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# **Afscheidsrede**

# Mathematics, the

Op 7 november 2019 nam Jaap Molenaar afscheid als hoogleraar Toegepaste Wiskunde bij Wageningen University & Research Centre, waaraan hij overigens als gastmedewerker verbonden blijft. In zijn afscheidsrede schetst hij hoe hij ertoe kwam wiskundig modelleren tot de kern van zijn onderzoek te maken. Hij beschrijft hoe je wiskundige modellen 1-dimensionaal kunt ordenen van 'bottom-up' tot 'top-down', maar pleit ook voor een integratie van beide benaderingen. Om de kracht van dimensieanalyse en wiskundig modelleren te demonstreren, besluit hij met een afleiding van de Law of Benford.

Delivering a farewell address is not only a nostalgic event, it is also a moment of relaxation and joy. To hear the organ of this Auditorium playing a cheerful melody; to see so many esteemed colleagues hoping to hear some wise last words; to realize that nobody in the audience expects you to impress him or her by an exposition of all your expertise—as is usually the case with inaugural lectures: it all contributes to the nice atmosphere evoked by again a member of the crew becoming old aged.

By the way, I like to play the organ myself, and my very first wise lesson is: don't forget to move the organ to the new Dialogue Centre on the Campus when this Auditorium will be abandoned...

A farewell address may have a high 'grandpa tells stories' character. That's why I feel free to start telling you how and why my career culminated in a position at Wageningen University.

#### Some personal history

Although mathematics eventually became the dominating science in my life, also physics and biology played an important role. The combination biology and mathematics seems a bit less natural than the marriage of physics and mathematics. For example, the physicist Einstein was able to develop the special relativity theory on his own, but the general relativity theory exceeded his powers and he was forced to invoke the skills of mathematician Grossmann. To his friend Sommerfeld he wrote:

"I have never worried so much, and received the greatest respect for the mathematics that I, simple soul, had hitherto regarded in its more subtle detail as pure luxury."

For the combination of biology and mathematics it may be more convenient to quote

a similar heart cry of good old Darwin in his diary around 1830:

"I have deeply regretted that I did not proceed far enough at least to understand something of the great leading principles of mathematics; for men thus endowed seem to have an extra sense."

I think this attitude of Darwin should be exemplary in the life sciences world, but practice is unruly. This is eloquently expressed by Gian-Carlo Rota, a famous mathematician at MIT, who wrote:

"The lack of real contact between mathematics and biology is either a tragedy, a scandal, or a challenge, it is hard to decide which."

In my opinion, this lack of contact between biology and mathematics is nor a tragedy nor a scandal, but a serious challenge. As a consequence of this opinion, I took two determining steps in my life. First, I married a biologist, and second I accepted a position in Wageningen. And, in spite of the worries expressed by Rota, I had a splendid time here. For a modeling minded mathematician as I am, Wageningen is the place to be. Here I was swimming in a pool with mainly biologists around me.



Jaap Molenaar

# science of my life

Wageningen has always respected its one and only chair in math, initially occupied by Van Uven in the period 1918–1950, who was famous for both his mathematical and musical activities and after whom a street in Wageningen has been named.

But still, it is always good to keep an eye on the perspective: the other technical universities — Delft, Eindhoven, Twente — have 8 to 12 math chairs each. Wageningen only one ...

That mathematics would take such a prominent place in my life was by far not self-evident. My secondary school was Christelijk Gymnasium Sorghvliet in The Hague. At that moment a very small school, of high quality, but unknown. Nowadays it's nationwide famous because the three royal princesses are visiting it.

When leaving Sorghvliet, I did not experience the slightest urge to choose mathematics as my study. To be honest I found it rather boring. The same applied to biology. It was in the sixties of the last century, and although the fascinating discovery of DNA had taken place already in the fifties, my biology teacher restricted the lessons to enumeration of the bones in the human skeleton and classification of the species in the plant domain. So, I started to study

physics, since that field seemed to provide one with a general understanding of the universe. It was only in the slip stream of physics that I also obtained a master degree in mathematics. I got so many exemptions for math courses, that I couldn't resist the temptation.

After graduating and having gained deep insight into the universe, I had to join the army. My official rank was 'vaandrig', i.e. 'vaandeldrager', so I had to bear the colors. It wasn't a bad time at all. In view of time limitations I skip this period today.

