Francesca Arici
Mathematisch Instituut
Universiteit Leiden
f.arici@math.leidenuniv.nl

Clara Stegehuis
Faculteit EWI
Universiteit Twente
c.stegehuis@utwente.nl

Proof by example Portraits of women in Dutch mathematics

Annegret Burtscher

Annegret Burtscher is assistant professor in Mathematics at Radboud University working on mathematical relativity. In 2019, she was awarded an NWO Veni grant to study the geometry of weak solutions of the Einstein equations.

When did you first realise you wanted to be a mathematician?
"I have always loved mathematics, starting right from 'Rechnen' in primary school. But still, I did not quite know what to do with this passion. This changed when at the age of fifteen I read Simon Singh's popular book on Fermat's last theorem. Immediately a whole new wonderful world opened up in front of me.

From then on I seriously played with the idea of becoming a mathematician. Initially, I still enrolled both in mathematics and in earth sciences in Vienna — the former for the fun and the latter to eventually get a 'real' job. That was my plan ..."

And then something changed...
"While attending university, I realized that I was actually quite good at math. And for the first time, I saw mathematicians in real life and learned what they do all day long. So, naturally, the mathematics became more central and I started doing geology as a hobby. I decided to do a PhD in mathematics because I wanted to work in academia."

Have you received any support in your decision to pursue a career in mathematics?
"To be honest, in the beginning, most of my friends and relatives discouraged me from studying math because they thought of it as an ivory-towerish concept with a dead end. But my parents never really pushed me in any direction and just let me explore my own world, for which I am very grateful."

You are working in the field of mathematical relativity, how did you end up in this field?
"I first heard about Einstein's equations in a differential geometry class. I was immediately hooked by this interaction between analysis, geometry and physics. You see, the Einstein equations describe the large-scale structure of our universe by relating its geometry to its matter content. The geometry is described in terms of the curvature of a Lorentzian manifold, so essentially a second derivative of the metric tensor, which from an analytic perspective turns the Einstein equations into a complicated system of non-linear partial differential equations.

This interaction of different types of math, physics and astronomy is something I still enjoy very much."


Photo: Radboud University/Dick van Aalst
Annegret Burtscher

Is there any result you are particularly proud of?
"That's quite a difficult question... Well, I started out studying the solutions to the Einstein equations from the perspective of hyperbolic partial differential equations. My most important result there was to show that trapped surfaces can form during evolution when the Einstein equations are coupled to perfect fluids.

Trapped surfaces are crucial to understand singularities and black holes in general relativity. But singularities here mean something geometric, namely geodesic incompleteness. In the sixties, Penrose and Hawking formulated several singularity theorems that essentially establish that geodesic incompleteness is natural and central in general relativity. But it then took several more decades to show that singularities can also form dynamically, including my work, and it is still not entirely clear how generic black hole singularities actually are. This goes under the name of weak cosmic censorship conjecture."

And how does this relate to what you are working on these days?
"While the singularity theorems provide some geometric motivation for my earlier work, they can not directly be applied to the weak solutions coming from the theory of PDEs. In fact, one already has to be very careful when interpreting curvature distributionally. Something is still missing: there is not enough geometric interpretation of what these weak metrics entail, and over time I became increasingly interested in filling this gap."

Can you tell us a bit more about some of these new geometric insights for Einstein's equations?
"The simplest way to understand the relation of curvature and regularity is via the geodesic equation, probably my most favorite ordinary differential equation. If the regularity of the metric tensor drops below the critical Hölder regularity $C^{1,1}$, then the Picard–Lindelöf Theorem no longer implies local uniqueness of geodesics, and below C^1 also the existence is at stake.

Since geodesics are crucial in general relativity, some of my work involves studying the implications and work-arounds of such issues. In Riemannian geometry, some nice properties of geodesics can be reinstalled by switching to the induced metric space structure and by imposing qualitative curvature bounds, e.g., via triangle comparison or by optimal transport.

