Boekbesprekingen

| Book Reviews

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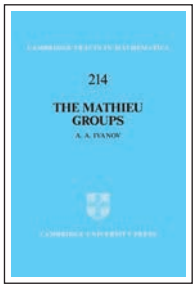


A.A. Ivanov

The Mathieu Groups

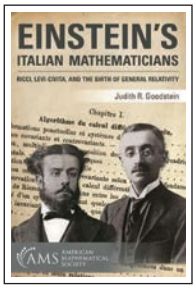
Cambridge Tracts in Mathematics 214
Cambridge University Press, 2018
xi + 171 p., prijs £95.00
ISBN 9781108429788

In het jaar 1873 (inderdaad, honderd-en-achtenveertig jaar geleden) zag het artikel 'Sur la fonction cinq fois transitive de 24 quantités' het licht in het *Journal de Mathématiques Pures et Appliquées* 18, pp. 25–46. De auteur was Émile Léonard Mathieu (1835–1890). Niet alleen 'zijn' groep M_{24} werd daarin geboren, maar ook over de kleinere M_{12} is daar het nodige te vinden. De groep M_{24} (met $2^{10} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$ elementen) opereert vijfvoudig transitief op een verzameling van 24 symbolen. In een kort voorwoord zet Ivanov zijn bedoelingen betreffende het boek uiteen. Namelijk, dat M_{24} en diens ondergroepen deel uitmaken van de zogenaamde Ivanov–Shpektorov-theorie van meetkunde, zie *Nagoya Mathematical Journal* 176 (2004), pp. 19–98. Die theorie is ontwikkeld om grote sporadische simpele groepen te identificeren en te construeren; men denke aan het Baby Monster, de vierde Janko-groep en het Fischer–Griess Monster. De theorie droeg ook bij aan het definitieve bewijs van het zogeheten Y-vermoeden voor het Fischer–Griess Monster, lang ontoegankelijk via andere technieken. Wat betreft M_{24}, de bewuste theorie verbindt deze met bekende structuren van Steiner-systemen op 24 punten en de Golay-code, aldus resulterende in een frisse, nieuwe en vernieuwende kijk op bekende oudere zaken en structuren.

Beste lezer, bovenstaande regels geven een vrije vertaling weer van vrijwel het gehele voorwoord van Ivanovs boek. Nu wat dingen als smaakmaker. In hoofdstuk 1 staat dat M_{24} optreedt als automorfismengroep van de heden ten dage welbekende Golay-code. Het boek is doordrenkt van zogenaamde 'incarnations', dat wil zeggen een gegeven abstracte eindige groep wordt onder de loep genomen via verschillende zienswijzen en daaruit worden dan structuren opgediept. Denk bijvoorbeeld aan de Cayley-representatie van zo'n groep, aan trouwe met zo'n groep isomorfe matrixgroepen, aan het optreden van zo'n groep als permutatiegroep op een zeker aantal symbolen, en wat dies meer zij. Zogeheten 'toevallige isomorfismen' worden opgespoord, ontrafeld en verklaard. Bijvoorbeeld: het is al lang bekend dat de groepen $\mathrm{PSL}(3,2)$ en $\mathrm{PSL}(2,7)$ isomorf zijn, men zie onder andere Satz II.6.14(4) in Hupperts boek *Endliche Gruppen, I*. Maar een betreffende isomorfie afbeelding vinden en maken is een heikele kwestie! Aanvankelijk weten we namelijk dat elk van de simpele groepen $\mathrm{PSL}(3,2)$ en $\mathrm{PSL}(2,7)$ 168 elementen heeft, maar aantonen dat er (op isomorfie na) precies één zo'n groep met die orde 168 bestaat is een ander verhaal; zie de details daaromtrent in Satz 6.15 in Hupperts boek. En wat betekent die isomorfie nu eigenlijk? Ivanov vertelt het u. Ook bespreekt hij in hoofdstuk 3 een bijzonder automorfisme van het semidirecte product $2^3 : \mathrm{PSL}(3,2)$, waarbij $\mathrm{PSL}(3,2)$ op een elementair abelse groep van orde 8 opereert onder conjugatie-actie. En, wat verderop, te denken van sectie 3.5: 'Another look at the isomorphism between $\mathrm{PSL}(3,2)$ and $\mathrm{PSL}(2,7)$'? Prachtig allemaal, maar niet eenvoudig. In hoofdstuk 3 wandelen we ook via de alter-

