Chris Peters
Leiden University, and
Eindhoven University of Technology
c.a.m.peters@tue.nl

In Memoriam Jacob ('Jaap') Murre (1929–2023)

A tribute to an expert in algebraic geometry

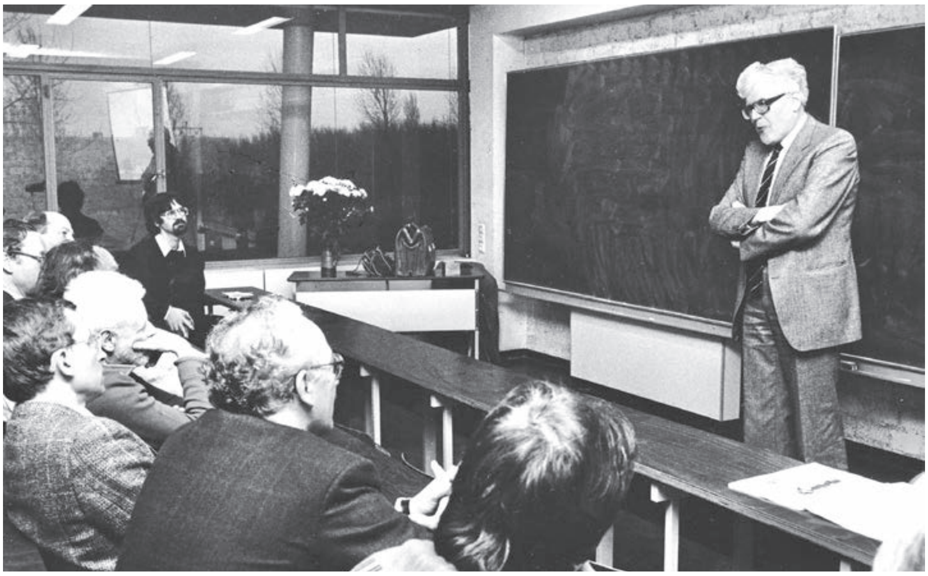
Jacob Murre, emeritus professor algebraic geometry of Leiden Univeristy passed away on 9 April 2023. His former colleague Chris Peters, who worked with Murre for a long time, looks back on his live and work.

With the passing away of Jaap Murre I lost one of the two mathematicians with whom I had the longest contact of my life, the second person being my thesis advisor Ton van de Ven who died nine years ago.

It all began in 1967 when, as a second year student I took Murre's advanced algebra course, and van de Ven's topology course. For the preparation of the master's degree only a few non-standard specialized courses were offered the next year, one of which was Murre's introduction to classical algebraic geometry, primarily centered around the Riemann–Roch problem for curves, and another was van de Ven's introduction to K-theory, based in part on Atiyah's lectures. These two teachers that knew each other from their student's time, had opposite personalities, but they generally got along remarkable well. Murre characterized van de Ven at the occasion of his commemoration at the KNAW (Royal Dutch Academy of Sciences) as follows: "His friends and colleagues knew Ton as a gourmand, he not only was knowledgable about surfaces but also of good food and famous wines." Ton came from a well-to-do family, his parents spoke French at home, while Jaap came from a modest middle class family: his father owned a potato farm in the province of Zeeland. A colleague characterized him as the "gentle giant with a full mop of hair and soft of speech, genuinely kind and considerate, with a certain mannerism of modesty". There were also similarities: both were excellent and inspiring teachers, with equally high demands and hence somewhat intimidating for us, students.

In the course of my twenty or so years at the Leiden Mathematics Institute I was in close contact with both, at first mostly with van de Ven, especially during the long period we worked with Wolf Barth, and later with Klaus Hulek on the book on complex surfaces [5,6]. During these years Jaap and I had several common PhD students. I got to know him better and we became friends. After I left Leiden University to accept a position at the Grenoble University we kept contact as local responsibles for three consecutive European algebraic geometry projects.

I have fond memories of two summer schools that took place in the new millennium. On the first, in 2001 in Grenoble, a three-week event I organized together with Stefan Müller-Stach, Jaap delivered a highly appreciated series of lectures on cycles and motives. He enjoyed the atmosphere even in the sometimes stiflingly hot class room. The mountains are close by in Grenoble, and so during the hottest week my wife Annie and I took him to a walk in the cooler scenic mountains north of the city. Jaap had some problems walking and I had to run back to fetch my car so that we all could be in time to enjoy the appointed lunch in a nearby restaurant. On the one in 2002 in Guanajuato, Mexico, a weekend trip to



Jacob Murre at the Mathematical Institute Leiden in 1986 on the occasion of his 25th anniversary as a professor the historical town Zacatecas will always remain in my memory: during the long ride in the rain Jaap told us detailed amusing stories about several legendary mathematical celebrities he knew personally. Some of these stories are going to appear below.

Jan Nagel, a joint PhD student of Jaap and me, having a job in France, like me spent the vacations and most of the summer with his family in the Netherlands. We thus could plan to visit Jaap together so that we saw him at least twice a year to talk about mathematics and our mathematical friends. The first decade we met at the Leiden Mathematics Institute, but in recent years, when his health gradually but steadily declined, we drove to his home, an idyllic house in the village of Wassenaar. We had arranged to visit him last April, but alas, we got a message that a few days before the planned date, he had died after a short illness. In our memories he'll live on, as I hope to testify through this tribute.

Early years (1929–1961)

Jacob Murre was born in 1929 in Baarland [1] as the son of a farmer. As for many people of his generation, the Second World War seriously affected the path of his secondary education. His school in Goes, a small town near Baarland, had to close near the end of the war. At the end of October 1944 the allied forces landed, an amphibious tank transported the schoolchildren across the Schelde waters to the already liberated Zeeuws-Vlaanderen [2], and they only returned towards the end of 1945. This intermission postponed their final exams till 1947.

Having obtained his diploma Jacob wanted to study mathematics. Instead, he complied with his father's wish who thought that being an engineer offered better prospects. So he started studying physical engineering at the Technical University Delft which at least offered some courses in mathematics. But after one year, inspired by the lectures of one of his teachers, the analyst Cees Visser [3], Jacob moved to Leiden University to pursue his true interest, mathematics. After obtaining his bachelor degree, he attended the lectures of Hendrik Kloosterman (1900–1968), which Jacob admired for their clarity and precision. No wonder he started his doctoral studies under Kloosterman's supervision after obtaining the degree of 'doctorandus' [4]. Jacob subsequently became Kloosterman's and J. Droste's [5] assistant.

Hendrik Kloosterman

Although Kloosterman's research was mainly in analytic number theory, he got interested in algebraic geometry because André Weil, using techniques from that field, could make the best estimates on exponential sums, a particular interest of Kloosterman. He advised Jacob to read *Einführung in die algebraische Geometrie* [31], a then standard 'modern' introduction by the famous Dutch mathematician Bartel Leendert van der Waerden (1903–1996). However, the 'bible' at that time was *Foundations of Algebraic Geometry* [32] by André Weil (1906–1998) written in 1946 and notorious for its extreme difficulty. Jacob, together with Tonny Springer (1926–2011), at that time assistant in Leiden, worked their way through the book. Jacob told me: "It was a joy to study Weil's book together with Springer. With his quick mind he often understood difficult passages more quickly then I did." As a consequence of their work, Jacob stumbled upon an open problem in Weil's book that seemed interesting enough to take up as a thesis topic.

