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Proof by example Portraits of women in Dutch mathematics

Palina Salanevich

In 'Proof by example', Clara Stegehuis and Francesca Arici portray women in Dutch mathematics. This edition portrays Palina Salanevich, assistant professor at Utrecht University. She investigates mathematical properties of signal processing problems that arise from optics, acoustics or speech recognition. In this interview she tells about her research and her motivation for mathematics.

How and when did you first get interested in mathematics?
"In school, I performed well in several subjects. But my mathematics teacher actually put in effort to give me some interesting extra mathematical problems, for example about logics. I liked this very much. From there I took the route which is quite standard in Belarus if you want to have access to extra math education: compete in math Olympiads. Once I made it to the national round, I was invited to a preparation retreat with all candidates. There I really felt like I fit well into this community. So I chose to study maths not only because I enjoyed it, but also because I really liked the people."

And how did you get into research from there?
"After my first year of university, I was invited to participate in a mathematical summer school. During this school, prominent mathematicians gave lectures on advanced math topics, and we also had opportunity to work together on research projects. I really enjoyed this much better compared to Olympiads: there was no time pressure, you can collaborate and you can apply much more creativity in these research problems. From there on, I always worked as a research assistant, until I finished my studies and continued in academia."


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So what is the research area you are working in now?
"In general, my research is in the area of signal processing and applied harmonic analysis. In many applications, *frames* are used to describe a signal. Frames are similar to the encodings of vectors in a basis of a vector space. But compared to a vector basis, frames allow for redundancy in the signal representation. This compensates for the loss of information caused by possible noise in the signal. In this way, frames form a stable, redundant way to encode signals.

One of the things my research focuses on is the *phase retrieval problem*, where we would like to reconstruct the original signal from measurements that are provided in the form of a frame. However, the measurements only provide the absolute values, or intensities, of the frame vectors, not the entire vectors. This problem actually arises in many applications. For example, in speech recognition, the frame vectors are usually so noisy that using them instead of their absolute values can prevent correct signal reconstruction. So even though I am working in theoretical mathematics, the problems I am working on are very much inspired by real applications."

What are the main mathematical challenges in this problem?
"Mathematically, phase retrieval leads to optimization problems that are typically non-convex, and therefore difficult to solve and analyze. If you use non-convex optimization, you often get stuck in local optima, which you would like to avoid. In my research, I study the properties of the optimization problems arising from phase retrieval using probability theory and harmonic analysis. Can we prove that signal reconstruction is possible for a specific set of measurements? And if the measurements contain some noise, can we still reconstruct the original signal accurately?"

What is the most interesting problem you have been working on recently?
"Recently, I have been investigating signals on graphs. Typically, we assume that signals are measured over time with a certain step size. This gives an easy structure on which we can apply time shifts, Fourier transforms and more. However, in some applications, the underlying domain is more complicated. For example, you may have several correlated measurements at the same time. This creates an underlying graph structure. Take for example weather prediction: weather stations are placed in different locations, but the measurements at neighboring stations are dependent. Analyzing signals on such graphs is much more difficult, because the standard Fourier methods do not immediately work on graphs. The questions I have been working on are: 'Is there an uncertainty principle for signals on graphs? And how does this depend on the graph structure and its eigenvalues?' In general, an uncertainty principle tells that you can only predict the position and speed of a particle at the same time within a limited accuracy even with many measurements. Similarly, for signals the uncertainty principle gives a bound on how accurately you can localize a signal on the time domain and the frequency domain at the same time. But what would an uncertainty principle look like for measurements on graphs? Knowing this is for example important if you do not want to measure all data, but rather sample only some measurements to work with."

And is there a particular result you are you most proud of?
"Recently I have worked on *frame order statistics* — a property that illustrates how well frame vectors cover all directions in a particular space. Most research in my area assumes that all frame vectors are independently drawn from some convenient distribution. This assumption makes it easy to prove many properties using probability theory. In most applications however, this assumption is not realistic at all. In acoustics for example, signals are usually represented by perturbations of one single vector, so that the frame vectors are extremely dependent. In these situations, we really needed new mathematical techniques, as virtually all existing ones assume this independence. So I am very happy that I was able to show that frame-order statistics provide a key to still derive properties of the phase retrieval problem in more application-relevant settings."

What do you like most about doing research?
"I love the sense of discovery. These moments when you finish a proof, and everything falls together. They do not happen every day, but they are definitely worth it. But also, I really like the connectedness of the mathematical community. It is not competitive at all, and we all like to solve new mathematical problems with a community spirit."

And what do you like less about doing research?
"In my particular area, many problems look very simple on first sight: they can be formulated in simple terms. However, they actually lead to very complicated mathematical questions. So at conferences, you are often approached by others who are less familiar with the particular question. They then often offer me an 'easy solution' to my problem using a method they know of. Then I have to explain them why the method they have in mind has been tried before, but was not powerful enough. Of course, I do not mind explaining, but sometimes it can be a bit frustrating when other scientists seem to think that I am making my research much more complex than necessary."