An enjoyable period of applying to positions of all kinds followed. I carefully avoided the ones with a mathematical flavour, and finally I started at a PhD project at the Free University in Amsterdam. In solid state physics, supervised by Adri Lodder. He infected me with the passion for research, and for deriving pleasure from endless manipulations with formulae to arrive in the end at a result of such an intrinsic beauty, that all nuisance is forgotten at the moment of triumph. Research requires a great deal of endurance. It is like giving birth to a baby: as soon as the sibling is there, most or even all misery is over.

I would like to give this address the flavor of a last public lecture. This lecture has

a simple structure: first, I'll tell you about the philosophy underlying mathematical modelling, then I'll show you the power of mathematics using an example, after which I'll close with some acknowledgements. To keep this address sufferable, I won't bother you with deep mathematics, nor will I make use of typical life science cases that presume a lot of expert knowledge.

# Modelling: what's inside the box?

As I already told you, physics had evoked with me a fascination for modelling. As I worked out in my two inaugural lectures, a model is a representation of some system, that helps us to analyze and predict its behaviour. It should at least mimic our observations of the system, but a model

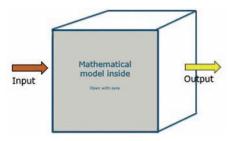


Figure 1 The mathematical modeling box.





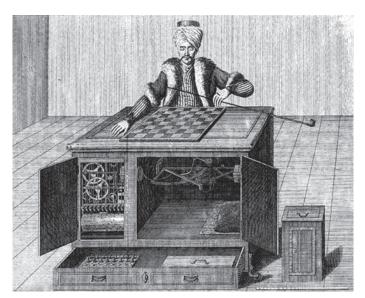


Figure 3 The modeling box of the chess Turk, containing clockwork like machinery.

without predictive power is like a lion without teeth. As an example you could think of the app 'Buienradar': given the present weather conditions the app predicts the rain fall for the coming hours. Bringing it back to its very essence, a model can be looked upon as a box with an input and an output, and some magic happening inside (Figure 1). The key question of mathematical modelling is: what's inside the box?

I like to play chess, so let us take an example from chess (Figure 2). This chess playing robot, the so-called 'chess Turk', was built in 1770 by Wolfgang von Kempelen for entertaining purposes. The robot was an enormous show success and the inventor traveled with it through Europe visiting many royal courts. The chess Turk was able to play chess like a human, moving the pieces, which on its own was already fascinating to see. But what's more, it defeated nearly all its opponents. We recognize the modeling scheme here: input is a move by a human opponent, upon which the robot comes up with a move as output. Here the question is literally: What's inside the box



Figure 4 The modeling axis, arranging models from bottom-up (left) to top-down (right).

below the desk of the chess Turk? During his shows, Von Kempelen opened the box to the audience, showing that there was only some clockwork like machinery in it (Figure 3).

For decades the secret of the machine, that nowadays would be referred to as being 'artificial intelligent', remained unraveled. A cliffhanger moment.

In science, mathematics turns out to be a very appropriate language to formulate models. And so, via the bypass of modelling, I became more and more entangled in the world called mathematics.

For Wageningen UR, world leading expertise center for the life sciences, modelling is a key expertise. That was for me—in addition to Wageningen being at cyclist distance from my home town Veenendaal—one of the main reasons to switch from TU Eindhoven to Wageningen University. Nearly all life sciences need mathematical modeling. It's a topic belonging to the heart of Wageningen research and education. That's why I would like to pay quite some attention to this activity.

Models can be ordered from so-called bottom-up models to top-down models. It seems reasonable to draw that ordering axis vertically, from bottom to top. Since this picture would suggest a sort of hierarchy, I shall use a horizontal presentation (Figure 4).

## Bottom-up modeling

First we focus on the left hand part of the modelling spectrum: the bottom-up models. This is the domain of my specialty:

systems biology. These models are suitable for systems that we know a lot: their components and the physical, chemical, and biological mechanisms governing their dynamics are more or less understood.

As an example I take this mechanical duck, constructed by Jacques de Vaucanson in 1739 (Figure 5). In this duck, the digestive tract was modelled in a mechanical way and the duck model really worked: one could feed the duck at the beak end and after some time the duck produced feces at the rear end.