Under Marshall Aid, scholarships were available to study in the United States, and Jacob was able to take advantage of such a scholarship to pursue his dissertation under the direction of A. Weil. In advance, Kloosterman had contacted Weil to obtain his permission. Together with Elly, whom Jacob had just married, he took the boat from Rotterdam to New York and then the train to Chicago. During his stay in Chicago, their first and only son, Jan, was born.

At that time Chicago was a center of algebraic geometry: Weil gave his famous lectures on abelian varieties attended by Armand Borel (1923–2003), Serge Lang (1927–2005), Teruhisa Matsusaka (1926–2006), Peter Swinnerton-Dyer (1927–2007), and a couple of Weil's students among whom Paul Cohen (1934–2007) [6] who later was awarded the Fields medal. Among Weil's circle were also Jean Dieudonné (1906–1992) [7] who had left France in 1952, and the complex geometer Shing-Chen Chern (1911–2004) who held a chair at the University of Chicago from 1949 on. In the course of his stay in Chicago, Jacob solved Weil's problem. He told me: "Weil asked me to explain my proof. The first time he did not believe it was right. A second time, after Lang's suggestion to adapt the notation, Weil still did not believe it, but the third time he was convinced."

Back in Leiden the thesis defense took place in May 1957. It led to the publications [54,55,56]. In 1959 Jacob was appointed as 'lector' [8]. The following academic year, 1959/60, his daughter Tineke had just been born, he accepted an invitation from Matsusaka, and with his wife and two young children took the boat to New York to travel to Evanston, a town not far from Chicago. There he worked on the behaviour of Picard varieties in families and on abelian varieties, ameliorating a result of Weil (cf. [59]).

His first student was Frans Oort (born 1935), who obtained his doctorate in 1961 on *The Riemann–Roch Problem and the Construction of the Picard Scheme for One-dimenional Schemes*} (suggested by A. Andreotti (1924–1980) during Oort's stay in Pisa). At that time only a full professor could award a doctorate and since Jaap was not appointed professor until 1961, Wil van Est (1921–2002) acted as Oort's formal supervisor.

After his promotion to full professor Jacob undertook various visits to institutes around the world that inspired and influenced his mathematical research. Without

André Weil

doubt the visits that influenced him most were those to Paris, where he worked with Alexander Grothendieck (1928–2014), about which I'll report below. His stay at the Tata Institute in Bombay from December 1964 to February 1965, accepting an invitation by C.S. Seshadri (1932–2020), was not only mathematically of interest but certainly also an exotic adventure: he talked to me about the famous Taj Mahal hotel where he stayed in an annex, his visits to Delhi, Agra, and the Corbett national park with the aim to see tigers. Unfortunately this did not happen, but he did see many elephants.

Grothendieck lecturing on SGA at the IHES

Influenced by Grothendieck (1958–1968)

Jacob has been deeply impressed and influenced by Grothendieck. Their first contact dates back to his stay in Chicago but then Grothendieck still worked and lectured on functional analysis. He broke through in algebraic geometry with his famous talk at the International Congress of Mathematics of 1958 in Edinburgh which Jacob attended. They briefly spoke with each other, and in the same year Grothendieck sent him a letter asking whether the results of his thesis could be transposed to the setting of *schemes*. Being unfamiliar with that theory Jacob could not provide an answer. In fact, having invested a lot of energy learning Weil's theory he was not sure to start all over again to learn schemes. But a little later, in the spring of 1960, Weil gave him a clear and pointed advice during a walk in the woods surrounding the Institute of Advanced Study in Princeton. "Grothendieck is doing things nobody else can, you are young and should quickly delve into these matters." This unexpected advice — it was well known that the relationship between Weil and Grothendieck was far from optimal — convinced him.

Jacob once told me of a conversation with Grothendieck in 1961 that very much impressed him. It took place in N. Kuiper's [9] bungalow in the woods near the Agricultural University of Wageningen where Grothendieck just had given a talk. During a long conversation with Jacob about the pathological behaviour of the Picard varieties in positive characteristics recently discovered by Jun-Ichi Igusa (1924–2013), Grothendieck remarked: "I have not yet accurately studied this point, but it will be explained in chapter 12 of EGA (*Élements de Géométrie Algébrique*). Other people [10] could not explain this behaviour because their assumptions are too strict. I'll assume less and prove more." Indeed, one year later, during a Bourbaki talk in Paris on the Picard functor he explained everything with schemes. Murre attended this talk, since Grothendieck had invited him for the spring of 1962. During this stay Grothendieck succeeded in convincing him of the need to use schemes. His main argument was that nilpotents are nature-given, they should be used; neglecting them is a mistake and leads to oversights and seemingly pathological phenomena. This was clearly true as shown by his work on the Picard functor. Algebraic groups in characteristic p also lead naturally to nilpotents: there are homomorphisms that are injective on the level of groups, but that are not necessarily isomorphisms onto their image; the reason is that, considered as homomorphism of schemes, they have a kernel. Another illustration is his approach to the algebraic fundamental group of curves in characteristic p: using nilpotents one can lift curves to characteristic 0 where one can employ the complex topology, a method that could not be used in the classical framework.

Jean Dieudonné once told Jacob: "Grothendieck always departed from a concrete problem and searched the most natural framework in which to place it so that the solution would present itself naturally." A fantastic example of this is his relative Riemann–Roch theorem. Also SGA (*Séminaire de Géométrie Algébrique*) 1 and 2 [2,3,4] are rather concrete, but the EGA are far more abstract and written in great generality as there are certainly situations where one can make use of this. Indeed, as an example, the theory of fibered categories as further developed by Michael ('Mike') Artin (born 1934), Pierre Deligne (born 1944) and Jean Giraud (1936–2007), is indispensable for an adequate treatment of moduli spaces via 'stacks'.

Grothendieck was not only generous with his ideas but also very hospitable. Jacob was always welcome in his house located on the *Ile-de-Jatte* just outside the center of Paris. Usually Jacob visited him in the afternoon and they talked many hours about mathematics after which he invariably was asked for dinner. This was usually followed by hour-long mathematical discussions, but now in Grothendieck's study upstairs. You could talk about any subject; for him there were no stupid questions.

This was the period when the IHES (Institut des Hautes Études Scientifiques) was founded by Léon Motchane (1900–1990) who was its first director. At first, it was still in Paris but it soon moved to the *Bois Marie* in Bures-sur-Yvette where the institute also had some apartments. The lively weekly seminar led by Alexander Grothendieck and Jean-Pierre Serre (born 1926) always drew a crowd, among whom the complex analysts Reinhold Remmert (1930–2016) and Hans Grauert (1930–2011). At that time there was no library at the IHES. One had to use the one at the IHP (Institut Henri Poincaré). Its librarian, Paul Belgodère (1921–1986), imposed strict rules. For instance, everyone who wished to make use of the library had to show him a letter of recommendation.

Jacob revisited Paris for a brief period in the fall of 1964; Grothendieck over lunch tried to explain the concept of *motives* to him. Later, in 1967 Grothendieck gave a series of lectures about these, attended by the Russian mathematician Yuri Manin (1937–2023) who understood the material perfectly and wrote a beautiful article [23] about it.

