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Event **Abel Prize 2022**

Dennis Sullivan is awarded the 2022 Abel Prize

Last year, the King of Norway awarded the 'Abelprisen' to Dennis Sullivan, who is most famous for his remarkable work on topology and dynamical systems. One of the hallmarks of Dennis' works is his emphasis on geometric structures, and his ability to see and exploit connections between a wide range of areas in mathematics. In this short note Sebastian van Strien and Edson de Faria look back on his career.

Dennis Sullivan was born in 1941 in Michigan, but grew up in Houston Texas where he stayed to study chemistry at Rice University. Although he always liked mathematics, he once told us that he initially did not know it was possible to have a career in mathematics. But in 1963 he did graduate in mathematics. Three years later he defended his PhD thesis *Triangulating Homotopy Equivalences* at Princeton. His 1967 paper on the Hauptvermutung, an important conjecture in geometric topology, led to his 1971 Veblen Prize by the American Mathematical Society.

In 1974 Dennis gave a plenary talk at the International Congress of Mathematicians in Vancouver, and became a permanent member of the Institut des Hautes Études Scientiques (IHÉS) in Paris. In 1981, Dennis became the Albert Einstein Chair in Science (Mathematics) at the Graduate Center, City University of New York, and reduced his duties at the IHES to a halftime appointment. In 1996 Dennis joined the mathematics faculty at Stony Brook University, where he is to this day.

Dennis has made huge contributions to a diverse range of topics in mathematics, often changing the focus of these fields entirely. His early work on geometric topology led to the creation of rational homotopy theory and his proof of the famous *Hauptvermutung*. In the late 1970's Dennis started focussing on the study of Kleinian groups. In the 1980's Dennis switched to the field of dynamical systems, proving a remarkable theory that rational maps do not have wandering domains and also giving a proof of Feigenbaum–Coullet–Tresser Renormalisation conjectures. He also had major contributions to string topology.

Photo: abelprize.org / Peter Bagde, Typos1

Dennis Sullivan

Rather than giving a summary of these mathematical achievements, for the remainder of this short note let us focus on the remarkable way Dennis influenced mathematicians through his outgoing personality. (Here we use the introductory section of our chapter on Dennis' contributions in dynamical systems for the fourth volume in a book series on the Abel Prize Laureates.)

Influencing other mathematicians

Both while at IHES and at CUNY, Dennis had an office which came with an anteroom. Our impression is that he would spend most of his time in this anteroom, talking about mathematics with whoever

