Assessment panel for the cluster WO Mathematics:
Onno Boxma [a], Sterre den Breeijen [b], Sabine Jongerius [c], Rainer Kaenders [d], Henk Prins [e], Duco van Straten [f], Lotte Weedage [g]

[a] *o.j.boxma@tue.nl,* [b] *sterre.denbreeijen@tno.nl,* [c] *s.c.jongerius@student.tue.nl,* [d] *kaenders@math.uni-bonn.de,* [e] *h.j.prins@marin.nl,* [f] *straten@mathematik.uni-mainz.de,* [g] *l.weedage@utwente.nl*

Education

The state-of-the-art of mathematics teaching at the Dutch universities

This is a slightly adapted version of the general report of the assessment panel for the cluster *WO Wiskunde*. In that report, the assessment panel for the cluster WO Wiskunde presented some impressions and observations regarding the state-of-the-art of mathematics teaching at the Dutch universities.

The general impression of the panel is very positive. The level of mathematics teaching overall is of a qualitatively high level. It features a very solid curriculum; teaching and assessments are organized in a professional and efficient way; and the lecturers are dedicated and competent. Furthermore, student numbers in the Bachelor and Master programmes of all involved mathematics departments have strongly increased in the past two decades (e.g., the total intake in the Bachelor Mathematics programmes was 153 in 2002, 673 in 2013, and 1007 in 2018). Nevertheless, all students still receive individual guidance in their Bachelor project and Master thesis and, without exception, students of all involved mathematics departments speak highly of their teachers, and in particular about their accessibility and eagerness to discuss mathematical problems. The quality of the Master theses ranges from reasonably good to excellent; quite a few theses gave rise to publications. After completion of their Master study, the students almost invariably find employment very quickly. A considerable percentage of them continues to do a PhD, either in Mathematics or in a closely related topic.

The panel has assessed 11 Bachelor and 13 Master programmes in nine universities; see the box below for the complete list. The panel is grateful to the various programme managements for the offered hospitality, the open attitude towards the

The assessed programmes

- Leiden University: Bachelor Wiskunde, Master Mathematics
- University of Groningen: Bachelor Technische Wiskunde, Bachelor Wiskunde, Master Applied Mathematics, Master Mathematics
- University Utrecht: Bachelor Wiskunde, Master Mathematische Wetenschappen
- Delft University of Technology: Bachelor Technische Wiskunde, Master Applied Mathematics
- Eindhoven University of Technology: Bachelor Technische Wiskunde, Master Industrial and Applied Mathematics
- University of Twente: Bachelor Applied Mathematics, Master Applied Mathematics
- University of Amsterdam: Bachelor Wiskunde, Master Stochastics and Financial Mathematics, Master Mathematics
- Vrije Universiteit Amsterdam: Bachelor Business Analytics, Bachelor Mathematics, Master Stochastics and Financial Mathematics, Master Business Analytics, Master Mathematics
- Radboud University Nijmegen: Bachelor Wiskunde, Master Mathematics

Composition of the assessment panel

- Prof. dr. ir. O.J. Boxma, full professor Stochastic Operations Research, Eindhoven University of Technology (panel chair)
- Prof. dr. R.H. Kaenders, full professor Mathematics and its Education, University of Bonn, Germany (panel member; panel chair for Eindhoven University of Technology)
- Prof. dr. D. van Straten, full professor Algebraic Geometry, Johannes Gutenberg University Mainz, Germany (panel member)
- Dr. ir. H.J. Prins, manager Research & Development, Maritime Research Institute Netherlands (panel member)
- Prof. dr. J. Molenaar, full professor Applied Mathematics, Wageningen University and Research (panel member for Eindhoven University of Technology)
- Drs. J. Poppelaars, senior manager, practice leader Advanced Analytics, BearingPoint (panel member for Vrije Universiteit Amsterdam, Business Analytics programmes)
- S.C. Jongerius BSc, student Master Industrial and Applied Mathematics, Eindhoven University of Technology (student member)
- S.R. den Breeijen MSc, recently graduated student Master Mathematics, Radboud University Nijmegen and presently junior scientist at TNO (student member)
- L. Weedage BSc, student Master Applied Mathematics, University of Twente (student member)

On behalf of the evaluation agency Certiked, drs. W. Vercouteren served as the process coordinator and secretary in the assessment process.

panel and the efforts to supply the panel with information. Preparing an extensive self-study as well as a site visit requires a huge effort. A positive effect of this effort is, as we have noticed, that those preparations oftentimes already expose some weaknesses in the curriculum, the teaching, the student guidance or the assessment procedures; quite often, such weaknesses were mentioned in the self-study, and improvement steps were outlined or already taken.

