Nelly Litvak
Department of Mathematics and Computer Science
Eindhoven University of Technology
n.v.litvak@tue.nl

Column Better than blackboard

We shouldn't give classroom lectures anymore

My colleague Frederic Schuller never planned to be a YouTube star. Some fans recorded his award-winning blackboard lecture courses on Differential Geometry, General Relativity and Quantum Mechanics [1] and launched a YouTube channel under his name. The style of these videos is exactly the opposite of what any course on educational videos teaches us: each lecture is two hours long, with just Frederic at the blackboard, telling his well-paced story, on most demanding level from the start, with admirable knowledge and humour. The videos are a stunning success. Each of the three 50-hour lecture courses has been watched more than 50 000 times by wide audience, from students all over the world to MIT professors. The loving comments say that Frederic is the best teacher ever.

What can be better than the magic of mathematics unfolding on a blackboard? I love to listen to blackboard lectures, this is how I learned mathematics. I love to give them even more. Many of my colleagues, academic mathematicians, feel this way. But is the blackboard lecture effective for teaching mathematics to the university students of today? This is an entirely different question.

Welcome to the new column 'Better than blackboard'! In this column we will talk about teaching mathematics at university. I will address problems that many university teachers face: students don't show up at classes, cannot concentrate, stay passive, learn by mimicking old solutions, show no deep understanding, get scared of the proofs, and have no reliable pre-knowledge in follow-up courses. Emotions aside, I want to talk about mathematics education in the same way as we talk about mathematics: stating definitions, questioning assumptions, and being very critical to our intuition. And I want to look for solutions.

I proudly introduce illustrations by Eline van Hove. Graduate of MSc Applied Mathematics as well as academy of arts, Eline says that she was formed by art and mathematics. In her daily work, Eline helps organizations to solve societal problems. I am delighted that she agreed to work with me on these articles! Check more of Eline's art work at www.elinevanhove.nl. This inaugural article is about classroom lectures. And my main statement is the title of this first column: 'We shouldn't give classroom lectures anymore'.

No classroom lectures anymore

We shouldn't give classroom lectures anymore. Not online, and not live either. In class, passive lecture should give place to active learning methods, when students do something themselves rather than listening to the teacher.

I know this is not a popular statement at university. Therefore I will write this article in a form of a conversation. I will state most common, in my experience, concerns and counterarguments in defense of classroom lectures, and I will try to give logical answers. Here we go.

What makes lectures ineffective?

Hundreds of studies, time and again, arrive to the same conclusion: classroom lectures are a terrible way for the students to learn. 'Terrible' is not my word choice, I quoted it from the book *Building the Intentional University*, about the experience of the innovative university Minerva, Chapter 11: 'The Science of Learning: Mechanisms and Principals' [5]. Why terrible? Below is an incomplete list of issues, that, most likely, sound familiar.

Low retention

In my experience, students almost never can accurately recall what was said in a lecture. This is of course an anecdotal evidence. The numbers in the literature vary, and are often up to interpretation. Yet, there is an overwhelming consensus that active learning methods yield much higher retention than a lecture. One may say: "We don't expect the students to recall the lecture. We expect them to read, solve problems, and then they will remember." Yes, and we hope that the students connect their reading and problem solving to the lecture, right? So, we do expect some retention after all, while in reality, this retention is much lower than we would like it to be.

