged. That is why it is also interesting as a cross-cultural phenomenon. In addition, it is a first report, that shows clearly why an astrolabe is useful for wetland surveying.

The arrival of the tangent

The history of trigonometry as written by Glen van Brummelen [4] begins with the use of the sine and the development of the sine-table in the context of astronomy. The tangent (and cotangent) appear in the study of sundials. They are used in the quantification of the length of a shadow cast by a stick, a *gnomon*, placed perpendicular to the surface of the earth and exposed to sunlight. The relevant quantity is the ratio of the length of the stick to the length of the shadow or the inverse. Van Brummelen discusses the topic further by describing the work of several scholars in the Islamic world, in particular Abū'l Wafā (AD 940–998) and al-Bīrūnī (AD 973–c.1048). Note that both lived at the turn of the millennium, a period in which Arabic-Islamic science flourished.

Shifting the attention to the developments in the Latin West, van Brummelen leaves astronomy as the main driver for trigonometry and moves towards practical geometry (p. 223). The words *Practica Geometriae* concern, in particular in the 12th century, the determination of heights and depths by focusing on the ratio of two lengths perpendicular to each other [24, 26, 29]. Intriguingly, in describing this type of measurements the use of an Islamic instrument, an astrolabe, is mentioned in the early 12th century by Hugh of St. Victor [26]. The mathematical content of the method is explained and taught using similar triangles with a common angle, which brings us also to tangents and cotangents. The earliest appearance of this type of surveying in the Latin West is in a text called *Geometria Incerti Auctoris*, which is attributed by Menso Folkerts [7, 8] to a Lotharingian author from the middle of the 11th century. This means that it coincides chronologically with a scientifically productive time in the Arabic-Islamic world.

These bits and pieces of information suggest that part of the history of trigonometry, including the cross-cultural exchange between the Arabic-Islamic world and the Latin West, may be revealed by studying practical geometry. The practice of surveying is often discussed in the context of determining areas of irregularly shaped fields, and interpreted as useful for taxation. However, surveying is also important to lay out a design to construct a cathedral and to monitor its integrity [31]. In this article I focus on the latter type of surveying, but instead of the construction of a cathedral, I focus on the construction of the geometrical pattern of the landscape in the western part of the Netherlands. These large patterns are unthinkable without the support of skilled work of surveyors. Their work on a large scale, which started in the first two decades of the 11th century, turned wetlands into agricultural soil. In my recent study [15] this landscape has been used as a source of historical information to reconstruct the specific surveying practice used in this application. It appears to be an early and influential intersection between the Arabic-Islamic world and the Latin West.

Exploiting the wetlands: the 'cope'

The estuary of the Rhine-Meuse delta was initially a vast inaccessible wetland, which was converted, from the early 11th century onwards, into new agricultural territory. The practice meant that excess water had to be removed from largely unused wetlands (peat) by interspersing them with drainage channels, which resulted in usable fertile soil. A striking element of the methodology were the long farming areas with identical width-to-length ratios, expressed in integers as, for example, $1:24$ [10]. This new approach had an advantage in providing equal economical starting conditions for several farmers at the same time. These two aspects together, meant that a large geometrical pattern had to be set out (Figure 1) in a swampy environment in preparation of the actual work of digging drainage-ditches. The undrained morass-like area is difficult to access on foot because of its non-uniform wetness with many sinkholes. Access to such a surface requires at every step the use of a stick to test the depth of the swamp before proceeding. Alternatively, one follows the natural creeks by canoe until one finds a relatively dryer patch. These conditions must have formed a major challenge for the wetland-surveyors, for which a new approach was a solution, which, as far as we know, was applied first in Western Frisia (presently called Holland). The resulting new geometrical patterns became known as 'cope', an often occurring word in the toponyms of the Holland-Utrecht lowlands [17, 18].