For each of the 24 reviewed programmes, the panel has given assessments for the programme as a whole, as well as for the following four standards:

Standard 1: Intended learning outcomes,
Standard 2: Teaching-learning environment,
Standard 3: Assessment,
Standard 4: Achieved learning outcomes.

The choice each time was between the four options *Unsatisfactory, Satisfactory, Good* and *Excellent*. In each case, the qualification was at least *Satisfactory*. In some rare cases, we gave the assessment *Good*. Since this may give the impression that there is much room for improvement, we would like to emphasize that we have given the qualification *Satisfactory* when everything was functioning well and 'upto standard', and that a programme receives the assessment *Good* for a standard if it systematically, and in full breadth, qualitatively exceeds the customary level for that standard.

The main purposes of the present report are (i) to present our global impressions of mathematics teaching at the Dutch universities, and in particular our views on a few topics that came up in almost all site visits, and (ii) to make a few statements regarding Mastermath, in view of its important role in the mathematics teaching at the Master level.

The remainder of this report is structured in the following way. The next section is devoted to Mastermath. Then follow a section that focuses on the Domain-Specific Framework of Reference, that lists the generic objectives and intended learning outcomes of the mathematics programmes under consideration; a section on professionalisation in the educational process; a section on intake, dropout and student success rates; a section on teaching in English; and a section on the education of mathematics teachers. We close in the final section with a few thoughts about the future of mathematics teaching.

Mastermath

The panel has visited Mastermath on April 26, 2109, before any of the site visits of the nine mathematics departments took place. Mastermath has often been called one of the crown jewels of Dutch mathematics, and that seems fully justified. Mastermath was started in 2003, at a time when student numbers in most mathematics departments were dangerously low. The mathematics departments joined forces, allowing Mastermath to offer a broad collection of national mathematics courses at the Master level, by leading Dutch experts. Those courses were given in Utrecht and Amsterdam, and students from all mathematics departments have been taking several such courses. As a side benefit, the need to prepare students for Mastermath courses has, to some extent, led to a national alignment of the Bachelor Mathematics teaching. Other benefits of Mastermath are the growing interaction between researchers, and also students, of various departments, and the fact that departments are now able to offer a significantly broader package of courses than before.

In the last decade, student numbers in the various Mathematics Master programmes have grown very significantly, and the present Bachelor inflow strongly suggests that this growth has not yet come to an end. In that same period, staff size has hardly grown, and Mastermath apparently now also plays a positive role in reducing the workload of staff members.

Despite all these remarkable accomplishments, there is presently a strong need for Mastermath to carefully design its medium-term strategy. There are several reasons for this. Firstly, because of the Sectorplan and some other developments, all mathematics departments are now hiring significant numbers of new staff members. This will allow the departments to offer a broader package of courses then before, reducing the need for Mastermath courses. Secondly, the large student numbers at some mathematics departments make it less desirable to encourage students to take mathematics courses outside one's own university. Thirdly, while all departments support Mastermath, there seems to be a growing uneasiness about Mastermath in a number of departments. Those departments are concerned about a lack of cohesion of their Master student population, as quite a few of those students spend part of the week attending Mastermath courses elsewhere. They also fear that their profile will be less pronounced because of the large role played by Mastermath in their Master curriculum.

Furthermore, some departments have indicated that the level of the Mastermath courses has increased in recent years, and that this is sometimes causing problems for their students. Several students, too, have some complaints, while overall being very positive about Mastermath; in particular, they sometimes have to travel long distances to Mastermath courses, also because for many Mastermath courses no good video recordings are available. Institutes located far away from Utrecht typically share this concern.