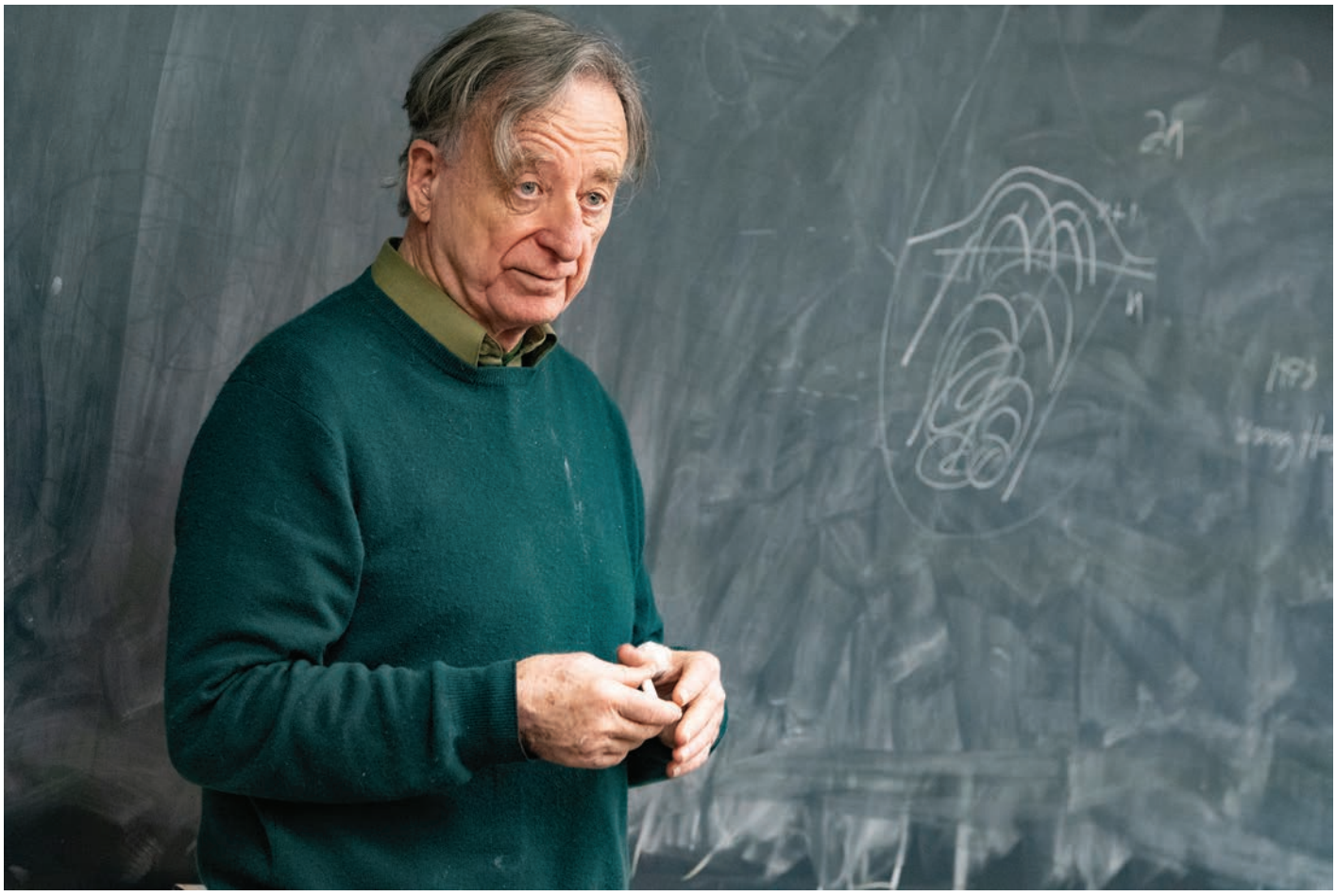

Photo: abelprize.org/John Griffin, Stony Brook University

Dennis Parnell Sullivan, 2022 Abel Prize laureate

he had invited or whoever was around. Quite often while listening to somebody, he would end up giving a new spin or a new interpretation to what they had been saying. Similarly, he would explain what he was working on, trying out new ideas, and also often explaining results of others. Spending time with him was always an incredible experience.

In this spirit, Dennis explained much of his work on renormalisation to Welington de Melo. In turn, Welington would then try to explain what he had heard and learned to SvS. When he could not convince SvS of some argument, Welington would go back to Dennis and this process would repeat again, sometimes many times. This is how the final chapter of the book *One Dimensional Dynamics* of SvS with Welington de Melo came into being. This chapter contains a full exposition of Dennis' remarkable renormalisation theory, arguably the only place in which it was published.

During talks, Dennis often asked questions not necessarily to know the answer himself, but because he knew that somebody else in the audience would find the answer helpful. In this way, Dennis took on the role of introducing two people to each other. His presence in the audience would usually make a talk much more accessible and interesting. His questions would often clarify connections that would have remained implicit otherwise.

When Dennis invented or learned about a new mathematical idea, he would push this idea to the limit. For him it was very important to understand what this idea would give, and equally important to find out what the limitations of this idea might be. Moreover, whenever possible, he liked to associate names to arguments such as the *dollar argument, smallest interval argument* or the *non-coiling argument* in order to synthesise a complex proof into its core ideas.

Often he mentioned that to understand a proof properly, you should treat it like a three dimensional object. You should not only look at it from one side, but from all sides. So in this sense, in his view, a proof was about mathematical understanding rather than about 'killing' a theorem.

Indeed, Dennis' interest in a result might not necessarily be in the power of the statement per se, but in the tools that are used in the proof of this result. Once he understands the tools and ideas, then he probably can recover the statement of the results by himself.