An example with an emphasis on the hydrological and agricultural aspects is shown in Figure 1. It is inspired by the analysis of Chris de Bont, a historian of geography [2]. The dark blue line indicates a canal which connects to a natural watercourse, which discharges ultimately into the sea. The light blue lines are ditches dug in the wetlands at a suitably chosen mutual distance d, which is the basic length-unit of the design. In the example, the ditches have an arbitrarily chosen length of $16d$ and the whole geometrical unit consists of eight farms, with at the end an additional ditch, parallel to the discharge canal. Through this extra ditch at the end one avoids that the area close to the end becomes unusable because of water emerging out of the uncultivated peat. The natural water-flow is from right to left, which should agree with the inclination of the peat bog. The peat itself is represented as a rugged, brownish landscape, representing the natural inhomogeneity of the swamp. The mutual distance d is based on the experience of the local farmer in using the soil. The purpose of the ditches is to drain the peatland to a level at which the soil is dry enough for agriculture for most of the year. Hence, there is a balance between the average rainfall and the diffusion of the excess water through the peat to the ditches. The distance is therefore determined by an empirical hydrological parameter representative for this water diffusion process. The parallel running ditches assure that over the whole length the soil experiences uniform wetness-conditions. If d is chosen too large, the farmer will encounter too wet soil conditions for more days of the year.

All blue lines were visible in the landscape and may even persist until the present day. They represent the hydrological functionality. The white dashed lines are included to assist in explaining the work of the surveyor, whose work is needed to prepare the peatland before constructing the geometrical block of eight farms. Note that the construction is quite different from constructing a building, which involves assembling and stacking material. In the case of peatland, the main action to be taken is removing material, which means digging a canal and the trenches in the right direction. But this digging has been done in such a way that it leads to a unit which is geometrically correct to fulfil the requirements of obtaining eight identical farms of the same degree of dryness and area, suggesting for this specific example a settlement which could be called in Dutch 'Achthoven' indicating eight identical farms. How does one proceed to achieve the 'sameness' [19]?

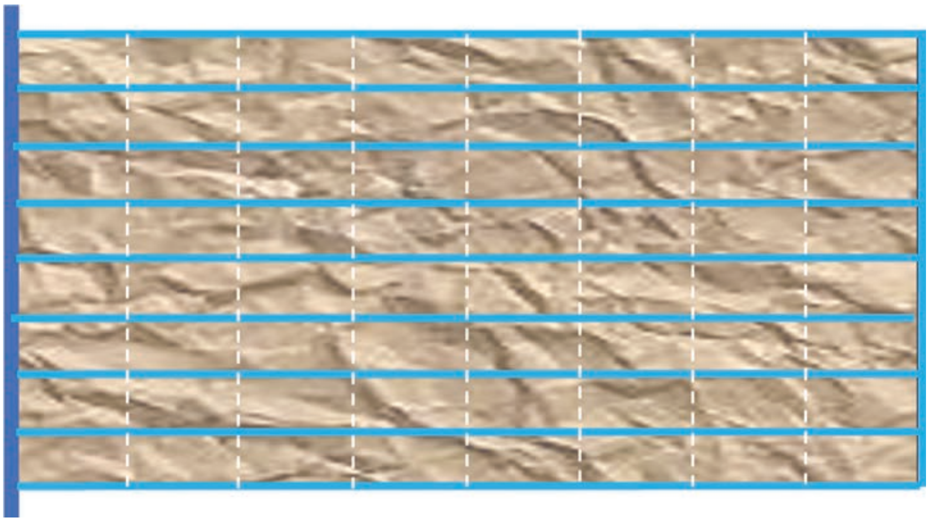

Figure 1 A sketch to represent a possible lay-out of eight farms in a peatland environment taking into account primarily hydrological and agricultural considerations [2].

Geometrical grids

In studying the early implementation of the method [15] it was inferred that the surveyors used units of a width-to-length ratio of $1:2$, which are depicted by the white dashed lines in Figure 1. In the example, one farm has a property with a length of 8 times a basic double-square unit. The ratio of $1:2$ occurs also for the dimensions of the whole block. As soon as one has taken into account the material properties, the methodology is quite generic, leaving freedom to take a larger area by making longer farms and of course making more of them in parallel. The purpose of this section is to explain how the preference for farms with a width-to-length ratio of $1:24$ is connected to an interlocked geometrical structure based on double-square units.

One of the challenges in creating a landscape with identical units is that the size exceeds the human scale. The commonly used instrument of the Romans to overcome this problem is the *groma*. It is a pole with a horizontal cross on top, made of two rigid rods perpendicular with respect to each other. At each end a plumb line is attached. The four arms of the cross are pulled downwards by identical weights, to ensure that the cross is horizontally level. By lining up visually two of these plumb lines, one direction is fixed, while a line perpendicular to the first is the line viewed along the two other plumb lines. The length along these two orthogonal directions is measured linearly by using some reference length, often called a *decempeda*, a length of ten feet.