Bottom-up modelling has a strong reductionistic character. With as consequence that one runs the risk of leaving out components, that later turn out to be essential. Or worse: that one ignores the holistic view that the whole may be more than the sum of its parts. For example, the appeal a racing motor has to some people will never be captured by

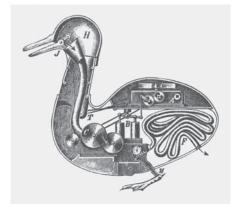
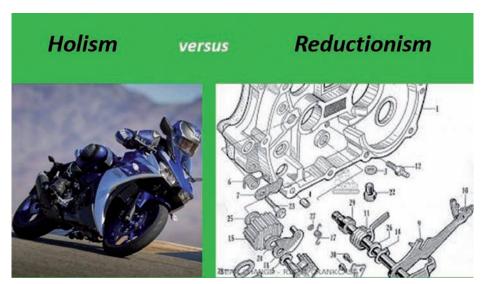


Figure 5 Mechanical model of the digestive tract of a



**Figure 6** The appeal of a holistic object like a racing motor stays in sharp contrast with the mechanical components that make it up.

only modelling its mechanical components (Figure 6).

We meet here with a very important modeling dilemma: which level of detail should be included. The answer is, of course, closely connected to the purpose you have in mind. This does not imply that bottom-up models consisting of relatively simple components can't give rise to complex behaviour. In the seventies of the last century the mathematical revolution leading to 'chaos theory' took place. I still remember how excited I was when following



Figure 7 Emergent behavior: simple agents like ants manage to build a bridge by cooperating.

these developments. Chaos theory shows that very small bottom-up models can already exhibit very complex, unpredictable behaviour. In the life sciences I came across the very similar notion of 'emergent behaviour', which, by the way, should not be associated with 'emergency exit' or so, but rather with 'Luctor et Emergo' where it means 'to pop up'. So, simple elements, provided that they interact effectively, may represent very intricate phenomena. As an example, look at ants (Figure 7). Each ant on its own has, as far as we know, no high level of intelligence. But together they are able to build a bridge. Isn't God's creation wonderful?

Until now I showed you mechanical models from centuries ago for illustrational purposes. At present, we have enormous computer power at hand and mathematical models live in the form of computer implementations. For example, how would we model the digestive tract of the duck in modern times?

#### Networks

Before going into detail, I need to give you a short general introduction in mathematical modelling. It always starts with identifying the essential components of the system at hand and representing them in the form of a network. The nodes represent the components, the arrows the interactions. Next step is the translation of the network into a set of mathematical formulae. In Figure 8 I show ordinary differential equations. There are other mathematical languages, but that's not relevant for this lecture.

This translation step is far from being trivial. Be aware that a network as such may still contain many ambiguities. Only if the formulae are specified, the model becomes unique and useful. The general lesson here is:

One picture tells us more than 100 words and

One formula tells us more than 10 pictures with the implication:

One formula tells us more than 1000 words.

The network framework is extremely useful thanks to its generality. For example, modeling the dynamics of a complex system of genes, the nodes stand for the levels of gene expressions and the arrows for the way genes may promote and/or inhibit each other's expression. And if we model a simple system such as a predator-prey system, the two nodes stand for the numbers of predators and preys, and the arrows for self-promotion via birth, decay via death, and, of course, mutual inhibition and promotion.

What would the network of a digestive tract look like? Well, we want to model the transformation from feed in the beak to feces at the back (Figure 9). In the beak the feed has a certain composition. Let's indicate the nutrients by  $N_1$ ,  $N_2$ , et cetera.

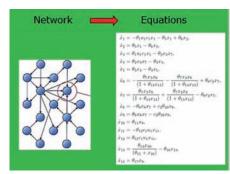
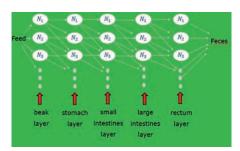


Figure 8 Network representation of a mathematical model and its translation into mathematical formulae.



**Figure 9** Network of a digestive tract. The feed contains nutrients  $N_1, N_2, \ldots$ , with fractions changing at the different stages of the tract.

The amounts of these nutrients are stored in the nodes in the first layer. For each stage of the tract—stomach, small intestines, large intestines, rectum—we have such a layer of nutrient nodes. From layer to layer the contents of these nodes change under influence of digesting enzymes and microbiome processes. Here, we will pay no attention to the translation step; for now it is only important to keep in mind the typical network topology, with layers and a unidirectional flow from beak to feces.