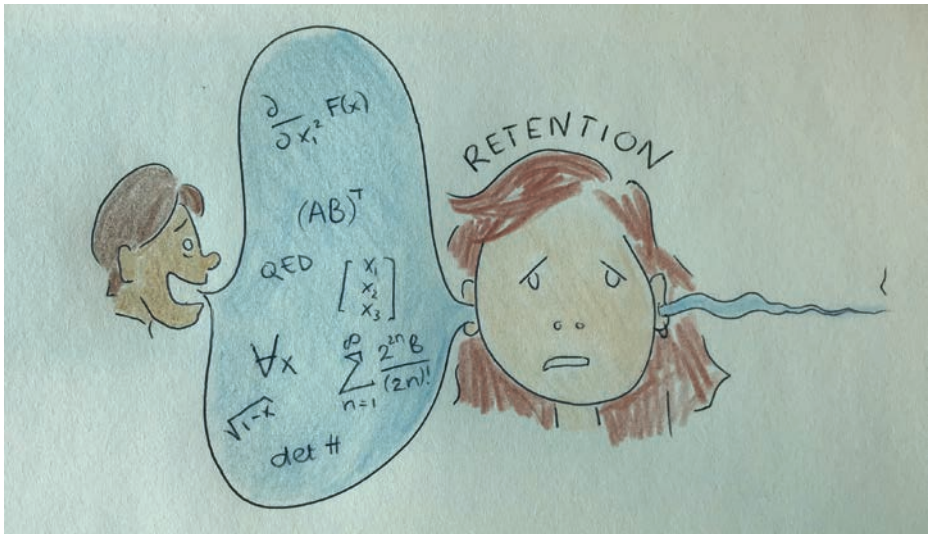

Illustration: Eline van Hove

Lack of interaction

Most teachers highly value interaction with students. This is why we were so eager to go back to campus after lockdowns. But how much interaction actually occurs during your lectures? Try to write down two numbers: (1) How many percent of time in your lecture do you spend on interaction? (2) How many percent of the students participate in the interaction?

Often in a classroom lecture only a couple of students answer teacher's questions or ask questions themselves. The rest stay silent. There is not much time for interaction either because the teacher must cover a sizable material.

One colleague argued that interaction is implicit: the teacher observes the students' reaction and therefore may slow down or speed up. Well, then videos are even better. Students can speed up or rewind the video, watch it at their own pace. What is the obvious value of an 'implicit' interaction, without actual communication between the students and the teacher? Can you name it? I can't.

Short attention span of the students

We all complain about it: students nowadays are not able to listen for half an hour at a stretch. True. Digital tools have greatly contributed to this. But have you ever checked out our digital competitors for the students' attention? For example, have you seen the YouTube channel '3blue1brown' [2]? If not, please have a look. And prepare to be amazed. Arguably, these videos will not teach students how to prove theorems or solve problems. But when material of this quality is freely available on the Internet, how can we even expect the students to patiently listen to a 90 minutes lecture?

Most importantly, even if our complaints are righteous, they are pointless. These are the students we have now. Digital tools are not going anywhere, they will only get better. We must learn how to work with real-life students in our class, not imaginary students we wish we had.

And by the way, the lack of students' concentration on a lecture doesn't mean that mathematics is doomed. Try to make this mental exercise: what, in your opinion, are the most important qualities for doing mathematics? In his brilliant book *Building Thinking Classrooms in Mathematics* [8], Peter Liljedahl reports on asking many teachers, and arrives at the following top-3 list:

1. perseverance,
2. taking risks,
3. collaboration.

Now think of 'listening for 30 minutes'. Will this quality make your top-3 list? Not mine, no. Honestly, I believe one can learn mathematics very well without even having this quality at all.

Altogether, I am convinced that it is not really a problem that the students cannot listen for too long. The problem is that we expect them to.

Writing dead notes

Ideally, we hope that the students listen actively and write notes. In reality, however, it is very hard for the students to listen to the teacher and write notes at the same time. I will again cite Peter Liljedahl [8], he talks about 'live notes' and 'dead notes'. The live notes are what the teacher writes on the board in real time while building up the story, they are the core strength of blackboard lectures. The dead notes are what is left on the blackboard after the story is finished. Unfortunately, when students try to make notes, they often lag behind, and end up copying the dead notes, without even understanding them. This is quite a mindless and not so useful activity (see also practice 11 on website [3]).

Sage on a stage

An obvious issue with a classroom lecture is that it is extremely teacher-centred. The teacher is the Sage-on-a-Stage. This is not ideal because students can learn mathematics only by doing it themselves. The learning process is inherently student-centred. The right place of the teacher is not the Sage-on-a-Stage, but the Guide-on-a-Side.

You may say, the students must receive some information first, before they can even start working themselves. This is right. My point is that the classroom time is not best spent on basic explanations. During the class, real-life students are right in front of you. This is your Guide-on-a-Side opportunity.