In covering a large area one creates a large grid of squares, which requires a rigid network of squares. Starting with one square one will find that in a first attempt one ends up with a square which deviates from the desired properties of an ideal square. The square is most likely slightly distorted, although this is not easily notable by the human eye in the field. A common solution [19] is to check the length of the diagonal. We know that it should be equal to $\sqrt{2}$, which is an inconvenient irrational number. However, fortunately it was known, also in the medieval Latin world, that it is quite accurately given by a ratio of two whole numbers: $\sqrt{2} \approx 17/12$. Given this insight it is convenient to construct a square grid of units with sides equal to 120 feet, 12 *decempeda*, and check, when expanding the grid, continuously the diagonal is 170 feet. Through this measuring and checking one expands the grid, giving a regular grid divided into equal square cells. When completed it may consist of a large rectangular unit, as if it has been laid down from a bird's eye perspective. All these measurements are linear measurements in units of a locally accepted measuring standard.

Although such a *centuration* is frequently encountered in Roman settlements, this is quite different from the medieval settlements found in the western part of the Netherlands. The difference is significant because the knowledge of the Roman *agrimensores* was taught routinely in the medieval abbeys and cathedral schools, because of the importance of agriculture for the economy of these institutions [8, 19]. The geometrical structure of long agricultural units separated from each other by carefully aligned ditches, running in parallel, is abundantly present in cultivated peatlands. Their interpretation based on hydrological arguments, as summarised in Figure 1, is quite generally accepted. However, these standard large scale geometrical structures are difficult to construct in wetlands by following the practice taught by the Romans. The wetness of the soil makes the process of walking around with a measuring rod, the *groma*, and checking frequently for the diagonal very cumbersome. An innovation is needed not only for the hydrological functionality but also for the surveying methodology.

In textual sources a number of extra pieces of information are available, which refer to or specify the dimensions of the geometrical structures. None of them appear to be the result from hydrological or agricultural requirements. They have been collected together in Figure 2 and I will argue that this gives information about the applied surveying methodology. Prior to discussing a number of details the main conclusion is that the surveyors used in practice a basic unit with a width-to-length ratio of $1:2$ (Figure 2), with a length which appears in texts with the word *vore lanc* or *voorling*. A particular striking piece of information is the Bremer contract from 1106, which speaks about the creation of hundreds of farms with a length of 720 rods and a width d of 30 rods, separated from each other by ditches [10, 17, 18]. This contract with *Hollandi* is clearly based on an established concept, traces of which are visible in the landscape northeast of Leiden and south of Delft. By combining it with information from textual sources [17, 18] this results in fields with a length of 60 rods stitched together in one direction to a maximum of 12, making a total length of 720 rods. This suggests that the original concept can be called *twaalfhoven*, meaning a total unit of 12 farms consisting of 12 units in parallel with a ratio of width-to-length ratio of $1:24$, each constructed from 12 units of a width-to-length ratio of $1:2$ (cf. Figure 2). This means that to construct a rigid geometrical grid the square of the Romans is replaced by a rectangle of two squares. As I will argue these specifics are compatible with the proposed surveying methodology.