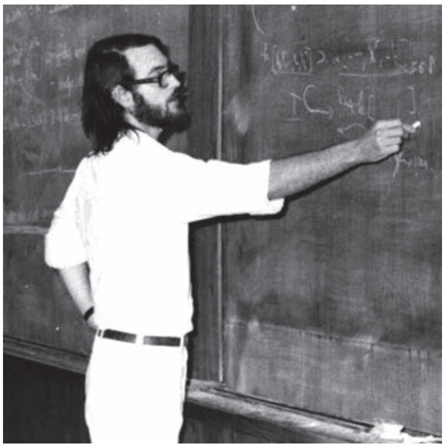
In 1964 Jacob also started to think about the *algebraic fundamental group*. Grothendieck was inspired by the article [24] by David Mumford (born 1937) on the local fundamental group of a normal surface, which he tried to mimic in characteristic p. Grothendieck, who already partly solved it, asked Jacob to look into it, since he was busy doing several other things. Back in the Netherlands Jacob indeed found a solution. Grothendieck told him to go ahead and publish it. However, since Jacob felt that Grothendieck had done an important part of the work, he asked him to be co-author. Grothendieck agreed (cf. [51] and also [79]). Jacob once told me: "This was a good move: I am one of the few people having published an article together with Grothendieck."

In 1969, after visiting Grothendieck at home, Jacob asked him about the *Weil conjectures*. He answered: "I think only one step is needed to solve them and it would not surprise me if one of the younger people like Enrico Bombieri (born 1940) or Pierre Deligne would make this last step." As one knows now, it was not a small step at all. Deligne solved it four years later in 1973 (see [11,12) for which he was awarded the Fields medal.

Jacob kept in contact with Grothendieck but instead of discussing mathematics, they exchanged ideas on e.g. the 'Survivre' movement, an eco-political movement led by Grothendieck in which, with his sharp insight, he was far ahead of his time. He was rather disappointed by the lack of support of his mathematical friends. The last letters date from 1988–1990 when Grothendieck had assumed a teaching job at the university of Montpellier, but the exchange stopped abruptly in 1991 when he had left for the Pyrenees.[11]

An expanding MI Leiden (1970–1994)

When Jacob started his studies in Leiden no local mathematical institute existed there. There were very few students, the exams were all oral and taken at the home of the professor. The results were communicated in the *leeskamer Bosscha* which later became the Lorentz institute for theoretical physics.[12] The first Mathematics Institute was at *Vreewijkstraat 12*, not far from the *Academiegebouw*, the oldest building of the Leiden university, but it did not have offices. In the 1970s the number of students exploded and the need for a larger mathematics institute became apparent. Two floors of an apartment building at the *Stationsplein* were designated to house the institute and math library. Some six years later the faculty of mathematics moved to a newly built three-storey building in which also the first computers of the university where installed.



David Mumford lecturing at Tata

Philip Griffiths

Philip Griffiths (born 1938), invited by Frans Oort, in 1970 gave a wonderful series of lectures at the University of Amsterdam. He gave talks on his newly found results regarding algebraic cycles [13] which Jacob followed with much interest. As a young PhD student I was encouraged to attend these lectures but, being a novice in algebraic geometry I did not understand all the details. Jacob, helpful as usual, answered many an elementary question I posed to him so that I ultimately caught up. These lectures as well as the influential article [10] on cycles on the cubic threefold by Herb Clemens (born 1939) and Philip Griffiths were an important source of inspiration for Jacob. This led the next year to discussions with David Mumford in Warwick[13] where a special year in algebraic geometry (1970/1971) was organized on the occasion of his visit, resulting in several publications: [62,63,65]. In Warwick he met Alberto Conte (born 1942) which led to joint research on topics related to the interests of the old Italian school. Conte would become a life-long friend. Below I'll report about their cooperation more in detail.

The number of students choosing algebraic geometry grew not only in the Netherlands, but worldwide. The first summer schools in algebraic geometry were organized with the aim to inform researchers on all levels about the latest developments in their field. Particularly influential were those organized by the AMS (American Mathematical Society), several of which Jacob attended.

At the one held in Arcata (1974) he met Alberto Collino (1947–2020) who, also on behalf of Alberto Conte, invited him to Turin the next year. In the library of Turin's mathematics institute Jacob admired the many books from Gino Fano's legacy[14]. This visit spurred further scientific cooperation between Jacob and Alberto Conte who was invited several times to Leiden. Later Marina Marchisio (born in 1969) after having obtained her PhD joined them for the papers they subsequently published together. Alberto and Jacob furthermore initiated the founding of three successful European projects 'GAV' (*Geometry of Algebraic Varieties*, 1991–1994), 'AGE' (*Algebraic Geometry in Europe*, 1995–1998), and 'EAGER' (*European Algebraic Geometry Research*, 2000–2004). He also cooperated with Collino (see [34,35,36]) who visited Leiden a few times. Thanks to his influence on the algebraic geometry in Turin, especially through his close cooperation with Collino, and with Conte and Marchisio, the University of Turin conferred in 2002 an honorary doctorate on Jacob, and in 2004 he was appointed foreign member of the *Accademia Delle Scienze di Torino*.

Spencer Bloch (born 1944) was the second young mathematician he met at Arcata and who would likewise play an important role in his mathematical life. They became friends and visited each other multiple times, discussing K-theory and exchanging ideas on algebraic cycles (see e.g. the joint publication [33]. Jacob also initiated the contact between Bloch and the group of K-theorists in Utrecht. This contact resulted in 1978 in the PhD thesis of Jan Stienstra under the supervision of Spencer Bloch

Spencer Bloch at Oberwolfach

and Jan Strooker (1932–2014). After the PhD Stienstra worked for two years with Bloch at the University of Chicago and for two years with Murre at the University of Leiden. A decade later Jacob introduced another promising young student to Spencer, Rob de Jeu from Leiden University. He was accepted by Bloch as a PhD student which led to a Chicago thesis in 1992.[15]

Let me skip back to the time of my doctoral studies (1970–1974). Then the central point of contact for the geometry group, consisting of a dozen persons, was the Leiden weekly Monday geometry seminar. Apart from the organizer, my thesis adviser, Professor van Ven[16], the senior members consisted of Jacob ('Professor Murre' for youngsters, 'Jaap' for his colleagues, and Professor van Est[17]. Van de Ven and Murre were fellow-students and had parallel careers: they completed their doctorate in the same year; van de Ven became lector at the Mathematics Institute of Leiden two years after Murre, and was subsequently promoted to professor in 1963. He remained at the institute until his retirement in 1996. Wil van Est in 1972 had taken up a position in Amsterdam and was succeeded by Wolf Barth (1942–2016), my second thesis adviser. At that time Jacob did not play a prominent role in the geometry seminar, but in later years that changed when he regularly invited renowned guests to the institute.

During 1980s and the beginning of the 1990s algebraic geometry at the Leiden institute was blooming. After the departure in 1976 of Wolf Barth to Erlangen, Joseph Steenbrink (born 1947) was hired as a lector (1978–1980) and subsequently became full professor (Leiden 1980–1988, Nijmegen 1988–2012) and had many successful students, several of whom later became professors[18]. In that period two major projects funded by the 'ZWO' (Dutch Foundation for Pure Research) were instrumental in building up a generation of promising young algebraic geometers. One of these projects, *Singularities*, was led by Dirk Siersma (born 1943), Eduard Looijenga (born 1948) and Joseph Steenbrink, the second, *Moduli*, starting a year later, was led by Gerard van der Geer (born 1950), likewise a former PhD student of van de Ven, Frans Oort and myself. The two-weekly Friday *Intercity Seminars* gathered PhD students from all over the country where they could freely discuss their problems and hear lectures about the latest developments in algebraic geometry, some of these given by foreign specialists who were invited as guest researchers.

In the course of the 1990s, after Hendrik Lenstra (born 1949) and Bas Edixhoven (1962–2022) were hired, the focus in the Mathematics Institute shifted more to number theory and arithmetic geometry, and algebraic geometry became less prominent.