nerende groep A_7 op 7 symbolen en het Fano-vlak naar sectie 3.7, geheten (vertaald): '$\mathrm{PSL}(4,2)$ is isomorf met de alternerende groep A_8 op 8 symbolen'. Verderop in hoofdstuk 3 komen dan 'point stabilizers' aan de orde, alsmede het zogeheten 'outer automorphism' van de symmetrische groep S_6 op zes symbolen, ook zo'n ding dat heel bijzonder is. In hoofdstuk 6 vinden we beschrijvingen van de maximale ondergroepen van M_{24}, en in hoofdstuk 8 een up-to-date gebrachte beschrijving van de Held-simpele groep He, genoemd naar Dieter Held (1936), die hem vond bij de classificatie van de verzameling van de eindige simpele groepen waarbij de centralisator van een involutie isomorf is met die van een transvectie uit $\mathrm{PSL}(5,2)$. Die verzameling bestaat uit de groepen $\mathrm{PSL}(5,2)$, M_{24} en He.

Het boek bestaat uit tien hoofdstukken en is overal doordesemd van constructies en optredende verbanden, zoals bijvoorbeeld ook met grafen, amalgamen, en universele completeringen. Het boek is te zien als een encyclopedie met uitgewerkte feiten. Zelfs $\mathrm{K3}$-oppervlakken worden daarbij niet vergeten. Definities van begrippen, op welk niveau dan ook, zult u nauwelijks aantreffen. Ik vond het boek uniek in zijn soort; geschikt voor specialisten en (zeer) gevorderde studenten; vergelijk het boek ook eens met M. Aschbachers boek *Sporadic Groups*, ook uitgegeven bij Cambridge University Press. Maar als men zich in Ivanovs boek wil verdiepen, kijk dan in de index en zoek alle aldaar vermelde begrippen elders eerst op, op een eenvoudiger niveau. Dat verrijkt ongetwijfeld uw kennis en waardering voor de materie. Doe zulks ook door de bronnen in de 'References' ter hand te nemen. *Rob van der Waall*



Judith R. Goodstein

Einstein's Italian Mathematicians: Ricci, Levi-Civita, and the Birth of General Relativity

American Mathematical Society, 2018
xiii + 211 p., prijs $35.00
ISBN 9781470428464

In November of 1915 Albert Einstein found his famous equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \varkappa T_{\mu\nu}$$

for the Lorentzian metric tensor field of physical spacetime (although in his original version the cosmological constant Λ was taken zero). This enabled him to explain accurately the observed discrepancy of 43 arcseconds per century in the perihelion precession of Mercury. Einstein's work is rightfully considered a landmark in the history of physics, comparable with Newton's work on classical mechanics (deriving for example the three Kepler laws for planetary motion) or the fundamental work on quantum mechanics by Heisenberg, Born, Dirac, Pauli and Schrödinger (explaining for example the Bohr–Sommerfeld quantization rules for the hydrogen atom).

The mathematical language Einstein needed for general relativity was the differential geometry of Gauss (his Theorema Egregium for surfaces in Euclidean space) and Riemann (a purely intrinsic approach for curvatures of metric spaces of arbitrary dimension). The habilitation lecture by Riemann in June of 1854 is like a beautiful impressionistic painting. It is brilliant, but there were no proofs and also hardly any formulas, partly due to the restrictions imposed on him to present a lecture for a fairly general audience.

In the autumn of 1858 the Italian mathematicians Enrico Betti, Francesco Brioschi and Felice Casorati (the former two mathematical friends and the latter student and assistent of Brioschi) traveled through Europe, visiting the mathematical centers of Berlin, Göttingen and Paris. The relationship between Betti and Riemann was deepened a few years later, when Riemann stayed for a longer period in the Mediterranean Pisa seeking relief for his declining health. The contact between Betti and Riemann has been a trigger in the formation of the famous Italian school of algebraic geometry (with illustrious names like Castelnuovo and Enriques), but likewise the Italian differential geometry was going to flourish (with protagonists like Ricci and Levi-Civita). Thus is suggested from the (partial) mathematical genealogy given in Figure 1.

The habilitation lecture of Riemann was published posthumously in 1867, and initial steps towards further understanding of what is now called Riemannian geometry were taken by Elwin Christoffel in 1869 before Ricci, Levi-Civita and Bianchi later joined. In 1875 Ricci completed in Pisa his dissertation on Fuchsian differential equations with Dini and a year later he finished his habilitation with Betti on a problem posed by Riemann concerning hypergeometric functions, as Levi-Civita writes in the obituary for Ricci. In 1878–1879 Ricci went to Munich as a postdoc with Felix Klein, and after another year in Pisa he got in 1880 a position as associate professor in Padova, where he was going to stay the rest of his life. The research of Ricci from that time on focuses on *The Absolute Differential Calculus (The Calculus of Tensors)* or *Der Ricci-Kalkül*, as the titles of the later text books by Levi-Civita from 1926 and by Schouten from 1924, respectively, used to call it.