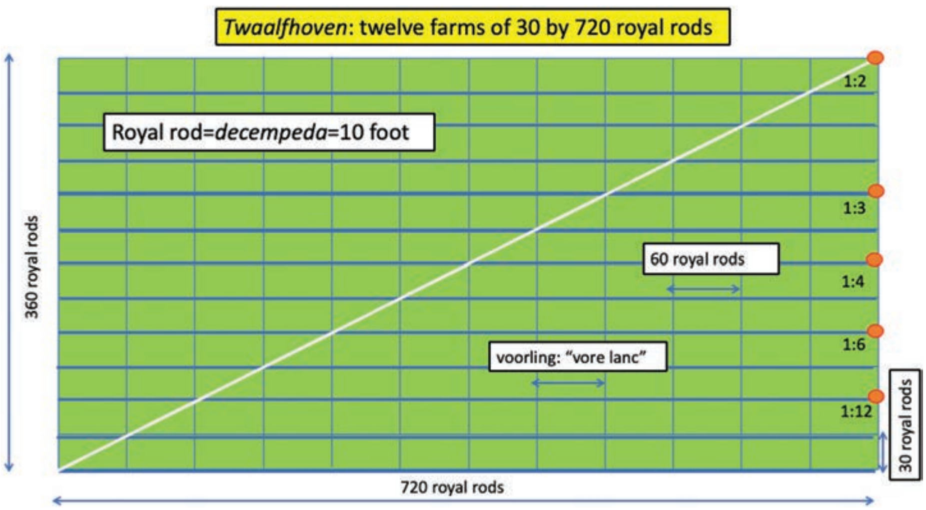

Figure 2 A sketch of the inferred lay-out of the initial approach to wetland-planning using the basic principles of the Roman *agrimensores*, but modified by the use of the optical method for angles with a fixed length to width ratio. Note that the horizontal separations between the parcels are drainage-channels. The vertical lines marks the end of each 'voorling', the length unit used in the textual material. The diagonal (white arrow) represents an angle of $\arctan(0.5) \approx 26.6°$, consistent with a length of the diagonal equal to $\sqrt{5}$ times the width.

A construction with basic units with a width-to-length ratio of $1:2$ means that it poses even more challenges to maintain the rigidity of the grid by repeatedly measuring the diagonal. In this case it should have, instead of a length of $d\sqrt{2}$, a length of $d\sqrt{5}$ rods. For $\sqrt{5}$ an approximation of comparable accuracy is $\approx 161/72$. Since this is certainly not a simplification in comparison to using squares, another reason must have facilitated this change. Fortunately, an indication is provided by the emergence in the 11th century of the subject called *Practica Geometriae* in the Latin world, which is based on the *quantitative* use of *visual* information [4, 24, 26, 29].

Early example near Delft

The specific lay-out with the emphasis on the numbers and ratios, shown in Figure 2, appears to be related to the earliest examples of systematic developments of wetlands, which can be partially inferred from textual material and accurate maps. The argument leading to the construction of a grid based on units with a width-to-length ratio of $1:2$ is based on the arrival of the subject of *Practica Geometriae*, which emphasises the use of a visual method advertised for the determination of heights of towers and depths of wells. However, *Practica Geometriae* was not taught in the Latin world in the context of surveying, although such a lateral application appears in texts from the Arabic-Islamic world. To obtain further clarity about the surveying methodology used in 11th century Holland. An indication is provided in a text called *De diversitate temporum* written between 1021–1024 by Alpertus of Metz. He reports that count Dirk III and "his men divide the land in a way that each one of them obtains a lot for agriculture" [21]. Our interest is in additional evidence for the early use of visual information. Fortunately, such evidence is apparent in the area around the city of Delft [15].

The beginnings of the systematic cultivation-history of the estuary in the western part of the Netherlands can be identified in Rijnland and Delfland [17, 18]. Both names are used, even today, to indicate the watermanagement bodies. The first one refers to the borders of the Old Rhine, which passes through Leiden. The area shows archaeological continuity dating back to the

Figure 3 Selection of the map of Kruikius of the area Delfland, just south of the city of Delft (1712). In the red circle the name St. Maertensrecht refers to a property of St. Maerten's church in Utrecht. The two yellow lines indicate edges of two reclamation-blocks, between them a piece called *Ruyven* indicating unreclaimed peatland 'ruvene'.

occupation by the Romans with the Rhine as the main artery for transport and also as a boundary marker for the Roman empire. In the following centuries the waterflow through the Rhine diminished considerably because of the formation of the *Lek* and the *Hollandse IJssel* who both cut across the vast peatland discharging the excess water in the Maas-Merwede system. Around the turn of the millennium this Maas-Merwede system had become the main traffic route from Northwestern Europe towards the North sea. It had raised the importance of the area called Delfland. In its name, meaning 'delved land', it already refers to human action. The name itself is currently used for its watermanagement body, a reminder of the fact that a large part is below sea-level and needs continuous pumping to prevent flooding. These artificial living conditions started with the creation of *'de Delf'* a more or less linear watercourse constructed in the 11th century [21] with a length of about one kilometer. From a mathematical point of view, its linearity is a pretty strong statement, which deserves further scrutiny. In addition, the statement speaks about a finite length expressed in modern units of measurements. The significance, or insignificance, of both must be discussed in the context of the 11th century.