In that period Jacob, through his many contacts abroad, invited renowned mathematicians like Uwe Jannsen, Christopher Deninger, Brent Gordon, Masaki Hanamura, Stefan Müller-Stach and Dinakar Ramakrishnan in order to exchange ideas and to collaborate. See the publications [46–50, 81].

Later years (1995–2023)

After his retirement Jacob remained professionally active. As mentioned above, in this period after the successful European grant 'GAV', he and A. Conte played an important role in assuring the two follow up grants 'AGE' and 'EAGER', which further stimulated cooperation between algebraic geometers in Europe.

During this period Jacob gave several series of lectures on algebraic cycles and motives, for instance during a summer school in Turin (1993)[19] [71], Grenoble (2001) [77], Guanajuato in Mexico (2002) and in Trieste (2010) [78], but also on conferences like the one in Banff (1998) [75]. In these lectures Jacob tried to convey older as well as recent results by giving the essential arguments without burying these in technical detail. At the same time he carefully explained the basic background so that one could reconstruct the full proofs without too much effort. Participants much valued this approach. Jan Nagel and I proposed to write up his lecture notes on motives. Discussing this with Jacob it turned out that he wanted us to assist in extending the notes by adding some newer material on mixed motives. This resulted in the monograph [80].

A few years earlier in 2004, Bas Edixhoven, Jan Nagel and I organized an EAGER conference at the Lorentz Center (Leiden) on the occasion of Jacob's 75th birthday (see [36]) and we were pleased that many first-class mathematicians took up the invitation. Jacob clearly enjoyed this present.

At Alberto Conte's (l) 70th birthday with Arnaud Beauville (r)

C.P. working with Jacob on the monograph [80]

75th birthday fest with E. Looijenga (l) and C. Deninger (r)

Mathematical works

I shall for the most part follow the *Summary of the contents of the selected publications of Jacob P. Murre*, communicated to me a few years before his demise.

1. On a connectedness theorem for a birational transformation at a simple point [54]
The famous connectedness theorem of Oscar Zariski (1899–1986) states that the total transform of a normal point by a birational morphism is connected and in [54] it is shown that for a birational morphism the total transform of a simple point[20] is rationally connected (i.e., any two points can be joined by a chain of rational curves). This result is on the one hand more precise than Zariski's, but on the other hand more restricted because it applies to a simple point instead of to a normal point. Murre's result 'explains', for instance, the well-known fact that a rational map of a variety to an abelian variety is regular at simple points. Grothendieck mentioned this fact in [15, Sem. Bourbaki 190, p. 26] and asked Jacob if he could extend his result to mixed characteristic which he could not at that time. However, Wei-Liang Chow (1911–1995) in [9] answered Grothendieck's question by proving the theorem in full generality.

In the subsequent paper [55] using his connectedness theorem, Jacob extended the notion of intersection multiplicity in the case of proper intersections, but also for maximal connected unions of intersections, solving a question posed by A. Weil in [32, p. 249].

2. *On contravariant functors from the category of preschemes over a field into the category of abelian groups (with an application to the Picard functor)* [60]
In [15, Sem. Bourbaki 232] Grothendieck constructed the Picard scheme for a scheme projective over a base scheme. He first defines the Picard functor and then goes on to shows its representability. His method for the construction of the Picard scheme is close to the method used originally in Matsusaka's construction of the Picard variety. However, Grothendieck replaced Chow varieties by Hilbert schemes. This uses in an essential way the projectivity over the base and so it leaves open the case of a scheme proper over a field. In the present article this gap has been filled, by constructing the Picard scheme for a scheme proper over a field. In rough outline the proof proceeds as follows. First, using all the fundamental tools created by Grothendieck in his previous Bourbaki talks (e.g. his theory of pro-representability, his comparison theorem and his famous existence theorem), necessary and sufficient conditions are established for the representability of a contravariant functor from the category of preschemes over a field to the category of abelian groups. Next it is shown that these necessary and sufficient conditions are satisfied by the Picard functor for a scheme proper over a field.

Further developments: In the fundamental paper [1] Michael Artin created the theory of algebraic spaces which 'settled' in a certain sense the problem of representability of functors over an arbitrary base. In general one can not expect representability by a scheme and Artin had the idea to use instead algebraic spaces. Under certain natural conditions Artin could show that such a functor (over a fixed base) is indeed representable by an algebraic space. Since, as also shown by Artin, over a field algebraic spaces endowed with a group structure are in fact schemes, this gives an improvement of [60].

3. The tame fundamental group of a formal neighbourhood of a divisor with normal crossings on a scheme [51]
Mumford in [24] studied the fundamental group of a normal point on a complex algebraic surface by considering a small tubular neighborhood of a suitable divisorial resolution of the singularity. As has been pointed out by Grothendieck in [16, VIII, Section 3] it is of interest to study this also in the abstract setting and this is the purpose of the present monograph, written jointly with Grothendieck. However, the final result is less nice and precise than Mumford's result. In this abstract algebraic case one has to work within formal neighborhoods of the exceptional divisor, and with the algebraic fundamental group. Moreover, one has to restrict to tame coverings. Hence there is not a characterization of simple points in terms of this fundamental group as in the complex situation as Mumford showed. See also the notes [61] of lectures given at the Tata institute.

4. Algebraic equivalence modulo rational equivalence on a cubic threefold [62]
One of the most striking results in 1971 in the field of algebraic geometry was the proof by Clemens and Griffiths of the non-rationality of the non-singular cubic threefold [10], solving a famous problem of classical Italian geometry. The crucial tool in their proof was the intermediate jacobian which one can associate to any odd-dimensional cohomology group of a nonsingular projective variety. In general this is an analytic torus constructed via Hodge theory. In the present case of the cubic threefold the intermediate jacobian of the middle cohomology is in fact an abelian variety. It has a geometric flavour since it is determined by the family of lines on the threefold. In the non-rationality proof these algebraic cycles of codimension two play a crucial role. Since, as discovered earlier by Mumford and Griffiths, divisors and algebraic cycles of codimension ≥ 2 behave fundamentally different with respect to various equivalence relations, in this setting a natural question arose: what is the nature of the *Abel–Jacobi equivalence relation*?[21]
In the present article that question was answered: for the cubic threefold the group of one dimensional cycles which are algebraically equivalent to zero modulo rational equivalence maps injectively to the intermediate jacobian. The method of proof is entirely algebraic and it works over every algebraically closed field of characteristic different from 2. In this algebraic setting the intermediate jacobian has to be replaced by a so-called *Prym variety* which itself is a principally polarized abelian variety (see also [63] which is discussed in work 5 below). To give an idea how the Prym variety enters the proof, first fix a (general) line L on the cubic X and a plane N not meeting L in the ambient 4-dimensional projective space. By taking the span $M_p = \langle p, L \rangle$ of the points $p \in N$ with L, the intersections $C_p = M_p \cap X$ form a family of conics (on X) and hence a conic bundle over N. Over some curve C in N these conics degenerate into two lines (both meeting L). On the other hand we consider the curve C' of the set of *all* the lines meeting the fixed line L. This defines an involution of C' over C by interchanging these two lines and hence an involution of the level of the jacobian. This is the Prym involution and its 'kernel', the corresponding Prym variety, takes the place of the intermediate jacobian of X.

