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Event

The KWG PhD Prize

Last October the Royal Dutch Mathematical Society (KWG) hosted the PhD Prize competition. Normally this event would have taken place during the Dutch Mathematical Congress (NMC) in April but due to the corona outbreak this meeting was cancelled and the competition postponed. As the prospects of organizing a real live event in the autumn slimmed, the organizers decided to hold a virtual zoom-meeting instead. On 6 October five candidates for the prize, selected from a longlist of fifteen, were each given the opportunity to present their work in a twenty minute online public talk followed by a five minute question and answer session. Based on their performances a jury of four mathematicians (Kees Oosterlee, Maria Vlasiou, Ronald de Wolf and Raf Bocklandt) awarded the prize to Emiel Lorist for his clear presentation, fitting use of multimedia and good handling of the questions. The jury was impressed by the quality of all five presentations and to give you an idea of the broad range of interesting topics that were covered, we asked the candidates to provide us with a nice picture drawn from their slides accompanied with a little explanation.

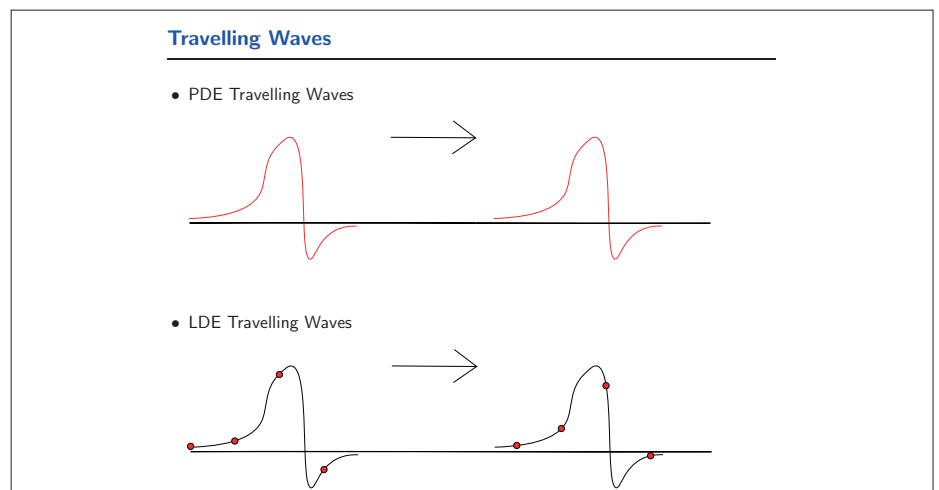
Willem Schouten-Straatman

Willem Schouten-Straatman finished his PhD under the supervision of Hermen Hupkes at the University of Leiden in the field of lattice differential equations. These differential equations are used to describe the dynamics of semiconductors, liquid crystals and networks of interacting neurons. In his thesis he established the existence and stability of travelling wave solutions for the

discrete FitzHugh–Nagumo equation.

Schouten: “Here you see a schematic view of travelling waves for a partial differential equation (PDE), i.e. a spatially continuous setting, and for a lattice differential equation (LDE), i.e. a spatially discrete setting. Even though the wave profile for LDEs remains continuous, we note that at fixed points in time we only see a discrete subset of the wave profile. In my

research, we established the existence and nonlinear stability of travelling wave solutions for several types of FitzHugh–Nagumo LDEs. These systems aim to model the propagation of electrical signals through nerve fibers. We constructed these waves by viewing them as perturbations of simpler systems, such as the corresponding FitzHugh–Nagumo PDE, for which the existence of travelling waves is already known.”



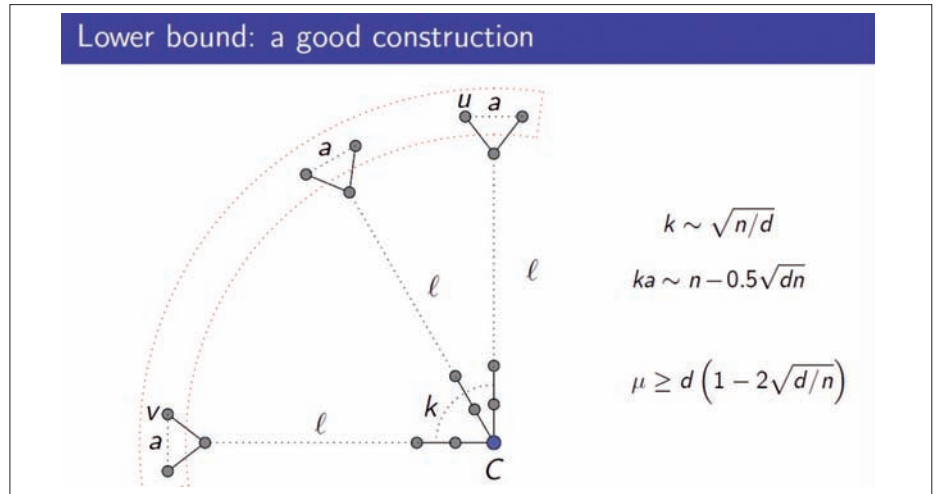
Stijn Cambie

Stijn Cambie is working on a PhD with Ross Kang at the Radboud University in Nijmegen. His main subject is graph theory and in his first year he gave an asymptotic solution to a longstanding problem by Jan Plesník about average distance in graphs.