Now, in the Lorentzian setting, none of this can really be done so easily, because the Lorentzian distance is not even a metric. But there are some exciting new ideas around that make use of other unique features that Lorentzian structures offer. Recently this study of non-smooth space-times has become a very active field of research."

Being in an interdisciplinary field, do you often work with people in different areas?
"Of course, I talk to other mathematicians and physicists, at conferences and at my institute, and collaborate with some directly. I enjoy this interaction because it gets me back to, you know, studying something meaningful, not getting lost in the details, but forces me to always keep the bigger picture in mind."

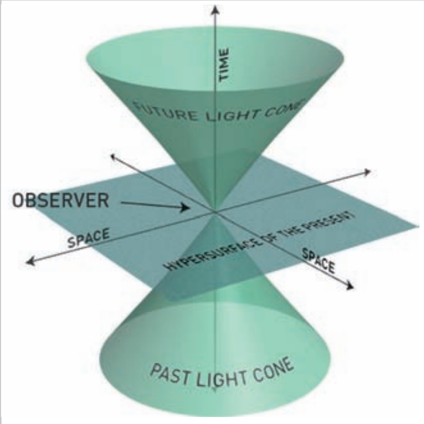

Illustration: Wikimedia Commons/Stib

Flat Minkowski spacetime $\mathbb{R}^{1,n}$ with the Lorentzian inner product $\eta(u,v) = -u_0v_0 + u_1v_1 + \cdots + u_nv_n$ is the simplest solution of the Einstein equations, and the Lorentzian analogue of Euclidean space. Due to the indefinite nature of η, one can distinguish time-like, space-like and null (light-like) tangent vectors u depending on whether $\eta(u,u)$ is negative, positive, or zero. This light cone structure captures the physical observation that the speed of light is finite.

What do you like about being a mathematician?
"In an ideal world, our job comes with a lot of flexibility. You can think about a research question, a problem, whenever and wherever you want. I like it when I conceive a crazy idea, think about it and it eventually works out and turns into something real and meaningful. It's also nice to see when researchers can use some of my insights, and vice versa... I find this whole process of creating new mathematics exciting and deeply satisfying.

Teaching is also an enjoyable aspect I would say. Basically I just like talking about mathematics on different levels to all kinds of people."

How has the current pandemic been affecting your research and work?
"Like most other people, in the first weeks I was totally overwhelmed. We had a full house and a lot of things to do: besides the abrupt switch to online teaching and supervision, I had too many collaborations going, and suddenly was supposed to homeschool the kids in Dutch. While having very little time for my own research at the same time I got more referee requests than ever before. Those were long days and weeks. This lockdown situation really made and makes me miss my quiet office and my colleagues, and also my family and friends in Austria. Of course, the pandemic also involves lots of cancellations — of seminars, conferences and some carefully planned research visits..."

Was this the reason behind the online mathematical relativity seminar you just started?
"Although it is rather timely now, some of us organizers had actually started to discuss such a joint online seminar already last year. It is not really intended as a replacement for something that usually takes place in person, but rather a new kind of unifying seminar so that people around the globe have continuous access to good talks."

You mentioned how the pandemic has also affected your work-life balance, having to care for your kids while teaching and doing research. How has your experience been otherwise, and what do you think still needs to change to increase the support for mathematicians with parenting responsibilities?
"I will start by first telling you about my experience... For me having kids at a relatively young age was a very deliberate choice, but of course, it was also exhausting, especially during my PhD and Postdoc years. The physical dependence alone, due to pregnancy and nursing, lasted for almost five years. On the other hand, having two kids never stopped me from actually doing anything: I moved a lot, travelled a lot, collaborated widely, et cetera. It was not always easy but I did it anyway. Now I am in a much more stable position which simplifies things a lot.

Generally, I think universities and research institutes should provide sufficient infrastructure, for example, for nursing mothers at work and during conferences, and full-time childcare on campus. And pay careful attention to dual-career couples. This is very important if we want to increase the number of female mathematicians and give progressive men a chance to equally share family responsibilities without having to sacrifice their own career.

But while on one hand, the practical support can be improved, I think what is even more important, particularly for young mothers, is to provide moral support."