I am a convinced proponent of a flipped classroom: give students a good textbook and videos to prepare, and spend classes on interaction. Below in this article I will share some specific ideas how such interaction may look like. And don't worry, this will not deprive you from explaining the material in your own way. Quite

the opposite. In a lecture, most time goes into covering the standard material, leaving only little space for truly original insights of the teacher. In a flipped classroom, the students have already studied the basics, and you are free to plan the entire class around a few most interesting questions. You may ask, how to make sure that the students watch videos (or read a book) before the class? In Minerva [5] they start classes with small low-stake quizzes. I like this idea but honestly I didn't yet have time to do this. Currently I simply assume that the students are prepared. Very soon they realize that it makes no sense to come to the class without preparation, they simply cannot understand anything. I believe, by the way, that this, too, is a useful feedback for the students. If they lag behind, they should have no illusion about it.

On a somewhat higher level, I think, it's downright arrogant to believe that what exactly I say is oh so important for the students' bright mind. Recently I saw a tweet from a fellow mathematician: "When I make a mistake on the board, students learn the most." I find this typical Sage-on-a-Stage talking. And I don't agree either. The students learn most when they make mistakes themselves. When you make a mistake, they are mostly confused and have messed up their notes. My motto is: it's not important what I say, it's important what students do.

Students not thinking

When assessing effectiveness of learning methods, Peter Liljedahl and his team measure only one thing: how much time students spend thinking [8]. They estimate thinking time by closely observing the students and interviewing them after the class. I like this approach because it is so elegant and pure. Indeed, no learning of mathematics can happen without thinking!

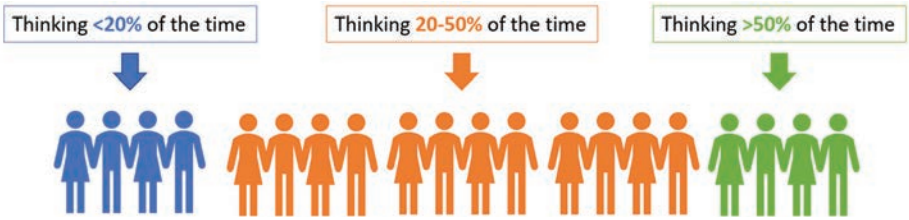

Figure 1 My own rough estimation of how much time students spend thinking in my lecture.

I imagine Liljedahl and his team sitting at the back in my lecture, observing the students. I wonder, what their results could be? Based on my own observations, I expect the results approximately as in Figure 1. This is not my dream situation. Obviously, I wish for more students in the green group. But unfortunately I suspect that my figure is too optimistic. Most probably, in reality, the blue group, that spends less than 20% thinking, is larger. What is your honest estimation for your lectures?

The problem is not a lecturing style. Personally I often receive compliments from the students for my lectures, and I won several teaching awards. According to Liljedahl's research, we cannot blame the students either. The students are merely humans reacting naturally to their environment. The problem is in the environment of classroom lectures. Besides the Sage-on-a-Stage setup and the lack of interaction, turns out, even neat rows of tables discourage thinking! (See practice 4 of the website [3].)