5. Reduction of the proof of the non-rationality of a non-singular cubic threefold to a result of Mumford [63]

Mumford has made a precise systematic study of Prym varieties in his paper [25]. Since the intermediate jacobian of a cubic threefold has an interpretation as a Prym variety (see work 4 above and the references given there), one could in this way get a proof (in characteristic zero) of the non-rationality of the cubic threefold. Using Mumford's results and methods from [62], and working with étale cohomology instead of singular cohomology the transcendental methods could be replaced by algebraic ones. Hence the non-rationality of the non-singular cubic threefold holds over every algebraically closed field of different from 2.

Further developments: In 1977 there appeared a truly beautiful paper [7] by Arnaud Beauville (born 1947), were he made a profound study of quadric bundles and their corresponding Prym varieties and which implied the results of [63] as a special case.

6. The Hodge conjecture for fourfolds admitting a covering by rational curves [41]
In this article, written jointly with A. Conte, the Hodge $(2,2)$ conjecture for a nonsingular complex projective fourfold is shown in case there exists a covering family of rational curves. In particular this applies to nonsingular cubic quartic and quintic fourfolds and also to quadric bundles of dimension 4. More examples were given in [42].

7. On the Chow groups of certain types of Fano threefolds [33]
In this joint article with S. Bloch the Chow groups of codimension two cycles on three types of Fano threefolds X are investigated, namely quartic threefolds in $\mathbb{P}^4$, intersections of a cubic and a quadric in $\mathbb{P}^5$, and the intersections of three quadrics in $\mathbb{P}^6$; more precisely, we study their subgroups of rational equivalence classes of cycles algebraically equivalent to zero. These groups are shown to be weakly representable in the sense that they can be dominated by the jacobian of a curve via a correspondence between the curve and the variety in question [22]. In fact, they are parametrized by the points of a generalized Prym variety, i.e., a principally polarized abelian variety associated to a pair consisting of a jacobian variety of a curve and an endomorphism satisfying a quadratic equation.[23] This result is greatly inspired by the beautiful paper [30] where such varieties (over $\mathbb{C}$) are considered via intermediate jacobians. Another important ingredient comes from Bloch's article [8], where it is shown that there exists a map between the torsion groups of the codimension two cycles to the third étale cohomology group. In the present situation these morphisms are isomorphisms. Using this, certain questions on the Chow groups can be reduced to known results in étale cohomology.

8. Applications of algebraic K-theory to the theory of algebraic cycles [67]
In this article it is shown that for codimension two cycles algebraically equivalent to zero there exists a universal regular map to an abelian variety (such a map is called regular if for every algebraic family T of such cycles the composite of the family and the map is itself a rational map — in the sense of algebraic geometry — from T to the abelian variety). Over the complex numbers this abelian variety is the image by the Abel–Jacobi map of the algebraic cycles of this kind in the intemediate jacobian.

The theorem answers a question of Mumford for such cycles [18, p. 143]. The proof uses results of Hiroshi Saito, Bloch and Bloch–Ogus in combination with the theorem of Merkurjev–Suslin from algebraic K-theory. In 2018 Bruno Kahn found a mistake in the proof of one the lemmas and fixed it in [20].

9. Abel–Jacobi equivalence versus incidence equivalence for algebraic cycles of codimension two [66]
As discussed in work 4 above, cycles of codimension j algebraically equivalent to zero are said to be Abel–Jacobi equivalent to zero if they are in the kernel of the Abel–Jacobi map to the j-th intermediate Jacobian. On the other hand there is also the subgroup of the cycles incident equivalent to zero, a concept introduced by P. Griffiths in [14]. The importance of this concept has also been stressed by Grothendieck in [17].

Griffiths proved that incidence equivalence is coarser than Abel–Jacobi equivalence and raised the question whether the two differ at most by an isogeny. For the case of codimension two cycles this question of Griffiths has a positive answer as shown in this article. The main tool of the proof is Hodge theory and so the result is only valid over $\mathbb{C}$.

10. On the motive of an algebraic surface [68]
One of Grothendieck's reasons to create a theory of motives was to understand better the similarity of the cohomology groups of an algebraic variety in the different settings (classical étale, crystalline). He focussed on homological motives, i.e., motives built from algebraic cycles modulo homological equivalence (or rather, numerical equivalence, but this depends on a conjecture). His construction works for every adequate equivalence relation. Jacob in this article, inspired by works of Yuri Manin and Christophe Soulé, and by discussions with Spencer Bloch, turned to Chow motives, i.e., motives built using rational equivalence instead of homological equivalence. As he states in the introduction of loc. cit.: "It is my conviction that Chow motives are more suited for the study of the Chow groups." In Grothendieck's theory one uses the cohomological Künneth components $p^j(X)$ of the diagonal of the variety X, but for Chow motives one replaces these with the finer Chow–Künneth components $\mathrm{ch}^j(X)$ belonging to the Chow group of $X\times X$ whose classes should give the usual Künneth components. These components act as projectors. For every nonsingular projective variety X of dimension d (defined over an arbitrary field) one has the two trivial projectors $\mathrm{ch}^0(X)=e\times X$ and dually $\mathrm{ch}^{2d}(X)=X\times e$, where e is a (rational) point on X. In the present paper for every such X the next Chow–Künneth projector $\mathrm{ch}^1(X)$ and its dual $\mathrm{ch}^{2d-1}(X)$ are constructed. The construction of ch^1 is based upon the work of A. Weil and S. Lang on the Picard and Albanese variety. Using a linear section of the variety one constructs first a morphism from the Picard variety to the Albanese variety which turns out to be an isogeny. An inverse of this isogeny gives a morphism from the Albanese to the Picard variety and therefore by the Weil–Lang theory one obtains a divisor class in the product $X\times X$. This is a refined algebraic version of the $\Lambda_{d-1}(X)$-class from the Lefschetz decomposition valid for cohomology. These ingredients allow to construct the Chow–Künneth projector $\mathrm{ch}^1(X)$, and its transpose yields $\mathrm{ch}^{2d-1}(X)$. The corresponding Chow motives are the motivic versions of the Picard and Albanese variety (similarly, in the case of a curve C the Chow motive $\mathrm{ch}^1(C)$ is the motivic version of the jacobian variety of the curve C).

In the case of a surface this yields a complete decomposition of the diagonal since for the remaining $\mathrm{ch}^2(S)$ one can take the difference of the diagonal with the previous projectors. The surface S is now as Chow motive completely decomposed into five Chow motives $\mathrm{ch}^j(S)$ each of which not only gives back the j-th cohomology but on the Chow level this gives a filtration, the so-called Bloch–Beilinson filtration.[24]

In his beautiful survey paper in the 1991 Seattle conference Tony Scholl [29] has given a more modern description (and some slight improvement) of this paper. See also work 13 below.

11. Motivic decomposition of abelian schemes and Fourier transform [46]
Already in 1974 Shermenev — a student of Manin — has shown, by intricate arguments, that the Chow motive $\mathrm{ch}(A)$ of an abelian variety A over an algebraically closed field can be completely decomposed. In this paper, joint with C. Deninger, the existence of a canonical motivic decomposition is proven, not only for an abelian variety over a field but also for an abelian scheme A over a base which is itself over a field. For this purpose the theory of Chow motives is extended to a theory of relative Chow motives over a given base. A. Beauville has used the work of S. Mukai on the Fourier transform to study the decomposition of the Chow groups of an abelian variety under the endomorphism of multiplication by n. Similar methods can be used on abelian schemes. The article [22] by D. Lieberman[25] then yields the required decomposition of the diagonal.

Further developments: K. Künnemann in [21] has extended these methods in a nice way and obtained a complete Lefschetz theory for Chow motives of an abelian scheme.