We are fortunate that the administration of Delfland has ordered around 1712 an accurate documentation of the landscape in the form of very high quality maps of the area [30]. They are based on data measured by triangulation and printed by the same professionals, named Kruikius, whose interest was in getting the maps as accurate as possible within the possibilities of the 18th century (Figure 3). Their accuracy can still be verified in present-day landscape structures. Nevertheless, it might come across as a leap of faith to assume that the landscape, that was accurately registered in 1712, contains also direct information from medieval times. In justifying such a claim, which I make, one needs to take into account that the watercourses of Delfland are constructed in the wet material called peat. The occupation of the swamps meant the creation of canals, both to get access to parts with more dry peat, and to obtain a surface suitable for agriculture (Figure 1). It is now well known that over the ages the surface erodes, which leads to its gradual lowering [2]. Fortunately, this vertically oriented erosion only rarely leads to lateral changes. So the direction of the early watercourses in the peatland may be preserved even if the surface, over the ages, got several meters lower. This means that, unless significant interaction with the sea closer to the coast provides disruptive forces, the watercourses of 1712 retain the geometrical structure chosen at their construction.

Figure 3 shows the area south of the city of Delft overlaid by a few differently coloured lines. The two yellow ones illustrate the extension of the lower side of the rectangle in which Hof van Delft is written, and the upper side of the part just below the name Ruyven, next to another rectangle called Abtsregt. The name Ruyven is a name derived from 'ruvene', meaning 'uncultivated peat', which signifies that it was between two cultivated blocks. The two yellow lines come together at a town called Schipluyde (present-day Schipluiden). This village is located at a meandering creek known as the Gaech (Gaag). The two extrapolated yellow lines signal the presence of one object which has been used as a visual beacon in the landscape to create the sides of the two structures south of Delft. This beacon could have been some dwelling at the western side of the Gaech. However, another possibility is that the line points to a farmstead on the eastern side of the Gaech, because it is connected to a long stretch of land extending eastwards, which is labeled as *St. Maertensrecht* (red circle). This name indicates that it was owned by the bishopric of Utrecht, in particular St. Martin's Cathedral. It is common in case of remote ownership also to own a farmhouse, usually called an *Uithof*. A few centuries later, a castle was erected at this location known as *Keenenburg*.

The two yellow lines clearly indicate the common use of visual information for the construction of straight lines in the landscape with a length of about two kilometres and a beacon at a distance of at least another three kilometres. In addition, we have highlighted a number of parallel lines as well as perpendicular lines (blue and white lines), which are associated with the northern block. The blue line indicates the part of the original and present *Delf*, whereas the white lines are boundaries of a block in the landscape. A search of possible locations that could have been used as a beacon to construct the blue and white straight segments turned out to be unsuccessful. This led to the conjecture, that the collection of four lines were part of a rectangular geometrical structure. The conjecture was also inspired by information about areas, which were developed later, where the 'cope' had become a dominant feature. The upper right hand corner of the rectangle is where currently the settlement *Delfgauw* is located. The road from there to the south is presently known as the *Zuideindseweg*. Note that this road, despite of a distance of 2.4 km, runs parallel to the canal the *Delf*. Another significant detail is that just above the upper left-hand corner an important archaeological site is present, known as *Koningsveld* [28]. At this location the Abbey of Koningsveld was created in 1246 by Ricardis, aunt of count Willem II of Holland, who was also emperor of the Holy Roman Empire. This property was donated to her while referring to it as the former *curtis* of the family of the count [20], which was since then relocated to the site of the city of Delft.

The conclusion is that these remnants, visible on a map of 1712, and also today, refer to the construction of a rectangle, as shown in Figure 2, in an area dominated by peat, with the fingerprints of the counts of Holland as the actors. The archaeological information collected at the site of the Abbey of Koningsveld and the supplementary textual information points towards an occupation in the early 11th century [28]. The dimensions of the rectangle are of the order of 1.2 km wide and 2.4 km long, which suggests that a large rectangular structure was created with outer dimensions, in agreement with the sketch of Figure 2, having a width-to-length ratio of $1:2$. Finally, it confirms that the dimensions were intended to provide identical areas for each farmer and were obtained with a surveying methodology designed to cover a large area based on a rigid grid-structure. Its orientation, which should support the desired waterflow, should be implemented with the needed geometrical rigidity for which a clearly visible object in Schipluiden was used. (The distance between the Zuideindseweg in Delfgauw and the Delftse Schie is according to Google Maps 2.32 km, which assuming a distance of 7200 foot implies that the surveyors have used as a standard a foot of about 32 cm. The empirically relevant distance between the original drainage trenches must have been about 96 m.)