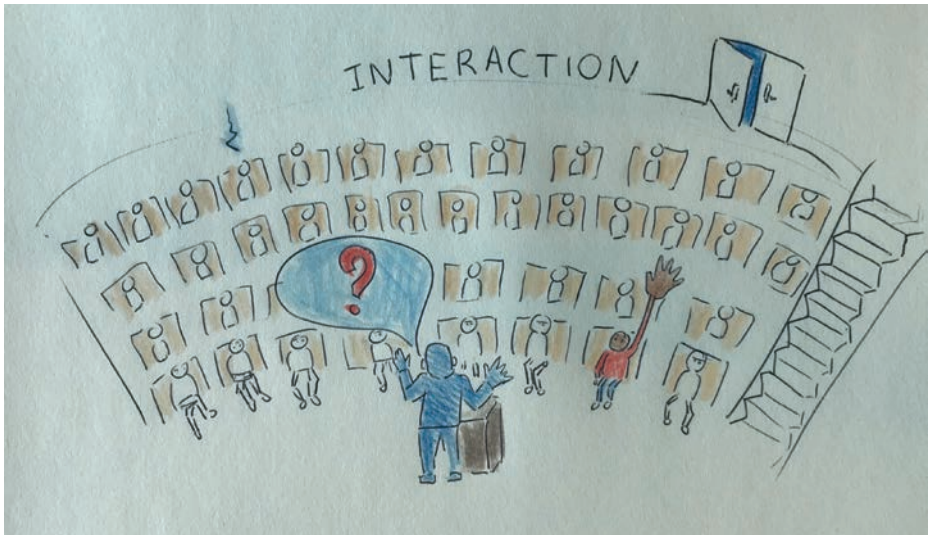

Illustration: Eline van Hove

Students at the back

We always have these students, leaning back passively, maybe looking at their phone, usually in the back rows. We may blame them for the lack of motivation but this is too easy and not necessarily fair.

To begin with, if a teacher doesn't use a microphone, then listening to the lecture is simply harder in the back rows. The teacher's voice dampens with the distance and interferes with classroom noises: turning pages, moving chairs, other students talking. The effort spent on merely hearing the teacher, goes on cost of cognitive capacity.

I believe however that there is more to the story than just hearing the teacher. I see the back rows as an embodiment of a larger problem in our higher education system — anonymity. Anonymity is opposite to agency and community. Agency means that students take responsibility for their studies, and academic community is a key value of the university. Anonymity means that students hide in the crowd, and the back rows are their hiding place. Agency and community are important topics by themselves, and we will come back to it in later articles.

Doubtful implicit goals: inspiration, general overview of the topic, et cetera

Many colleagues actually agree on all I said above. They say: "Yes, students don't learn much at a lecture, but lecture is also not for learning, it has different goals. It gives the students motivation for the topic, exposes them to inspiring professors, and gives general ideas on the subject…" I agree, these are noble goals, and I don't believe that a standard classroom lecture is the best way to achieve them. If we want to inspire, maybe better to give short inspirational talks? If we want to give general understanding, maybe better ask students to write summaries? And stating the question very pragmatically: are the inspiration and the general ideas in the form of a classroom lecture worth 1,5 hours per week of students' valuable time? I seriously doubt that.

This is just your opinion

I have explained the above arguments, and more, in different forms, many times, to many different people. I don't know what's wrong with me or with these arguments, but the most common reaction I get is: "This is just your opinion." I find this dismissive if not hurtful, and highly inaccurate, too.

Of course, a statement "lectures are ineffective" is not a hard mathematical fact as in "the determinant of an invertible matrix is non-zero". But it is also not an opinion as in "watching sitcom is relaxing". My opinion that classroom lectures are ineffective is not grounded in my personal taste. Quite the opposite, I love giving lectures myself, I miss them even. But I won't give them again, because I have read a lot about it, I intentionally tried many different things in my classes, and I came to the conclusion that lectures are ineffective. If you insist that this is just my opinion, then let's make it symmetric: your opinion is that lectures can be effective. Great. What else have you tried? What is your opinion based on? Below are most common arguments, and honestly I don't find them very strong.

"... I loved it when I studied. It worked for me." Yes. Me too. This is why you and I ended up teaching at university. Projecting our experience on today's students is a perfect example of survival bias, selection bias, and a problematic time translation.

"... We have always done it this way." Yes. So, what? We don't do almost anything the same way we did thirty years ago. Think about communication technology and healthcare. The fact that we didn't change our teaching habits for so long might not be an argument for change, but it is definitely not an argument against the change.

"... It's nice what you do. But do we all have to do this? Isn't variety good for students?" Yes, variety is good. But is this comment really about the variety? Sometime I feel that people who say this actually are saying, "Good for you. But I will continue in my old ways, and look, the students will benefit from the variety!" I want to answer this very directly: passive learning is not variety, it's an ineffective education. Will you use a computer from 1980s for the sake of variety? Will its terrible running time, memory overflow, and painful neon green font benefit you in any way? Good education requires active learning methods. And don't worry, active learning has plenty of variety.