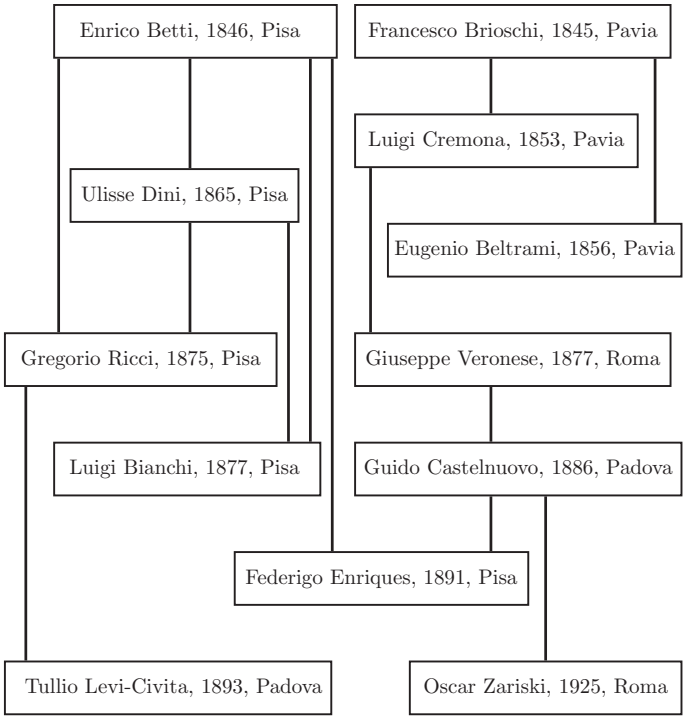


Figure 1

"The Ricci tensor was introduced by Ricci in 1904 with the hope that the integral curves for the principal directions of the Ricci tensor relative to the metric tensor would yield nice curves on the manifold, analogous to the lines of curvature on a surface in $\mathbb{R}^3$. For abstract Riemannian manifolds however Ricci was quite disappointed to find no good geometric interpretation of these Ricci curvature lines", according to Marcel Berger in his *Panoramic View of Riemannian Geometry* of 2003. The differential (or second) Bianchi identities for the Riemann tensor were found (but not published) by Ricci in 1888, and were later rediscovered and published by Bianchi in 1902. In fact they had already been found before by Aurel Voss in 1880. The contracted Bianchi identities for the Ricci (or rather the Einstein) tensor were an important ingredient in understanding the ultimate form of the Einstein equation, because of the conservation of the energy-momentum tensor, as explained by Einstein in *The Meaning of Relativity* from 1921.

Levi-Civita wrote his dissertation on absolute invariants with Ricci in 1893. He was the only PhD student that Ricci would have. In 1897 Felix Klein invited Ricci to write a survey paper on absolute differential calculus for the *Mathematische Annalen*, which in fact appeared in collaboration with Levi-Civita in 1900. It would be the only joint paper for Ricci. In 1898 Levi-Civita was appointed as full professor in Padova, where he stayed until 1918 when he moved to Rome. Levi-Civita had a broad interest in mathematics and did for example important work in celestial mechanics (the Levi-Civita regularization for collisions in the three-body problem).

The focus of the book by Goodstein under review is primarily on the personal lives of Ricci (1853–1925) and Levi-Civita (1873–1941). The overall impression of Ricci is a family man (married with Bianca Bianchi in 1884, they got three children) and mathematically an original but also a rather introvert nature. Levi-Civita was mathematically the more extrovert one, with many international contacts, notably with Einstein in that remarkable year 1915, in a certain sense as successor of Marcel Grossmann, who had helped Einstein a few years before with mastering the necessary differential geometry (leading to the Entwurf theory). Levi-Civita married fairly late in 1914 with his former PhD student Libera Trevisani, who did work on the three-body problem. Despite her wish they did not have children together.

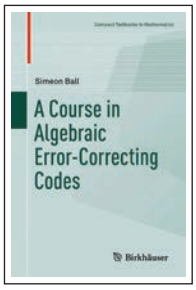
During his life Ricci had a rather difficult time getting recognition for his mathematics. He became full professor fairly late (age 37), and he engaged in several prize competitions, but was never successful. Levi-Civita had an easier start with his career and became full professor at the early age of 26. He travelled abroad quite often, and he had many international contacts. But things changed for both men towards the end of their lives.