12. On a conjectural filtration on the Chow groups of an algebraic variety. Parts I,II [69,70]
One of the famous standard conjectures is that the Künneth decomposition of the diagonal of a nonsingular projective algebraic variety X can be given by algebraic cycles on the product $X \times X$. Based on some examples Jacob suggested an even stronger conjecture namely that those required cycle classes can in fact be lifted to rational equivalence and then give an orthogonal decomposition of the diagonal in the Chow ring, i.e., that there is not merely a Künneth decomposition but a Chow–Künneth decomposition which gives a filtration on the Chow groups (see work 10 above).

In [69] a number of conjectures were announced describing how this filtration should be determined by those Chow motives. In [70] part of these conjectures were verified in a (rather limited) number of cases. In particular for surfaces these are true (see work 10 above); moreover they are also true for threefolds of the type $S \times C$ where S is a surface and C is a curve. For abelian surfaces part of the conjectures is true and the remaining part is equivalent to a conjecture of A. Beauville.

To this day the evidence for the truth of the conjectures is still meagre (the cornerstone in this story remains of course the famous Standard Conjecture itself about the algebraicity of the Künneth components of the diagonal themselves — the so called *standard conjecture CK* — for which unfortunately no progress has been obtained for a long time).

In his beautiful lecture in the Seattle Conference on Motives (1991) Uwe Jannsen [19] discussed these conjectures and he showed that they are equivalent with the so-called *Bloch–Beilinson conjectures* (in one of their forms) and that the corresponding filtrations (if they exist) also coincide.

13. On the transcendental part of the motive of a surface [52]
This is a joint paper with Bruno Kahn and Claudio Pedrini. A warning is in order: in this paper Voevodsky's[26] convention is used which is homological rather than cohomological and uses a covariant version of Chow motives which is the dual of Grothendieck's contravariant (cohomological) version. In all other papers of Jacob he adhered to Grothendieck's convention.

Recall that in the work 10 above a complete Chow–Künneth decomposition of the Chow motive of a surface S was given. Here the mysterious middle motive $\mathrm{ch}_2(S)$ is further decomposed into an obvious algebraic part (carrying the algebraic cohomology) and a transcendental part $t(S)$. The algebraic part is a sum of twisted Tate motives. However, the transcendental part is the truly mysterious part carrying the transcendental cohomology as well as the Albanese kernel. After the introduction there are three sections, each written by a different author. In the first part (written by Jacob) the validity of his conjectures is investigated for the motive of the product of two surfaces. In the second part (written by C. Pedrini) the birational motive of a surface is treated as well as the relation between Bloch's conjecture and the so-called finite dimensionality conjecture by Kimura–O'Sullivan. Finally, in the last part (written by B. Kahn) again birational motives of a surface are studied and also the step is made from passing from pure motives to the triangulated category of Voevodsky.

Honors
1971 Member Royal Dutch Academy of Science (KNAW)
2000 Silver Medal *Unione Matematica Italiana*
2002 Doctor Honoris Causa Università di Torino
2004 Foreign Member *Accademia Delle Scienze di Torino*

Murre's doctoral students
R. van der Waall (with H. Kloosterman, 1972), F. van Schagen (1972), J. Bioch (1975), W. de Bruin (with M. van der Put, 1976), J. Brinkhuis (1981), G. Welters (with F. Oort, 1981), D. Epema (with C. Peters, 1983), R. Aerdts (with F. Oort, 1986), B. Ruitenburg (with G. Heckman, 1988), J. Mulder (with C. Peters, 1993), J. van Beelen (with B. Edixhoven, 1994), J. Nagel (with C. Peters, 1997).

Acknowledgements
This article heavily draws upon an interview from 2012 with Jacob which is available (in Dutch) on the author's webpage https://www-fourier.univ-grenoble-alpes.fr/~peters. The author also wants to thank A. Conte, C. Deninger, G. van der Geer, E. Looijenga, J. Nagel, F. Oort, U. Persson, H. Sterk, J. Stienstra, who provided him with corrections and further information, and S. Bloch, Ph. Griffiths, D. Mumford for also allowing to use their pictures.

Notes

1 A small village in the province of Zeeland. Baarland at that time had only 800 inhabitants.

2 The Dutch part of Flanders.

3 C. Visser was professor in Delft from 1946–1956, and in Leiden from 1956 until his retirement in 1976.

4 Roughly the equivalent of a masters.

5 Johannes Droste (1886–1963) was a Dutch mathematician and theoretical physicist.

6 Paul Cohen changed his adviser to Antoni Zygmund (1900–1992).

7 Jean Dieudonné was one of the founders of the Nicolas Bourbaki group.

8 Comparable to Reader or Associate Professor.

9 Nicolaas Kuiper (1920–1994) at that time held a chair at the Agricultural University of Wageningen, later moved to the University of Amsterdam, and from 1971–1985 served as director of the IHES.

10 He meant Weil, Chow and Matsusaka.

11 See also [27,28].

12 Johannes Bosscha (1831–1911), assistant at the Physics Lab Leiden (1855–1860), professor at the Royal Military Academy (KMA), Breda (1860–1865), and at the Polytechnical University Delft (1873–1885).

13 In 1965, a new mathematics research institute was founded in Warwick by Sir Christopher Zeeman (1925–2016).

14 Gino Fano (1871–1952) was one of the founding fathers of the Italian school of algebraic geometry.

15 Rob de Jeu is now professor at the VU, Amsterdam.

16 Antonius (Ton) van de Ven (1931–2014).

17 Wil van Est was mentioned above, he was the formal promotor of F. Oort.

18 For example D. van Straten and T. de Jong, now professors at Mainz University, and Jan Stevens at the university of Gothenburg.

19 This was just before his retirement.

20 This is older terminology for 'non-singular point'.

21 Two codimenson 2 cycles are Abel–Jacobi equivalent if their difference maps to zero in the intermediate jacobian under the Abel–Jacobi map.

22 This implies that they are not of infinite dimension in Mumford's sense.

23 This indeed generalizes the Prym situation explained for work 4.

24 The mysterious part is the Chow motive $\mathrm{ch}^2(S)$ which carries the Albanese kernel and the entire second cohomology group-both the algebraic and transcendental part-and therefore also gives the rank of the Brauer group.

25 David Lieberman (1931) is an American mathematician, at first professor at Brandeis University, later, from 1977 on employed at the Institute for Defense Analyses (Princeton, USA).

26 Vladimir Voevodsky (1966–2017).

References

1 M. Artin, Algebraic approximation of structures over complete local rings, *Inst. Hautes Études Sci. Publ. Math.* 36 (1969), 23–58.

2 M. Artin, J.E. Bertin, M. Demazure, P. Gabriel, A. Grothendieck, M. Raynaud and J.-P. Serre, *Schémas en groupes. Fasc. 1: Exposés 1 à 4*, Institut des Hautes Études Scientifiques, Paris, 1963/1964. Deuxième édition, Séminaire de Géométrie Algébrique de l'Institut des Hautes Études Scientifiques, 1963, dirigé par Michel Demazure et Alexander Grothendieck.

3 M. Artin, J.E. Bertin, M. Demazure, P. Gabriel, A. Grothendieck, M. Raynaud and J.-P. Serre, *Schémas en groupes. Fasc. 2a: Exposés 5 et 6*, Institut des Hautes Études Scientifiques, Paris, 1963/1965. Deuxième édition, Séminaire de Géométrie Algébrique de l'Institut des Hautes Études Scientifiques, 1963/64, dirigé par Michel Demazure et Alexander Grothendieck.