## Top-down modelling

Let's switch to the right hand side of the modeling axis (Figure 4). There, we meet with, amongst others, classical statistical models, but I will pass them to leave room for the farewell address of my colleague Fred van Eeuwijk. On the utmost right hand side we enter the domain of artificial intelligence and big data science where neural networks and other machine learning approaches are used. They are designed to cope with systems that are way too complex for us to understand. Think of the human brain, our economy, or the climate. Can we achieve modelling progress in such cases? Yes, we can, but only if we accept a paradigm shift.

If we have no idea of the inner structure of a system, the only thing we can do is to observe its behaviour as response to as many different inputs as possible. That's to say, to gather data. The more, the better. And from these data, we may try to mimic the system. This approach seems a bit poor, but may be very powerful, as I will show you.

A system far beyond our bottom-up modeling abilities is typically the human brain. Our brain is nearly unbeatable as it comes to pattern recognition. Expose a human, e.g., one of my grandsons, to dozens of pictures of cats and dogs in all kinds of poses, and he may flawlessly classify the dogs and the cats, putting the dog pictures on one pile and the cats on another.

#### Networks, deep learning

Given enough data, top-down models are amazingly good in imitating this kind of abilities. But, if we have no idea what's happening in the brain when recognizing patterns, what kind of network should we put in the modelling box (Figure 1) to imitate the brain? What about its topology?

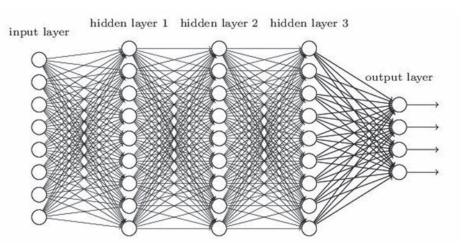
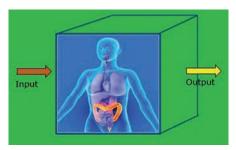


Figure 10 General structure of a neural network, characterized by a layered structure and a unidirectional flow of information.



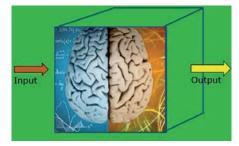


Figure 11 The structure of a neuronal network rather reflects the structure of our digestive tract than that of our brains.

What about the interactions? Since we have no clue here, we are free to take a fantasy network with fantasy interactions. One would expect a neural network to mimic the neuronal network in our brains. So, with neurons interconnected in a complicated fashion. But terminology is misleading here. In practice, neural networks have a layered topology and a unidirectional flow (Figure 10).

Does this ring a bell? Of course, we recognize the structure of a digestive tract (Figure 9). So, we conclude that neural networks rather reflect the structure of our digestive tract than that of out brains.

Recently, so-called deep learning networks have become popular. Roughly speaking they are very similar to neural networks, but then with many more layers. So, one could say that a standard neural network reminds us of the digestive tract of a carnivore, whereas a deep learning network is more like the digestive tract of a herbivore. These similarities should, of course, not evoke the suggestion that these networks deliver only feces. Not at all.

How does a neural network function? You remember that the second step in mathematical modeling is the translation of a network into formulae (Figure 8). In these formulae parameters are present. These are numbers that are typical for the specific system under study. In bottom-up models the parameter values are either dictated by physical laws or deduced from experiments. In top-down models we do not have that information; the values of these parameters have to be estimated from fitting the model to the data. Therefore, as said before, the more data, the better. People are used to say that they "train a network" — like a dog —, simply meaning that its parameter values are estimated from data.

# Artificial intelligence

For certain applications top-down models are indeed the way to go. To come back to the chess example, in the artificial intelligence world it has been a long-standing discussion whether computers could ever play inherently complicated games like *Chess* and *Go* at world-champion level. The Dutchman Max Euwe (1901–1981) and the Russian Michael Botwinnik (1911–1995) — chess world champions in the thirties and fifties of the last century, respectively — were heavily involved in developing computer chess algorithms. They



**Figure 12** In 1997, for the first time in history, a world champion, Kasparov, was beaten by a computer programme: a turning point in artificial intelligence.

followed the bottom-up approach and implemented strategies they thought humans apply when playing chess. In their days, computers were still simple, so the level of playing remained relatively low. In later years the computers got such an enormous computational power, that in 1997 world champion Kasparov was beaten by the computer program Deep Blue (Figure 12).