"... Educational sciences have nothing to say about how I should teach my courses." I understand the sentiment. I felt the same when I was taking my teaching qualifications twenty years ago. And even now I keep a healthy dose of scepticism. I believe that educational sciences failed miserably in disseminating their knowledge to university teachers, and there is plenty of bad educational science, too.

Yet, I will stand by educational sciences and listen to them. And not only because there is a lot of excellent educational science such as Peter Liljedahl's thinking classroom. But mainly because in any complex human activity practitioners adopt their practices based on new science. For instance, quite recent results from mathematics are adopted in the practice of logistics, finance, cybersecurity, imaging, et cetera. Higher education is a complex human activity. I am a practitioner of higher education. Educational sciences are the only source of evidence-based scientific approach in my classroom practice. I don't have a choice but listen, this is the only sustainable way forward.

"... Students like the lectures." Yes, they do. Today's students like them even more than, say, thirty years ago. One explanation, offered by a colleague of mine, is that the quality of lectures has improved a lot. Long gone the times when a lecturer was standing with their back to the class mumbling something, scribbling messy formulas with no beginning and no end. Many of today's lecturers are skilled presenters, our lectures are simply very good! But the point is not whether students like the lectures. The point is how much students learn from the lectures. And even if they believe they learned a lot, can we really trust their judgement?

Here is a fantastic paper [7] published in 2019 in PNAS, a journal respected in all areas of science. The authors conducted a very clean experiment. They gave students two sorts of classes. One was a standard lecture. Another one was a class where students worked on a question first, and then received explanation from the teacher. In evaluation, on the questions like "I learned a lot" and "I wish all my classes were taught this way", standard lecture clearly wins. Except, the test results came out exactly the other way around! The authors suggest three explanations to this, and I think you will recognize all three. First, the students confuse the fluency of the teacher with their own fluency. They have an impression that if they could follow the explanation, they can explain, too, but this is simply not true. Second, students are novice to the topic, so it is really hard for them to evaluate how much they really learned. And third, when students work on a new difficult question, they make errors, get stuck, and therefore feel that they learn nothing. But exactly the opposite is true: they learn most exactly when they make errors and get stuck!

By the way, similarly, students believe that they learn more from a lecturer who is fluent and has great presence, rather than from one who is not so fluent and avoids eye contact, but study [6] found that appeal of a lecture had no effect on retention.

It is not so surprising that students are often simply wrong about what works for them in a class. Daniel Kahneman has received the Nobel Prize basically for demonstrating that humans don't like to think and are very poor in evaluating their own cognition. What students like is not necessarily effective for their learning.

What can we do instead of lectures?

I believe that we often hold on the lectures simply because we don't exactly know what to do otherwise, and our academic jobs are so terribly busy that we have truly no time to figure it out. I will write a lot about active learning in further articles, but for now I will explain two simple ideas.

Quiz

In my courses, I have all material covered in videos, and in the class, instead of lectures, I do quizzes. Usually I prepare 5–6 questions, that emphasize fundamental concepts, and common difficulties. Maybe somewhat unexpectedly, multiple choice questions often serve these purposes very well. In the class I let students answer the questions anonymously on their devices using an online tool. University of Twente uses Wooclap (and I love it, it is very easy), but every university has some tool at their disposal. I like to use questions that students

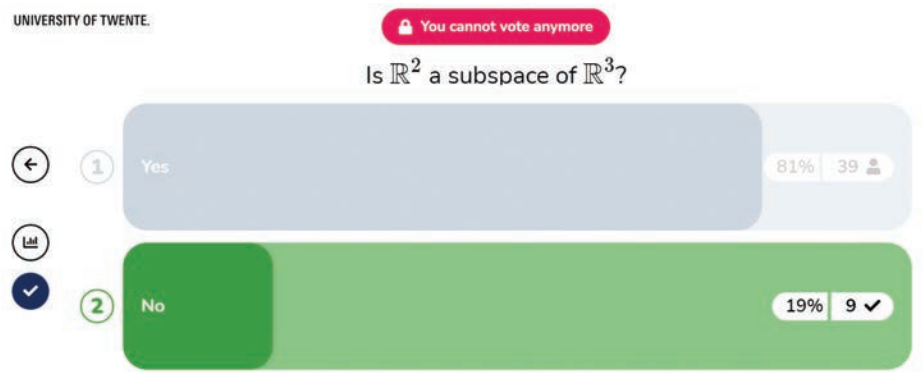

Figure 2 First year students in applied mathematics, in the second week of their study, learning about subspaces by making a quiz.