Cambie: “The problems presented originated from 1984. They asked for the

minimum and maximum average distance a graph can attain when you know the order (number of dots) and the diameter of the graph. For the minimum, this was solved in 1984. But the maximum problem was harder. Over 35 years, many people tried to solve it, but only minor progress was made. Due to that and as it only deals with fundamental

concepts, it has been recognized as one of the central problems on graphical indices (a subfield of combinatorics). In the talk, I gave an idea about the asymptotic solution of that problem, i.e. a rough sketch of the optimal structure was given and the intuition about the proof for the upperbound was given as well.”

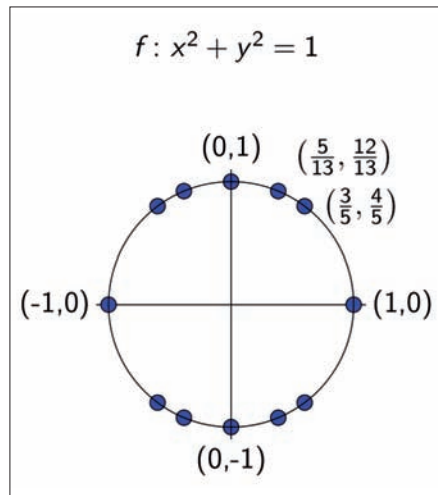


Rosa Winter

Rosa Winter completed her PhD in alge-

braic geometry at the University of Leiden under the supervision of Martin Bright and

Ronald van Luijk. Together with Julie Desjardins she proved the density of rational points for a specific family of del Pezzo surfaces of degree 1.



Winter: “In the picture you see some rational points (points where both entries are fractions) on the circle with radius one. These rational points are related to Pythagorean triples. This circle contains infinitely many rational points, and they are ‘nicely’ distributed everywhere on it. While this is well-known for objects like the circle, for certain surfaces we do not know if the rational points are similarly distributed everywhere, or all clustered together. In my research I studied rational points on a certain type of surfaces. I gave necessary and sufficient conditions for the rational points on these surfaces to be distributed everywhere.”

Emiel Lorist

During his PhD at the Delft University of Technology under the supervision of Mark Veraar, Emiel Lorist worked on an open problem in stochastic differential equations and harmonic analysis. He developed stochastic versions of the

Calderón–Zygmund theory and the famous A_2 -theorem.

Lorist: “If we zoom in on a polycrystalline material, for example an alloy like bronze, we can see that the material is not uniform. It actually consists of microscopic grains or crystals. In each of these grains

the atoms form a lattice, but the lattices between two grains are not compatible, which is illustrated on the slide. A prototypical equation that models the growth of such grains is the Allen–Cahn equation. It is a phase field model, in which we assume that there are two phases, or

in the case of grain growth two crystal orientations. When we start with a mixture of these two phases, these two phases first quickly separate into two distinct regions. Afterwards, over a longer period of time, the length of the boundary between the two phases is minimized. A simulation of this process can be seen on the slide.

The Allen–Cahn equation is a nonlinear partial differential equation. When one wants to account for thermal fluctuations,


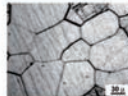
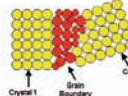
one can add a noise term to the equation. This gives rise to the stochastic Allen–Cahn equation, which is a nonlinear stochastic partial differential equation. One way to analyse such equations, is to reformulate it as a stochastic evolution equation. One then disguises the equation as a stochastic differential equation (without the ‘partial!’) and employs fixed point arguments to obtain existence and uniqueness of solutions. The crucial ingredient for these fixed point arguments is a deep understanding

of the linearized problem, which can be expressed in terms of the boundedness of certain integral operators with singular kernels. While these operators have been thoroughly studied in the deterministic setting, an abstract theory for such singular integral operators in the stochastic setting was completely absent. In my work I made the first steps towards such a theory, which has implications for stochastic partial differential equations like the stochastic Allen–Cahn equation.”

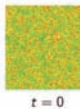
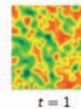
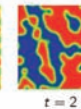



Singular stochastic integral operators and their applications to SPDEs

Emiel Lorist






The Allen-Cahn equation is a prototype for the growth of grains:

Theorem (Dure '00, L., Veraar '20)

If our deterministic and stochastic questions hold for some $p_0 \in [2, \infty)$, then they hold for all $p \in [2, \infty)$.



Harry Smit

Harry Smit completed his PhD at the University of Utrecht under the supervision of Gunther Cornelissen. His main area of research is arithmetic geometry, in particular L -series for global fields, but he also works on graphs and complexity theory.

Smit: “For our purposes, elliptic curves are equations of the form $y^2 = f(x)$, where

$f(x)$ is a cubic polynomial with algebraic coefficients. Sometimes two nonisomorphic elliptic curves have the same number of solutions modulo p for almost every prime p . In that case the elliptic curves are isogenous, which means there exists a surjective map with finite kernel between the elliptic curves. An example is given in the picture, where the black box

is an oracle that counts all the solutions modulo p . In my research, I looked at a related invariant called the L -series of an elliptic curve and asked the question what the L -series tells us about the elliptic curve. I proved that two elliptic curves for which the L -series of their quadratic twists match in a nice way are necessarily isogenous.”




Same output does not imply isomorphic elliptic curves!

$$E: y^2 = x^3 + x^2 + x$$

$$E': y^2 = x^3 - 2x^2 - 3x$$

↓



p	$\#E(\mathbb{F}_p)$
3	.
5	8
7	8
11	16
13	16
17	16
19	16
23	16
.	.
.	.

E and E' are not isomorphic: they have different j -invariants.