As seen in Figure 12, this was quite a dramatic event, since Kasparov had claimed that this was impossible. Deep Blue was based on a combination of strategic rules and brute force, since at that time the computer could already calculate the consequences of many, many moves in a split second. However, this bottom-up approach turned out to have limitations, even with supercomputers, since it failed when applied to a much more complex game like Go.

Euwe, Botwinnik, Kasparov, they all couldn't foresee the paradigm shift caused by deep learning. In December 2018 the so-called 'AlphaZero' team reported (DOI: 10.1126/science.aar6404) that they had trained a deep learning network that could beat the world champion Go. A network with the same topology could also play Shogi and Chess at that very high level. Only the

parameter values were different for each of these games. Taking notice of these developments, Kasparov made a comparison that will resonate with biologists:

"Much as the Drosophila Melanogaster fruit fly became a model organism for geneticists, chess became a Drosophila of reasoning".

What kind of data did the AlphaZero team use? Data from the immense literature on Chess or Go? No. They simply got their data from letting a computer playing Chess or Go against itself. In this way they could practically obtain infinitely many data. The deep learning network was not explicitly fed with strategies but, on the contrary, deduced these strategies from the data and stored them in an implicit way, namely in the values of parameters.

It is clear that these techniques will deeply influence our society, for example via robotics. It also evokes philosophical questions. Are we creating real intelligence, consciousness, monsters? I don't think so. Although the achievements with deep learning are very impressive, from a modelling point of view this approach leaves us with an unsatisfactory feeling. After the training stage, the deep learn-

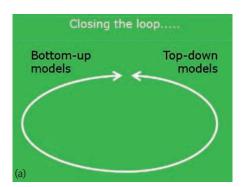
ing network may mimic the system under consideration perfectly well, but we haven't gained any insight in the system. The full training information has been stored in the values of parameters, but these parameters don't bear any interpretable relationship with the functioning of the system itself.

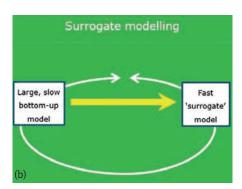
#### Closing the loop

I think that the challenge of the coming years is to close the loop. The modelling axis should be bent and closed in itself, so that the extremes meet each other (Figure 13a). A first, partial attempt is a research topic within my group. The idea is to start with a complex bottom-up model. Because of its complexity, analysis in the form of, for example, sensitivity analysis is obstructed by extremely long computation times. A top-down model is then trained, a so-called surrogate model, that mimics the original model but is ways faster to handle and analyze (Figure 13b).

From a more fundamental point of view, I expect that in some way it must still be possible to interpret neural networks (Figure 13c). The learning process during training must contain information to deduce how the system internally works, at least globally. This seems me one of the most intriguing problems artificial intelligence presents us nowadays.

By the way, do you remember our cliff hanger: the thrilling secret of the chess Turk. As you may have guessed, inside the box a human was hidden, undetectable for the audience, thanks to an ingenious system of mirrors and a moveable seat (Figure 14). So the mechanical Turk, representing a human, was modeled by a human. It's like a digital twin, but then not digital. More bottom-up is simply not possible!





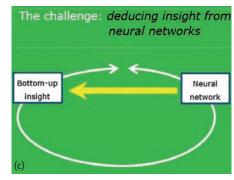


Figure 13 The modelling axis should be bent, such that bottom-up and top-down modeling fertilize each other (a). A large and computationally hard to handle bottom-up model can be replaced by a surrogate model, trained on data from the bottom-up model (b). Deducing insight in the functioning of a system by studying the neural network by which it is modeled is the challenge we are facing nowadays(c).

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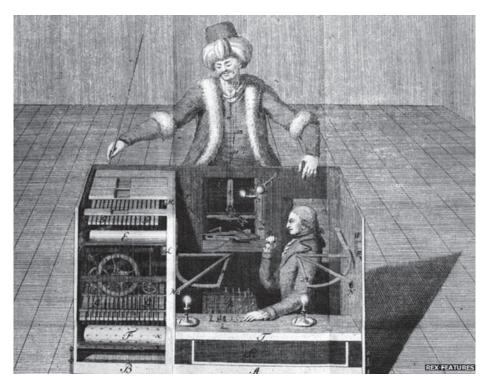


Figure 14 The secret of the chess playing robot was a human hidden inside the box: the utmost form of bottom-up modeling!

#### Law of Benford

I promised to explicitly show you the beauty and power of mathematics via an example. The example I have chosen is an appetizer for a course we developed about the extremely important role played by 'scaling' in nature and thus in modelling.