4 M. Artin, J.E. Bertin, M. Demazure, P. Gabriel, A. Grothendieck, M. Raynaud and J.-P. Serre, *Schémas en groupes. Fasc. 2b: Exposés 7a et 7b*, Institut des Hautes Études Scientifiques, Paris, 1965. Deuxième édition, Séminaire de Géométrie Algébrique de l'Institut des Hautes Études Scientifiques, 1963/64, dirigé par Michel Demazure et Alexander Grothendieck.

5 W. Barth, K. Hulek, C. Peters and A. van de Ven, *Compact Complex Surfaces*, Ergebn. der Math. 3. Folge, Vol. 4, Springer, 2004, second edition.

6 W. Barth, C. Peters and A. van de Ven, *Compact Complex Surfaces*, Ergebn. der Math. 3. Folge, Vol. 4, Springer, 1984.

7 A. Beauville, Variétés de Prym et jacobiennes intermédiaires, *Ann. Sci. École Norm. Sup. (4)* 10(3) (1977), 309–391.

8 S. Bloch, Torsion algebraic cycles and a theorem of Roitman, *Compositio Math.* 39(1) (1979), 107–127.

9 W.L. Chow, On the connectedness theorem in algebraic geometry, *Amer. J. Math.* 81 (1959), 1033–1074.

10 C.H. Clemens and P.A. Griffiths, The intermediate Jacobian of the cubic threefold, *Ann. of Math. (2)* 95 (1972), 281–356.

11 P. Deligne, La conjecture de Weil. I, *Inst. Hautes Études Sci. Publ. Math.* 43 (1974), 273–307.

12 P. Deligne, La conjecture de Weil. II, *Publ. Math. I.H.E.S.* 52 (1980), 137–252.

13 P.A. Griffiths, On the periods of certain rational integrals. I, II, *Ann. of Math. (2)* 90 (1969), 460–495, 496–541.

14 P.A. Griffiths, Some transcendental methods in the study of algebraic cycles, in *Several Complex Variables, II (Proc. Internat. Conf., Univ. Maryland, College Park, MD, 1970)* , Lecture Notes in Math., Vol. 185, Springer, 1971, pp. 1–46.

15 A. Grothendieck, Fondements de la géométrie algébrique [Extraits du Séminaire Bourbaki, 1957–1962], Secrétariat mathématique, Paris, 1962.

16 A. Grothendieck, Cohomologie locale des faisceaux cohérents et théorèmes de Lefschetz locaux et globaux (SGA 2), Advanced Studies in Pure Mathematics, Vol. 2, North-Holland and Masson et Cie, 1968. Augmenté d'un exposé par Michèle Raynaud, Séminaire de Géométrie Algébrique du Bois-Marie, 1962.

17 A. Grothendieck, Hodge's general conjecture is false for trivial reasons, *Topology* 8 (1969), 299–303.

18 R. Hartshorne, Equivalence relations on algebraic cycles and subvarieties of small codimension, in *Algebraic Geometry, Humboldt State Univ., Arcata, CA, 1974*, (Proc. Sympos. Pure Math., Vol. 29, Amer. Math. Soc., 1975, pp. 129–164.

19 U. Jannsen, Motivic sheaves and filtrations on Chow groups, in *Motives (Seattle, WA, 1991*, Proc. Sympos. Pure Math., Vol. 55, Amer. Math. Soc., 1994, pp. 245–302.

20 B. Kahn, On the universal regular homomorphism in codimension 2, *Ann. Inst. Fourier (Grenoble)* 71(2) (2021), 843–848.

21 K. Künnemann, A Lefschetz decomposition for Chow motives of abelian schemes, *Invent. Math.* 113(1) (1993), 85–102.

22 D.I. Lieberman, Higher Picard varieties, *Amer. J. Math.* 90 (1968), 1165–1199.

23 J.I. Manin, Correspondences, motifs and monoidal transformations, *Mat. Sb. (N.S.)* 77(119) (1968), 475–507.

24 D. Mumford, The topology of normal singularities of an algebraic surface and a criterion for simplicity, *Publications mathématiques de l'I.H.É.S.* 9 (1961), 5–22.

25 D. Mumford, Prym varieties. I, in *Contributions to Analysis (a collection of papers dedicated to Lipman Bers*), Academic Press, 1974, pp. 325–350.

26 J. Nagel and C. Peters, eds., *Algebraic Cycles and Motives*, London Mathematical Society Lecture Note Series, Vol. 343 and 344, Cambridge University Press, 2007.

27 W. Scharlau, *Wer is Alexander Grothendieck?*, Vol 1 and 3, Books on Demand, 2010, 2011.

28 L. Schneps, Who is Alexander Grothendieck?, Vol 2, https://webusers.imj-prg.fr/~leila.schneps/grothendieckcircle/Mathematics.html, 2023, in progress.

29 A.J. Scholl, Classical motives, in *Motives (Seattle, WA, 1991)*, Proc. Sympos. Pure Math., Vol. 55, Amer. Math. Soc., 1994, pp. 163–187.

30 A.N. Tjurin, Five lectures on three-dimensional varieties, *Uspehi Mat. Nauk* 27(5) (1972), 3–50.

31 B.L. van der Waerden, *Einführung in die algebraische Geometrie*, Dover Publications, 1945.

32 A. Weil, *Foundations of Algebraic Geometry*, American Mathematical Society Colloquium Publications, Vol. 29, Amer. Math. Soc., 1946.

Publications of J.P. Murre

33 S. Bloch and J.P. Murre, On the Chow group of certain types of Fano threefolds, *Compositio Math.* 39(1) (1979), 47–105.

34 A. Collino and J.P. Murre, The intermediate Jacobian of a cubic threefold with one ordinary double point; an algebraic-geometric approach. I, *Nederl. Akad. Wetensch. Proc. Ser. A* 81 / *Indag. Math.* 40(1) (1978), 43–55.

35 A. Collino and J.P. Murre, The intermediate Jacobian of a cubic threefold with one ordinary double point; an algebraic-geometric approach. II, *Nederl. Akad. Wetensch. Proc. Ser. A* 81 / *Indag. Math.* 40(1) (1978), 56–71.

36 A. Collino, J.P. Murre and G.E. Welters, On the family of conics lying on a quartic threefold, *Rend. Sem. Mat. Univ. Politec. Torino* 38(1) (1980), 151–181.

37 A. Conte, M. Marchisio and J.P. Murre, On unirationality of double covers of fixed degree and large dimension; a method of Ciliberto, in *Algebraic Geometry*, de Gruyter, 2002, pp. 127–140.

38 A. Conte, M. Marchisio and J.P. Murre, On the unirationality of the quintic hypersurface containing a 3-dimensional linear space, *Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur.* 142 (2009), 89–96.

39 A. Conte and J.P. Murre, On quartic threefolds with a double line. I, *Nederl. Akad. Wetensch. Proc. Ser. A* 80 / *Indag. Math.* 39(3) (1977), 145–160.

40 A. Conte and J.P. Murre, On quartic threefolds with a double line. II, *Nederl. Akad. Wetensch. Proc. Ser. A* 80 / *Indag. Math.* 39(3) (1977), 161–175.

41 A. Conte and J.P. Murre, The Hodge conjecture for fourfolds admitting a covering by rational curves, *Math. Ann.*, 238(1) (1978), 79–88.

42 A. Conte and J.P. Murre, The Hodge conjecture for Fano complete intersections of dimension four, in *Journées de Géometrie Algébrique d'Angers, Juillet 1979 / Algebraic Geometry, Angers, 1979*, Sijthoff & Noordhoff, 1980, pp. 129–141.