Fortunately, Mastermath is led in a professional way by a lean-and-mean Executive Board plus support staff, and is overviewed in a constructive way by the *Regieorgaan*, a committee composed of the nine educational directors of the participating institutes. Therefore, this panel is confident that Mastermath will be able to adapt its strategy in such a way that it will remain a strong asset of Dutch mathematics. We also would like to mention that Mastermath was evaluated in 2017 and that we, broadly speaking, support the recommendations made in the corresponding evaluation report.

Modus operandi of the assessment panel

All panel members and the secretary confirmed in writing being impartial with regard to the programmes to be assessed and observing the rules of confidentiality. NVAO have given their approval to the panel composition prior to the assessments. The panel studied the self-assessment reports of each of the programmes as well as the final projects of fifteen graduates of each of the programmes. On the days of the site visit, the panel studied course material and the examinations of a number of courses of the programme. Before the site visit dates, the panel met to go over the preliminary findings concerning the quality of the programmes. During these meetings, the preliminary findings of the panel members, including those about the final projects, were discussed. The procedures to be adopted during the site visit, including the questions to be put to the programme representatives on the basis of the list compiled, were discussed as well.

On the days of the site visits, the panel was given the opportunity to meet with faculty representatives, programme management, examinations committee members, lecturers and final projects examiners, students, and alumni. At the end of the site visit, the panel chair presented a broad outline of the considerations and conclusions to programme representatives. The panel members and the programme representatives met to conduct the development dialogue, this meeting being separated from the process of the programme assessment. The assessment report, finalised on the basis of the panel findings and considerations, was presented to programme management to be corrected for factual inaccuracies. Having been corrected, the report was sent to the University Board to accompany their request for re-accreditation of the programmes.

Domain-Specific Framework of Reference

The joint Mathematics programmes in the Netherlands have drafted the Domain-Specific Framework of Reference (DSFR) for both Bachelor and Master Mathematics programmes. In this DSFR, the generic objectives and the generic intended learning outcomes for these programmes have been listed. These objectives and intended learning outcomes meet the international standards for mathematics of ASIIN in Germany. They also correspond to the Dublin descriptors and the Meijers' criteria. In addition, they are largely comparable to those of the mathematics programmes of some renowned universities abroad, such as ETH Zürich, KU Leuven and the University of Padua. DSFR allows some space for differences between the various programmes.

Each programme has drafted its own intended learning outcomes; mostly in line with the DSFR, but sometimes featuring minor adaptations, perhaps made to emphasize the unique character of a programme or the presence of a particular specialism or track. During the site visits, this has led to some discussions with programme managements.

Attention for ethical aspects in mathematics was in some cases not included in the local intended learning outcomes for Master studies — although it is included in the DSFR. The panel strongly believes that such an omission should be corrected, and that each mathematics curriculum should pay proper attention to ethics. Staff and students should realize that mathematics is potentially harmful, the wrongful or careless use of data being just one aspect of this complex topic. In addition, plagiarism and proper referencing to used sources deserve ample attention.

Furthermore, we encountered the following problem. Several programmes offer a Major in *Science in Society* or in a comparable topic. Typically, about one of the two years in a Master programme is devoted to such a Major, and the Major involves an internship and/or Master project devoted to a non-mathematical topic, performed under the main responsibility of another department. Such a Master programme may have excellent merits and deliver valuable alumni to our society, but it should be observed that, formally, such a Master programme also cannot satisfy MC2 of the DSFR, viz.: *Graduates are able to act and behave like mature and professional mathematicians. They are able to acquire sufficient knowledge including of recent developments in subdisciplines, other than their own specialization*. The panel has come to the conclusion that, to take care of this problem, it is necessary to make an adaptation to the DSFR. The purpose of such an adaptation, a diversification, is to maintain the profile of mathematics and make a clear distinction with programmes and tracks which are less mathematically oriented. This distinction should also be reflected in the text on the certificate.

Professionalisation

In almost all visited departments the panel has observed an increased professionalisation in the educational process. In fact, having read the detailed procedure descriptions in the self-studies, we feared that those procedures would lead to too much bureaucracy and too little freedom for individual lecturers. The site visits have shown us that, fortunately, that stage has not yet been reached — but it is important to be aware of the risk. A related issue is that the boards of some universities are imposing procedures which must hold in all departments — procedures which are not always suited for mathematics studies. The panel holds the opinion that the unique role of mathematics justifies a certain freedom of the mathematics department to make its own decisions.