The shadow quadrant

The early 11th century wetland lay-out of Holland, as suggested in Figure 2, is more complicated than the one transmitted from the Roman *agrimensores* but would be compatible with the principles of the *Practica Geometriae*. However, the *Practica Geometriae* appears in Latin texts, which are from a later date and do not refer to the surveying as described here. But in these texts, which describe the measurements of heights of towers and depths of wells an instrument, called an astrolabe [3, 14, 22], is used. Therefore it is worth to study how an astrolabe might have been used to create the pattern shown in Figure 2.

In turning the attention to an astrolabe, in the spirit of the *Practica Geometriae*, one must focus on the backside of the astrolabe as depicted in Figure 4. It shows a drawing from a medieval manuscript of the *Bibliotheca Vaticana* and a more detailed one from the historian of astronomy John North [22]. The latter can be used to explain the use for surveying. The crucial ingredients are the rotatable *alidade* and the *shadow quadrant*. The markings on the periphery of the circular plate are ignored, because they are not relevant for the present discussion but rather for astronomy [23]. The alidade rotates around a sharply defined point in the center. The hands of the alidade are constructed in such a way that one side lines up exactly with the center of rotation. At the end of these hands a visor is mounted which allows for the selection of a line of sight which goes through the center of rotation and along the two aligned sides of the hands. The red arrow has been added to identify one particular direction for a ray of light. It can be approached by rotating the alidade until the point is reached where the sides of the hands are in the direction of the red arrow. The second item is the shadow quadrant of which in the left drawing, in a medieval manuscript, only one is sketched whereas in the more accurate modern drawing, two are present. Each of the sides of the quadrants is divided into 12 units. They are called shadow quadrants because they can be used to determine the hour of the day, but they are also used for measuring in *Practica Geometriae*. For example, the direction of the red arrow cuts the vertical part of the quadrant at 6 units, whereas the distance from the central point to this vertical is 12 units. The direction of the red arrow is at an angle for which the ratio between the opposite side and the adjacent side, in other words the tangent, is $6:12$ or $1:2$. This is true for any similar triangle, which makes it possible to scale a length at a distance to a smaller length nearby. Taking these divisions one notices that one has available a number of angles with suitable ratios: $1:2$, $1:3$, $1:4$, $1:6$, $1:12$ and even $1:24$. Rather than using the full range of values of the tangent one has a few selected angles available with convenient ratios of whole numbers.

With this information it is easy to understand the lay-out shown in Figure 2. For practical reasons one has chosen to construct with a grid for which the width-to-length ratio is $1:2$. By using the shadow quadrant the verification for $1:2$ is, in comparison to $1:1$, reduced to choosing simply another angle for the alidade. The length of the $1:2$ diagonal is about 190 meter, which allows for both voice- and vision-communication in the field. Using such a unit one can stitch together the full pattern, with various ways to check the rigidity. For example, in Figure 2 with the red dots at the right hand side the various points are indicated corresponding to the different angles and ratios mentioned above. Through these one can check and ensure the rigidity of the whole block and also the rigidity of the various parallel running lines, as

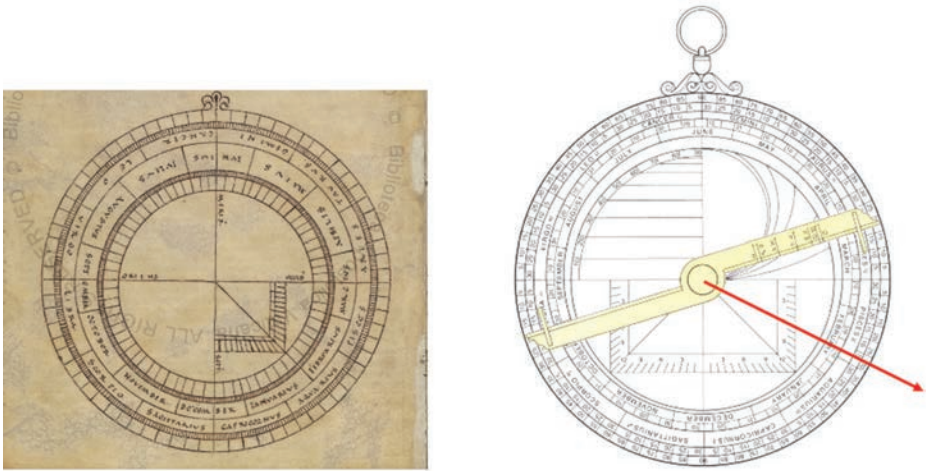

Figure 4 Left: Drawing of the backside of an astrolabe showing a shadow-quadrant in a manuscript dating from before 1054, written by Hermann of Reichenau (*Bibiliotheca Vaticana*, lat. 598 f. 119'). Roght: Accurate sketch of the backside of an astrolabe including the alidade. The red arrow indicates a possible direction of the line of sight [20] representing an angle for which the horizontal length is twice as long as the vertical length.