It is generally believed that with each formula in a book or presentation one loses half of the audience. This goes back to Stephen Hawking who wrote:

"Someone told me that each equation I included in the book would halve the sales".

I challenge you to belie this claim. This story starts with a guy called Newcomb who consulted a book with logarithm tables in the library of his institute (Figure 15).

To his surprise, he observed that the pages in the first half of the book were much more worn and dog-eared than the later pages. Why were these initial pages used so frequently? As you remember, these pages are related to numbers that start with the lower digits. So, numbers starting with 1, 2 or 3, occur much more often than numbers starting with 7, 8 or 9. He started to think about this strange phenomenon, reported it in 1881, but couldn't find an explanation. Later, in 1938, Benford rediscovered the phenomenon and understood the essence.

I will explain this remarkable phenomenon to you in a my own, very condensed way, showing how powerful math can be.

Assume you generate a data set by measuring the size of randomly picked objects. From the size of a bacterium, to the length of your nose, to the height of your house, to the height of the Eiffel tower, et cetera. This results in a set of numbers spanning many orders of magnitude.

There must be a probability distribution, p(x) say, that tells you what the probability

is for a number to be chosen. For the time being we have not the slightest idea how p(x) looks like, but this ignorance will not last long. I think most of you still remember that the probability P(a,b) of picking a number in the interval [a,b] is given by the integral of p(x), from a to b (Figure 16).

Now the scaling argument comes in. All of a sudden, we realize that all the numbers in our dataset have been measured in meters. But there was no reason to select meters as unit of length. We could have used decimeters, centimeters, micrometers, kilometers, or whatever unit as well. This implies that the horizontal x-axis could have been arbitrarily squeezed or stretched. So, if I replace the interval [a,b] by the interval  $[\gamma a, \gamma b]$ , with  $\gamma$  an arbitrary positive constant, the chance to find a number in that new interval must be the same as in the original interval. This implies that the form of p(x) must be such that the two shaded areas in Figure 17 are equal.

A picture is informative, but as I already said above, a formula is better. Scaling leads to the condition

$$P(\gamma a, \gamma b) = P(a, b),$$

or

$$\int_{\gamma a}^{\gamma b} p(x) dx = \int_{a}^{b} p(x) dx.$$

It is hard to believe, but this condition is enough to derive the distribution p(x) and thus the probability P(a,b). The mathematicians among us will immediately see that if we differentiate the integrals on both sides with respect to b, apply the chain rule, and set then b equal to one, we find

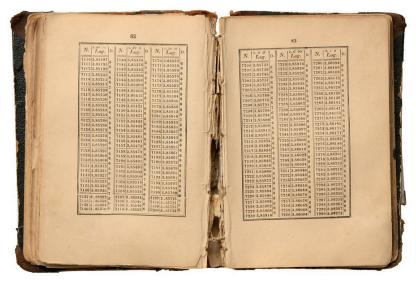
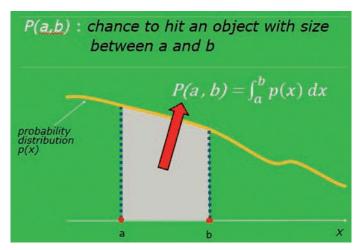
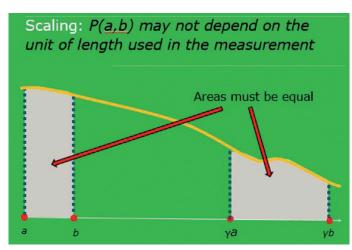


Figure 15 The first pages of the book with log tables were much more worn than the last pages, as Newcomb observed.



**Figure 16** The probability P(a,b) to randomly pick a number in the interval [a,b] is given by the integral of p(x) from a to b.



**Figure 17** If we stretch or squeeze the horizontal axis, the shaded area, representing P(a,b), must be conserved.

that p(x) behaves as:

$$p(x) \sim 1/x$$
,

which implies that

$$P(a,b) \sim \log(b/a)$$
.

Since P(a,b) only depends on the quotient of b and a, we find that the chance to pick a number between 1 and 2, say, is equal to pick a number between 10 and 20, or between 100 and 200, et cetera. So, this distribution of chances is for each decade the same. In words:

- chance of picking a number starting with 1 is proportional to  $\log(2/1)$ ,
- chance of picking a number starting with 2 is proportional to  $\log(3/2)$ ,
- et cetera.