Below we mention four types of educational procedures in which the panel noticed an increased professionalisation.

(i) The guidance of students has become more professional. Candidate Bachelor students are offered lots of opportunities to discover what studying mathematics comprises; they can take part in matching events, attend some classes, et cetera. In the first Bachelor year, students often get a mentor — a staff member or a more senior student. Furthermore, a study advisor is actively monitoring the study progress of individual students. Despite all these positive actions, the panel was informed in several departments that students have psychological problems, of various nature. Where appropriate, such problems should be handled by a professional.

(ii) Almost all staff members now are UTQ-certified (University Teaching Qualification), while new hirings have to obtain their UTQ within a few years after their appointment. This is a major change with respect to the previous accreditation.

(iii) The departments have refined their procedures for the assessment of exams. In most departments, the examinations and assessments are governed by the principles of *constructive alignment*, linking the course examinations to the intended learning outcomes of the programme. This development is a decision in favour of an output-oriented approach to learning processes, that fits well into administrative processes of learning management. However, individual development in students and student-teacher interactions are much less reflected and stimulated in this approach. It is important to keep some room for experimentation.

These assessment procedures typically are more strict than at the previous accreditation, routinely using four-eyes principles like demanding that another staff member carefully inspects a written exam before it is given, and that a second staff member is present at an oral exam. In only a few instances we noticed that procedures were not quite adequate, or that the procedures were not strictly adhered to. In a few institutes, student assistants who themselves still are Bachelor students are asked to check exams beforehand, or to mark exams without immediate supervision of the examiner. In our view, that is inappropriate.

(iv) The procedures regarding the assessment of Bachelor projects and Master theses also have been sharpened. In some institutes, a member of the examination board is routinely part of the examination committee of a Master thesis, or is even chairing that committee. In our view, that is a very good procedure.

Each Mathematics department now uses assessment forms for the Bachelor project and the Master thesis. Almost invariably, the forms contain rubrics (in some cases with over a hundred boxes) but very little text. This is an example where we feel the procedure sometimes has become too bureaucratic. The panel strongly feels that it would be much better to produce here a quite extensive text (at least one A4 page) in which, at least, the following topics are being discussed in an informative way: the choice of the project topic, the knowledge the student already had about the topic before the start of the project, the literature provided by the advisor, the main results of the project, the student's own contribution, the creativity and depth of the work, and the quality of the writing and of the oral presentation. The mark should then naturally follow. Such a text is valuable for everybody involved — in particular for the student and the examination board.

Intake, drop-out and student success rates

Table 1 contains intake numbers and student success rates for all 24 programmes. Here student success rate is defined in the following way. For Bachelor students: percentage of students re-entering in the second year, who complete the programme within a total of four years; for Master students: percentage of intake who complete the programme in three years. There are minor differences in the number of years over which the averaging took place

Programme	Intake	Success
Leiden Bachelor Wiskunde	108	74%
Groningen Bachelor Technische Wiskunde	28	67%
Groningen Bachelor Wiskunde	110	54%
Utrecht Bachelor Wiskunde	130	63%
Delft Bachelor Technische Wiskunde	216	62%
Eindhoven Bachelor Technische Wiskunde	120	64%
Twente Bachelor Applied Mathematics	64	70%
UvA Bachelor Wiskunde	74	61%
VU Bachelor Business Analytics	119	80%
VU Bachelor Mathematics	58	50%
Radboud Bachelor Wiskunde	99	55%
Leiden Master Mathematics	40	50%
Groningen Master Applied Mathematics	6	70%
Groningen Master Mathematics	15	50%
Utrecht Master Mathematische Wetenschappen	34	59%
Delft Master Applied Mathematics	80	84%
Eindhoven Master Industrial and Applied Mathematics	40	84%
Twente Master Applied Mathematics	25	70%
UvA Master Mathematics	15	57%
UvA Master Stochastics and Financial Mathematics	12	58%
VU Master Business Analytics	59	70%
VU Master Mathematics	12	71%
VU Master Stochastics and Financial Mathematics	7	73%
Radboud Master Mathematics	37	60%

Table 1 Intake numbers 2018 and student success rates per programme; success rates are averaged over the last few years.

and in the underlying calculations; we have included the numbers because they present a rather informative overall picture.