well as of the boundaries. The initial orientation of the block is to be based on the inclination of the local landscape to use the natural direction of the waterflow. The beacon at Schipluiden is most important at the initial stage when the first unit is being constructed. It can also be used for a check when several units are placed in a row. After the surveyor has laid out the desired pattern the actual construction consists of the removal of peat along a number of straight lines, i.e. just digging the trenches. This sheds light on the fact that in text about the early settlements of Esselijkerwoude and Rijnsaterwoude northeast of Leiden, the size of the farms is expressed with a length of 12 'voerlinc lanc' later becoming the word 'voorling'. It is a measure for a length of the basic unit, used to construct the whole block. The practice of the construction means digging a 'voor' in the peat of that length. The specific length in the medieval text is a remnant of the practice of laying out a grid in wetland by using the shadow quadrant. Similarly, the requirements of the size of the farms, expressed as $1:24$ through 30 rods by 720 rods, mentioned in the Bremer contract of 1106, follows from this practice.

Lotharingia

The identified wetland-technology contains several specific geometrical elements, which tie the early history of the western part of the Netherlands, through the astrolabe, unexpectedly strong to the Arabic-Islamic knowledge. Also strikingly, the local ruler Dirk III became in 1018 the subject of a raid at the instigation of Emperor Heinrich II by a coalition of bishop Adelbold II of Utrecht, bishop Balderik of Liège and count Godfried of Verdun (nicknamed 'the prisoner'). The raid failed, but the composition of the coalition suggests that the local ruler Dirk III was taken seriously in his actions in the wetlands. He was obviously expanding his territory in a way which drew widespread attention and invoked a perception of power. Given the early stage of the development of the area it is unlikely that military or economic power was the main source of concern, although control of the traffic by merchants to the North Sea did play a role. The present analysis of the actions of count Dirk III indicates, at least in retrospect, that his power might have been, perhaps partially, the effective use made of the newly arrived Arabic-Islamic knowledge. The effectiveness in supporting the large-scale expansion of territory into peatland may have been a striking message. How does Dirk III fit into the *upper strata* of the educated community of North Western Europe?

In the 10th century a leading intellectual in Lotharingia was Gerbert of Aurillac, who would later become pope Sylvester II (999–1003). Gerbert was born around 946 in Auvergne and died in Rome on May 10th, 1003. He studied in Spain, from where he moved to Rome. Here he attracted the attention of the German emperor Otto the Great, who chose him to educate his son Otto II. A few years later Gerbert was appointed to lead the cathedral school of Reims, a position through which he became an educator of quite a few intellectuals and sons of the ruling elite. In his teaching he clearly deviated from the standard program of the medieval *quadrivium* in particular with respect to astronomical and geometrical topics. For example, in 984 he writes to a certain Lupitus of Barcelona to request a copy of his Arabic-to-Latin translation of a book on astronomy [21]. A large fraction of Gerbert's correspondence is extant, including an answer to a question raised to him by the scholar Adelbold II, who would become bishop of Utrecht in 1010 and who played a key role in the events of 1018. Adelbold, born in 975, was asking about how to determine correctly the value of the area of an equilateral triangle with height h and base a (see [9] pp. 48-49), for which Gerbert provides the correct route to the answer. They clearly know each other well and Gerbert respects Adelbold's qualities as a scholar. It meant that in 1018 archbishop Adelbold II was well-prepared to appreciate the new knowledge, which was apparently used effectively in Frisia. A few years after this collision of forces, in 1025, two scholars Ragimbold of Cologne and Radolf of Liège exchange letters discussing the difference between acute, right and obtuse angles [12, 27]. This discussion is very well known in the history of science, most often quoted as an indication of the low level of mathematical knowledge in the Middle Ages. In one of these letters Radolf also refers to an astrolabe in his possession, which he likes to copy, but offers to show it to Ragimbold [3]. Details are missing but the context of their exchange is geometry, which suggests that the use in surveying may have been part of the conversation, rather than astronomy. The use of an astrolabe in astronomy becomes prominent in the texts attributed to a scholar in the Abbey of Reichenau, Hermann the Lame, born in 1013 and deceased in 1054. He is credited for having written the first understandable text in Latin about the construction and function of an astrolabe for astronomical purposes [13] in the 3rd or the 4th decade of the 11th century. This sequence of events suggests that the astrolabe may have arrived in Lotharingia first and put to use in surveying of the wetlands as testified by the landscape. The subsequent appearance in the exchanged letters and in explanatory texts is most likely a testimony of a learning process of scholars in response to proven practice.