43 A. Conte and J.P. Murre, Algebraic varieties of dimension three whose hyperplane sections are Enriques surfaces, *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)* 12(1) (1985), 43–80.

44 A. Conte and J.P. Murre, On the definition and on the nature of the singularities of Fano threefolds, *Rend. del Sem. Mat. Univ. Pol. Torino* 43 (1986), 51–67.

45 A. Conte and J.P. Murre, On a theorem of Morin on the unirationality of the quartic fivefold, *Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur.* 132 (1998), 49–59.

46 C. Deninger and J.P. Murre, Motivic decomposition of abelian schemes and the Fourier transform, *J. Reine Angew. Math.* 422 (1991), 201–219.

47 B.B. Gordon, M. Hanamura and J.P. Murre, Chow–Künneth projectors for modular varieties, *C.R. Math. Acad. Sci. Paris* 335(9) (2002), 745–750.

48 B.B. Gordon, M. Hanamura and J.P. Murre, Relative Chow–{Künneth projectors for modular varieties, *J. Reine Angew. Math.* 558 (2003), 1–14.

49 B.B. Gordon, M. Hanamura and J.P. Murre, Absolute Chow–Künneth projectors for modular varieties, *J. Reine Angew. Math.* 580 (2005), 139–155.

50 B.B. Gordon and J.P. Murre, Chow motives of elliptic modular threefolds, *J. Reine Angew. Math.* 514 (1999), 145–164.

51 A. Grothendieck and J.P. Murre, *The Tame Fundamental Group of a Formal Neighbourhood of a Divisor with Normal Crossings on a Scheme*, Lecture Notes in Math., Vol. 208, Springer, 1971.

52 B. Kahn, P. Murre and C. Pedrini, On the transcendental part of the motive of a surface, in *Algebraic Cycles and Motives, Vol. 2*, London Math. Soc. Lecture Note Ser., Vol. 344, Cambridge Univ. Press, 2007, pp. 143–202.

53 S.-I. Kimura and J.P. Murre, On natural isomorphisms of finite dimensional motives and applications to the Picard motives, in *Cycles, Motives and Shimura Varieties*, Tata Inst. Fund. Res. Stud. Math., Vol. 21, Tata Inst. Fund. Res., 2010, 227–241.

54 J.P. Murre, Intersection multiplicities of maximal connected bunches, *Amer. J. Math.* 80 (1958), 311–330.

55 J.P. Murre, On a connectedness theorem for a birational transformation at a simple point, *Amer. J. Math.* 80 (1958), 3–15.

56 J.P. Murre, On a uniqueness theorem for certain kinds of birational transformations, *Nederl. Akad. Wetensch. Proc. Ser. A* 62 / *Indag. Math.* 21 (1959), 129–134.

57 J.P. Murre, On divisors on products of three factors, *Nieuw Arch. Wiskunde* 3/8 (1960), 129–133.

58 J.P. Murre, On Chow varieties of maximal, total, regular families of positive divisors, *Amer. J. Math.* 83 (1961), 99–110.

59 J.P. Murre, On generalized Picard varieties, *Math. Ann.* 145 (1961/62), 334–353.

60 J.P. Murre, On contravariant functors from the category of pre-schemes over a field into the category of abelian groups (with an application to the Picard functor), *Inst. Hautes Études Sci. Publ. Math.* 23 (1964), 5–43.

61 J.P. Murre, *Lectures on an Introduction to Grothendieck's Theory of the Fundamental Group*, Tata Inst. Fund. Res. Lectures on Math, Vol. 40, Tata Inst. Fund. Res., 1967. Notes by S. Anantharaman.

62 J.P. Murre, Algebraic equivalence modulo rational equivalence on a cubic threefold, *Compositio Math.* 25 (1972), 161–206.

63 J.P. Murre, Reduction of the proof of the non-rationality of a non-singular cubic threefold to a result of Mumford, *Compositio Math.* 27 (1973), 63–82.

64 J.P. Murre, Algebraic hypersurfaces, *Nederl. Akad. Wetensch. Verslag Afd. Natuurk.* 83 (1974), 11–13.

65 J.P. Murre, Some results on cubic threefolds, in *Classification of Algebraic Varieties and Compact Complex Manifolds*, Lecture Notes in Math., Vol. 412, Springer, 1974, pp. 140–160.

66 J.P. Murre, Abel–Jacobi equivalence versus incidence equivalence for algebraic cycles of codimension two, *Topology* 24(3) (1985), 361–367.

67 J.P. Murre, Applications of algebraic K-theory to the theory of algebraic cycles, in *Algebraic Geometry, Sitges (Barcelona), 1983*, Lecture Notes in Math., Vol. 1124, Springer, 1985, pp. 216–261.

68 J.P. Murre, On the motive of an algebraic surface, *J. Reine Angew. Math.* 409 (1990), 190–204.

69 J.P. Murre, On a conjectural filtration on the Chow groups of an algebraic variety. I, The general conjectures and some examples, *Indag. Math. (N.S.)* 4(2) (1993), 177–188.

70 J.P. Murre, On a conjectural filtration on the Chow groups of an algebraic variety. II, Verification of the conjectures for threefolds which are the product on a surface and a curve, *Indag. Math. (N.S.)* 4(2) (1993), 189–201.

71 J.P. Murre, Algebraic cycles and algebraic aspects of cohomology and K-theory, in *Algebraic Cycles and Hodge Theory (Torino, 1993)*}, Lecture Notes in Math., Vol. 1594, Springer, 1994, pp. 93–152.

72 J.P. Murre, On the work of Gino Fano on three-dimensional algebraic varieties, Algebra and Geometry (1860-1940): the Italian contribution, *Rend. Circ. Mat. Palermo (2) Suppl.* 36 (1994), 219–229.

73 J.P. Murre, Representation of unramified functors. Applications (according to unpublished results of A. Grothendieck), in *Séminaire Bourbaki*, Vol. 9, Exp. No. 294, Soc. Math. France, 1995, pp. 243–261.

74 J.P. Murre, Introduction to the theory of motives, *Boll. Un. Mat. Ital. A (7)* 10(3) (1996), 477–489.

75 J.P. Murre, Algebraic cycles on abelian varieties: application of abstract Fourier theory, in *The Arithmetic and Geometry of Algebraic Cycles Banff, AB, 1998)*, NATO Sci. Ser. C Math. Phys. Sci., Vol. 548, Kluwer, 2000, pp. 307–320.

76 J.P. Murre, Fano varieties and algebraic cycles, in *The Fano Conference*, Univ. Torino, 2004, pp. 51–68.

77 J.P. Murre, Lecture on motives, in *Transcendental Aspects of Algebraic Cycles*}, London Math. Soc. Lecture Note Ser., Vol. 313 Cambridge Univ. Press, 2004, pp. 123–170.

78 J.P. Murre, Lectures on algebraic cycles and Chow groups, in *Hodge Theory*, Math. Notes, Vol. 49, Princeton Univ. Press, 2014, pp. 410–448.

79 J.P. Murre, On Grothendieck's work on the fundamental group, in *Alexandre Grothendieck: A Mathematical Portrait*, Int. Press, 2014, pp. 143–167.

80 J.P. Murre, J. Nagel and C.A.M. Peters, *Lectures on the Theory of Pure Motives*, University Lecture Series, Vol. 61, Amer. Math. Soc., 2013.

81 J.P. Murre and D. Ramakrishnan, Local Galois symbols on $E\times E$. in *Motives and Algebraic Cycles*, Fields Inst. Commun., Vol. 56 Amer. Math. Soc., 2009, pp. 257–291.