Traditionally, a rather large percentage of Bachelor freshmen drops out during the first year of study or, with the present BSA (Binding Study Advice), is not able to collect enough EC to continue his/her studies. In most Bachelor programmes, the drop-out rate in year 1 still is above 30%, despite very significant efforts in providing candidate students with enough information, and in tutoring students in the first year of their studies. One explanation is that, with the increasing student numbers, it is likely that a smaller percentage than before is gifted enough in mathematics to pass the first year. Furthermore, there is a large gap between highschool mathematics and the mathematics taught in mathematics departments. For a sizable percentage of the students, the level of mathematical abstraction at the university is unattractive or simply too high. We have decided not to include drop-out rates in the table, because these rates are hard to specify, and may give the wrong impression; the drop-out rate may for example be relatively high when students have the option of combining a Bachelor in Mathematics with a Bachelor in another discipline, or when they can switch between Applied Mathematics and Mathematics (like in Groningen).

Students who pass the first year typically will obtain their Bachelor degree — although only few complete their Bachelor's within three years. After obtaining this degree, most students opt for a Master study; often a Mathematics Master in the same university, but they also sometimes switch to another Master programme in the same university, or move to another university. The drop-out rates in the Master programmes are in most cases rather low, but students often take considerably more time than two years to finish their studies. We have heard many different explanations for this, like: (i) most Master students have a part-time job (often as student assistant), (ii) the level of some courses, in particular from Mastermath, is much higher than that of the typical Bachelor course, (iii) students very much enjoy their studies and hence take several extra courses, and (iv) students often do not know yet what they would like to do after their studies, and therefore postpone the decision by delaying their studies.

Alumni invariably expressed their enthusiasm for their alma mater. In quite a few cases, they were not on a daily basis using the mathematics that they had learned, but they claimed to benefit a lot from the structured way of reasoning that was instilled in them in their studies. In almost all cases, the ties between department and alumnus seem rather loose — both parties do not make a big effort to regularly stay in touch. One can think of various reasons why it would be beneficial for a department to strengthen the ties with its alumni; e.g., the department could get feedback from alumni that is useful in fine-tuning the curriculum, and students could get a more accurate picture about the daily activities of a professional mathematician.

Teaching in English

All assessed Master programmes now are in English, and these programmes are attracting an international population of students. Most Bachelor programmes still are in Dutch, but everywhere the option of switching to the English language has been, or still is, a topic of discussion. The main reasons for switching to English are the following: (i) Education is research-driven, and English is the universal language for research. (ii) Most mathematics textbooks and literature are in English. (iii) A growing proportion of the staff — the potential lecturers — is non-Dutch speaking. (iv) An international programme could attract students from outside the Netherlands. (v) Opening the programme for international students results in a student population with a wider cultural and educational background, which not only poses challenges but also offers opportunities when it comes to training various skills and creating a stimulating study environment.

The main reasons for *not* switching to English are: (i) The added difficulty, in particular at the early stages of the study, that students will experience in grasping new mathematical concepts. (ii) Dutch teachers may find it hard to express themselves fluently and precisely in English. (iii) The feeling that one is forsaking his/her own language. (iv) To offer better opportunities to foreign students to get in touch with the rich Dutch mathematical culture and its history.

The panel has mixed opinions on this choice and hence presents no recommendation; in fact, we welcome the diversity offered. For example, because the Vrije Universiteit Amsterdam has opted for English, we are pleased that the University of Amsterdam sticks to Dutch — although, as in many institutes, courses in the third Bachelor year often are given in English.

First degree and second degree education

Almost all Master programmes offer a specialization to mathematics teacher education leading to a first degree teacher qualification (*eerstegraads bevoegdheid wiskunde*) for prospective teachers. The inflow of these specializations is disappointingly small in almost all programmes.

The academical teacher education in mathematics is of great importance for Dutch mathematics in general. All Bachelor programmes are struggling with aspirant students having no clue what university mathematics is about, and there might be an even larger number of school graduates who would be interested in studying mathematics and would possibly be capable of it, if they would have had a role model of a high school teacher being an academical mathematician. The crisis of Dutch mathematics around the millennium change, when almost no students came to study mathematics, has shown the vulnerable dependence of Dutch mathematics from the mathematical culture at school. Internatonalization is valuable, enriching and necessary, but it cannot compensate the need for an inflow from the Dutch highschools.