Given these traces of increasing interest and awareness of the Arabic-Islamic heritage the biographical details about the upbringing of the Frisian count Dirk III are important. His grandfather died in 988 while having received from king Otto III in 985 the rights to own and develop the land between the Liora (currently De Lier) and the Hollandse IJssel, the area which was to become Delfland. In this arrangement in which the widow of Otto II, the empress Theophanu (of Greek descent), played a key role, diplomacy of Gerbert and archbishop Egbert of Trier was crucial. In the negotiations count Siegfried of Luxemburg worked together with Egbert on behalf of the diocese of Trier. Notably, archbishop Egbert was the second son of Dirk II, while the oldest son Arnulf, father of Dirk III, was married to Liutgarde, the eldest daughter of Siegfried of Luxemburg. After the death of Dirk II in 988 the rightful heir was Arnulf, which meant that the Luxemburg family as well as the Frisian family had through their family-ties a strong interest in the whereabouts of Frisia. Unfortunately, in an attempt to implement his rule Arnulf was killed in battle in 993, which leaves Dirk III, still a minor, as the heir to the property in Holland (at the time still called Frisia). It means that the assigned ownership of the land between Lioria and the Hollandse IJssel was resting on the shoulders of a minor, Dirk III, under custody of his mother Liutgard of Luxemburg. In comparison to possible alternatives the most likely place for the educational upbringing of Dirk III in 993 was the Abbey of Echternach in present-day Luxemburg [15]. The cathedral school of Reims was no longer in the secure hands of Gerbert and Liège had not developed yet into prominence as the Athens of the north. In contrast Siegfried of Luxemburg had been consistently building a well functioning school and library at the Abbey of Echternach, in collaboration with the archbishop of Trier, Egbert.

Is there any sign of Arabic-Islamic influence on the intellectual conditions of the Abbey of Echternach from 993 onwards? Echternach's abbey is part of the diocese of Trier, which includes also the Abbey of Mettlach, which had close ties with the school in Reims. In 993, the year in which Arnulf died, a remarkable transfer occurred at the initiative of archbishop Egbert of Trier by moving the abbott Lioffin from Mettlach to Echternach. Lioffin had been active as builder of a chapel in Mettlach, with remnants still visible today. Upon Lioffin's entry from Mettlach into Echternach several books entered the library. One of those books was recently found to be bound in a piece of parchment, which had served as an abacus-table as used by Gerbert of Aurillac. This abacus-table contained the Arabic numerals as recently described by Charles Burnett [5]. If indeed the minor Dirk III was prepared for his future role in Frisia through an education in Echternach he must have been aware of the practical significance of the Arabic-Islamic science. Meanwhile Echternach's active interest in the settlements in Holland and Friesland in the early 11th century is apparent from a list of names of settlements discovered in another manuscript from Echternach and discussed by Blok [1, 6, 16, 25]. Given the analysis of the landscape of Frisia, which can be dated in the early 11th century, and is to be attributed count Dirk III, it is reasonable to conclude that a shadow-quadrant and alidade as present on an astrolabe has been used. The medieval surveying of the wetlands of Frisia provides, an extra indication of the physical presence of the Arabic-Islamic instrument called the astrolabe, prior to 1018. This observation is consistent with its availability in Liège in 1025 [3, 12, 27]. Although the latter textual information is often quoted to illustrate the low level of mathematical knowledge in the Middle Ages [12], it is worth noting that the application of the shadow quadrant and alidade is a mathematically correct way to use the linear propagation of light to lay-out a large scale area. It is an early demonstration of the power of reliable knowledge.

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