In 1921 Einstein was invited by Enriques to lecture on general relativity. His three lectures took place in the prestigious University of Bologna. A few days later Einstein took the train to Padova, and gave a fourth lecture in the presence of Ricci, who had not been able to attend his talks in Bologna. "Introduced by Ricci, Einstein spoke for an hour, slowly, accurately, in Italian, with a scientific precision", according to the local newspaper *Il Veneto*. Levi-Civita makes in his obituary for Ricci the comparison between Apollonius of Perga and Ricci. "The work of Apollonius on conics had to wait eighteen centuries before Kepler, Galileo and Newton made it of use for physics, while Ricci was more fortunate, in that he was able to witness the superb way in which his theories were put to work."

For Levi-Civita the last years of his life were very difficult. In 1938, due to the racial laws of the Italian fascist regime of Mussolini, Levi-Civita was banished from the university for being Jewish. He found himself cut off from all that made life interesting to him. "The thing that left the strongest impression on my mother", Libera Levi-Civita's daughter Susanna Silberstein Trevisani (adopted by Libera in 1945) said decades later, "is that many mathematicians, even those with whom there had been a close bond of friendship, had ceased all contacts with Levi-Civita, as if they had never known him." This was a hard blow for Levi-Civita, his health started declining and he died in December of 1941.

Susanna Silberstein was born probably in 1942, and died recently in April of 2020 in Rome according to her Memoria. Her entire family was murdered in Auschwitz in 1944. Through her Judith Goodstein got access to the scientific correspondence and private documents of Levi-Civita, as mentioned in the preface of the book.

I can certainly recommend the book by Judith Goodstein to those who are interested to read more about the personal lives of Gregorio Ricci and Tullio Levi-Civita. On their mathematics I found the book somewhat meager. But for example the obituaries for Ricci by Levi-Civita (luckily translated into English for people like me) and for Levi-Civita by William Hodge, added to the book in appendices, helped to compensate for that. *Gert Heckman*



Simeon Ball

A Course in Algebraic Error-Correcting Codes

Birkhäuser, 2020
xiii + 177 p., prijs €43,59
ISBN 9783030411527

The book offers an outstanding first course in algebraic coding theory and makes a very valuable contribution to the existing literature. It is conveniently organized into ten short chapters, each of which covers a different topic and has very distinctive learning objectives. The dependencies among chapters are summarized in a flow diagram at the beginning of the book, which greatly facilitates the lecturer in the course design process.

While the book was mainly designed as a first course, three of the ten chapters cover more advanced topics (codes from algebraic geometry, low density parity check codes, and codes over rings). These chapters can be efficiently used to spice up lectures or to encourage independent reading among students, making the book very 'flexible' and suitable for a large variety of curricula and for a broad readership. The students can easily determine which chapters can be read in autonomy thanks to the dependency diagram in the preface.

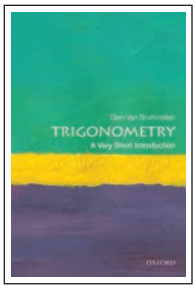
The book opens with a concise chapter on Shannon information theory, introducing the notions of entropy, channel, mutual information, and capacity. These foundational and fascinating concepts, which are often neglected in courses on algebraic coding theory, serve as a strong motivation for the study of error-correcting codes. They also show the link between probabilistic and alge-

braic methods in information technology, implicitly stressing the importance of interdisciplinary approaches to applied research and helping the students develop an open mindset.

The chapter on finite fields (Chapter 2) is particularly handy to ensure that students have the necessary background to understand and appreciate the theory of error-correcting codes. Even at universities where algebra dominates the undergraduate curriculum in mathematics, students greatly benefit from a recap on finite fields before seeing them into action.

The book is very clearly written and revisits even classical topics in an original way, offering concise and neat proofs. Both students and lecturers will greatly benefit from the clarity of exposition.

The author, Simeon Ball, is a prominent expert in coding theory, finite and real geometry, semifields, graph theory, and in the interactions among all these research areas. *Alberto Ravagnani*



Glen Van Brummelen

Trigonometry

A Very Short Introduction

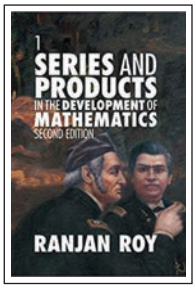
Oxford University Press, 2020
xxii + 163 p., prijs £8.99
ISBN 9780198814313