Therefore, the panel values that nearly all universities created the possibility to become first degree mathematics teacher within the different mathematics Master programmes. However, offering a specialization in a Master programme has to be based on a research background in the respective specialization. Internationally, the specialization of mathematics education is a vivid academic research area with sizable national and international communities of researchers. Mathematics educational research on a scientific level goes back to the beginning of the 20th century. The epistemology of mathematics with its abstractions and concept development makes mathematics education different from general science education (*Bèta didaktiek*) and it cannot be covered by experts in science education as several Dutch universities try to do. Like any other specialization, mathematics education, if it is offered in a Master programme, needs principal researchers, which are usually full professors.

Mathematics education is, next to history and philosophy of mathematics, one of the three reflective mathematical sciences and can be supported by the other two as it happens (with history) in Utrecht, Amsterdam (VU and UvA) and Twente. The panel is positive about any attempt to relate these reflective mathematical sciences to the mathematics programmes as they broaden the general mathematical formation, especially but not exclusively in teacher education.

Finally, in several of the Bachelor programmes there is the possibility to also do a minor in mathematics teacher education that can lead to a second degree teacher qualification (*tweedegraads bevoegdheid wiskunde*) for the Dutch school system. This option allows students who explicitly want to become maths teacher to just start with a study of mathematics. Those who discover during their Bachelor study that their interest in higher mathematics goes not as far as a Master in mathematics would require, then still have the possibility to follow their pedagogical ambitions and to become a mathematics teacher of second degree. These academically educated teachers are certainly especially qualified to teach in havo and in the early years of vwo, as these school forms lead to pre-university education. For these reasons, the panel recommends this engagement in mathematics teacher education.

The future of Mathematics teaching

Our society is presently witnessing revolutionary changes. We mention two changes which might affect mathematics teaching.

Firstly, the way people communicate with each other and exchange knowledge is changing in a dramatic way. Remarkably, this is only to a quite limited extent reflected in the teaching methods and curricula of the visited departments. Everywhere we saw a strong emphasis on classical methods of knowledge transfer. Most teachers and also almost all students expressed a preference for blackboard-and-chalk lectures (although several teachers experimented with concepts like ‘flipping the classroom’); and in guiding students in a Bachelor project or Master thesis, the classical one-on-one master-pupil relation invariably was in place, apparently to everyone’s satisfaction. The panel in principle strongly supports this; there are good reasons why the above-described common practice has been so effective in the past centuries.

A second revolutionary change is that data of various types are becoming available in many different forms and in unprecedented amounts. It is obvious that mathematics can make major contributions in dealing with such data: their storage, processing, statistical analysis and their use for optimization purposes (among others, in Artificial Intelligence) all give rise to deep mathematical problems. At the various site visits, however, we were often struck by the robustness of the mathematics curriculum. We saw a ‘cathedral’ with strong pillars (learning lines) like Analysis, Algebra and Geometry, Scientific computing and Stochastics. Again the panel had a natural tendency to support this. The mathematical building indeed can be viewed as a cathedral, with pillars which have proven their strength through the ages, and it is essential that a student of mathematics acquires fundamental knowledge about the key topics and methodologies, as well as becoming well versed in mathematical reasonings and proof techniques.

And yet, and yet… one cannot escape the feeling that quite possibly, twenty years from now, completely different methods of knowledge transfer will be in use in mathematics teaching, and a number of completely new topics will have been included in the curriculum — at the expense of some traditional topics. But one thing must and will definitely survive the times: the ability of mathematics students to think in a well-structured way, applying strictly logical lines of reasoning, will always be an important added value.

As a postscript we would like to add that twenty weeks after the completion of our general report, instead of twenty years, Covid-19 forced all mathematics departments to drastically change their teaching methods. The efficiency and effectiveness in which the various departments have dealt with this is truly impressive, and a testimony of the dedication, flexibility and quality of teaching directors and teachers alike. We hope that, after the crisis, the teachers can resume their proven teaching standards with face-to-face encounters, and enrich them by some new insights gained during this involuntary experiment.