often answer wrongly because I want them to make errors in a safe way, and learn from these errors. For instance, every year, in the second week of their studies, 80% of my first year mathematics students vote $\mathbb{R}^2$ to be a subspace of $\mathbb{R}^3$, see Figure 2.

After students have answered the question, I usually do the simplest thing: I explain which answers were wrong, which were correct, and why. This part actually resembles a lecture, although there are differences. First, since the students already tried to answer the question themselves, they want to know the answers. They usually listen well and often ask further questions. Second, I see from the answers what was difficult, and I can spend more time there.

This year in evaluations some students wrote that they didn't find quizzes useful because "there was no new material". However, judging by plenty of wrong answers in the quiz, I have enough reasons to believe that quizzes are very useful.

Quizzes have many forms with more interaction. The innovative university Minerva [5] has a standard quiz format that works as follows. (1) Let students answer a question individually. (2) Let students discuss in groups. (3) Let students answer the question again. (4) Discuss with the entire class and the teacher. There is also a similar method with game elements called Team Based Learning [9] that works as follows. (1) Students individually answer several multiple choice questions, each with four possible answers. (2) Students discuss all questions in groups. (3) Each group chooses the answers. If they answer a question correctly in the first attempt, they get 4 points. If not, they try again. In they answer correctly in the second attempt, they get 2 points, and in the third attempt 1 point. The points may be used for grade, but not necessarily, it can be just a fun way of getting feedback.

Creating questions

This idea comes from a colleague in Germany, I didn't use it myself yet. This teacher, too, has all her material on videos. In the class, she divides students in groups and asks each group to come up with a question about the material. Then they write these questions on a board and discuss together. She says it works great in her MSc courses. In BSc courses, however, it didn't work very well.

Will blackboard lectures disappear?

No, of course not, what a ridiculous question! A blackboard lecture is a craft perfected by mathematicians through centuries. It will stay with us forever. But not per se in the same function and context. Think of listening to a music. In early days, everybody had vinyl disks. Now most people use digital devices, but vinyl disks didn't disappear, they became exclusive. Likewise, blackboard lectures may (and should) give place to active learning in everyday classrooms, but they will not disappear, they will become exclusive events.

A masterful blackboard tutorial is perfect for a specialized workshop, with motivated fellow researchers in the audience. I saw many such lectures and enjoyed them a lot. In education, too, videos of blackboard lectures by teachers like Frederic Schuller or MIT's Gilbert Strang [4], are invaluable. Students need these explanations before active learning can even start. I believe that creating such series of videos is comparable to writing a textbook or shooting a show. In fact, we may also invite a live studio audience like in (my favourite) sitcoms. And you know what? For some of these lecture recordings, I will gladly buy a ticket!

Replacing classroom lectures by active learning doesn't mean the end of lecturing. Rather, it is a new beginning for the lecture genre and for the future-proof education of today's students.

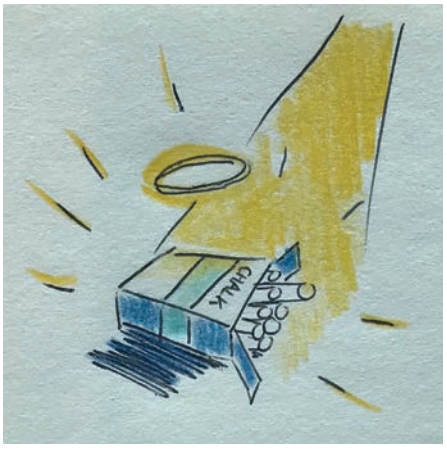

Illustration: Eline van Hove

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