Dit boekje (het is inderdaad klein van formaat) is verschenen in de reeks *Very Short Introductions*, een reeks waarin al honderden titels op allerlei gebied zijn verschenen. Op wiskundegebied gingen in de reeks diverse wiskundigen Van Brummelen voor, zoals Timothy Gowers en Ian Stewart. Het boek is geen schoolboek goniometrie, zoals de titel misschien suggereert, maar een boek dat aan de hand van (wetenschappelijke) vragen uit het verleden ingaat op de rol en ontwikkeling van goniometrie en goniometrische functies. Het schakelt daarbij soepel tussen wiskundige en historische aspecten. De wiskundige uiteenzettingen schuwen de details niet en zijn geschreven in een heldere en goed toegankelijke stijl, ondersteund met vele illustraties. Bij elk hoofdstuk geeft de auteur per deelonderwerp aan waar de lezer verdere informatie kan vinden.

Het boek start met problemen bij Hipparchus (verduisteringen), Bressieu (de hoogte van een toren) en Sir William Thomson/Lord Kelvin (getijden), en bespreekt in de daarna volgende hoofdstukken de ontwikkeling van de relevante delen van de goniometrie, consequent de historische setting erbij betrekkend: onder meer goniometrische formules, de rol van Taylorreeksen, de relatie met complexe getallen, bolmeetkunde, doorgaans inclusief elegante afleidingen.

Van Brummelen heeft al enkele uitvoeriger werken op zijn naam staan over verwante onderwerpen en mag wel tot de experts gerekend worden. Het is opmerkelijk hoe informatief dit boek in zo'n kort bestek weet te zijn. Ik denk dat het vooral van direct nut is voor leraren wiskunde. Het biedt hen op toegankelijke wijze (historische) achtergrond en context bij de leerstof waar ze vaak mee bezig zijn. In dit verband is vermeldenswaard dat de auteur bespreekt hoe rekenmachines goniometrische (functie)waarden berekenen. Ook kan het boek als ideeënbron voor leerlingenwerkstukken dienen. Het is voortreffelijk uitgegeven door Oxford University Press voor een uiterst vriendelijke prijs. *Hans Sterk*

Recent verschenen publicaties. Als u een van deze boeken wilt bespreken of als u suggesties heeft voor andere boeken voor deze rubriek, laat dit dan per e-mail weten aan reviews@nieuwarchief.nl.



Ranjan Roy

Series and Products in the Development of Mathematics

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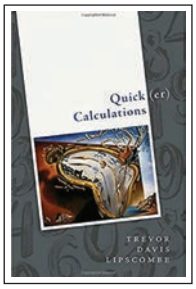

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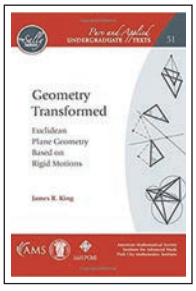

James R. King

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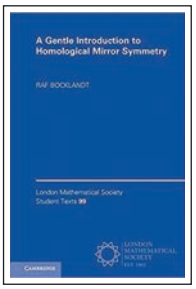

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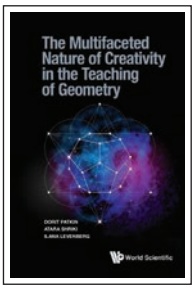

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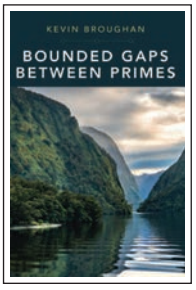

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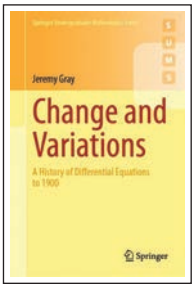

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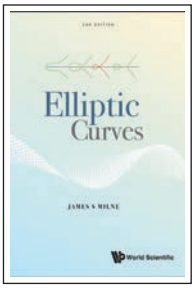

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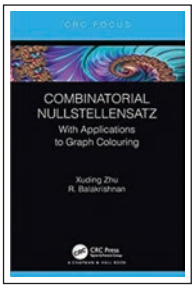

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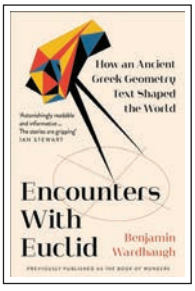

Benjamin Wardhaugh

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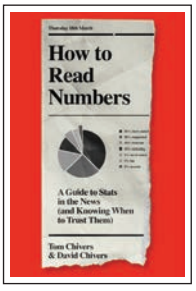

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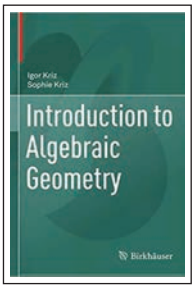

Igor Kriz, Sophie Kriz

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