If we plot these chances (Figure 18), we find a distribution that completely explains the observations by Newcomb and Benford. The chance to find 1 as first digit is about 6 times higher than the chance to hit 9 as first digit.

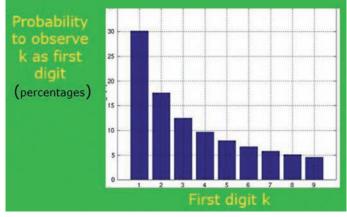
This phenomenon does, of course, not only hold for data with the dimension of length. The only essential issue is that the data span several orders of magnitude. All kinds of data turn out to follow the Law of Benford. For example, the numbers of inhabitants of USA towns obey, unexpectedly, this law nearly perfectly.

In practice, the Law of Benford is used to detect fraud: if someone is manipulating data or even generating artificial data, he/she usually forgets to take into account this law. From that omission the malversation can easily be detected. You may ask: has this relevance for Wageningen? As for fraud, I don't hope so. But Wageningen UR is a huge data generator. In applying statistical regression models, one often applies the log transform. It is good to know that this is not a calculation trick, but is based on sound scaling arguments.

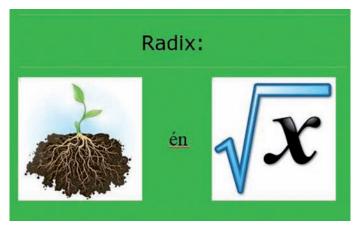
I wonder whether I lost half of my audience. Time for another shock: we switch to Dutch.

#### Tot slot

De uitslagen van de MedewerkerMonitor - een regelmatig afgenomen werktevredenheidsonderzoek - spreken boekdelen: Biometris eindigde de laatste twee keren van alle wetenschappelijke WUR-groepen bovenaan. We begonnen 13 jaar geleden vrij klein. Experimenteel ook: een fusiegroep bestaande uit één leerstoel Wiskunde en Statistiek en een DLO-groep, oftewel de commerciële afdeling Biometrie. Fred van Eeuwijk en ik bezetten samen die éne leerstoel. Dat was een beetje naïef bedacht door het toenmalige management. Met twee kapiteins op één schip had dat wel eens geweldig fout kunnen gaan. Maar in de praktijk bleek het een geweldig geluk dat we qua persoonlijkheden gigantisch verschillen. Na een periode van zoeken vonden we de perfecte rolverdeling. Fred werd minister van Handel en ik minister van Binnenlandse Zaken. Dit uiteraard naast ons dagelijks onderzoek en onderwijs. Fred heeft zijn handelsmissies



**Figure 18** Probability distribution to find k as first digit according to the Law of Benford.



**Figure 19** De naam Radix van onze huisvesting heeft een dubbele betekenis: het gebouw huisvest zowel plantenwetenschappers als wiskundigen.

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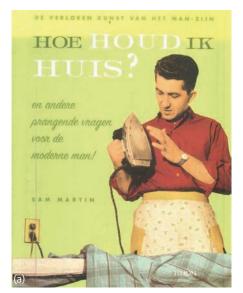






Figure 20 Handleiding "Hoe houd ik huis" voor de moderne man, al of niet met pensioen.

op fantastische wijze tot een succes gemaakt. Biometris groeide gestaag dankzij drie factoren: de vele externe projecten, de stijgende studentenaantallen, en de overname van collega's van andere groepen - soms zeven medewerkers tegelijk die kwantitatief bezig waren, zich niet op hun plaats voelden, en zich daarom graag bij ons aan wilden sluiten. Daardoor werd binnenlandse zaken steeds belangrijker. Echter, doordat we financieel nogal goed boerden, heb ik mijn taak altijd als relatief licht ervaren. Medewerkers vragen wel eens: wat gebeurt er met al die miljoenen in onze reserves? Wel, daarmee worden armlastige groepen, die toch wel waardevol zijn voor de wetenschap, maar niet zo gemakkelijk geld binnenhalen, gesteund. Dat doen we graag, want zoals de Bijbel al zegt: "Het is zaliger te geven dan te ontvangen."

Dankzij Biometris heb ik me hier in Wageningen altijd als een vis in het water gevoeld, of met een toepasselijker beeldspraak: als Arabidopsis Thaliana in Radix: een onooglijk plantje, maar waardevol en gewaardeerd in de wetenschap. Toen onze huisvesting, het gebouw met de fraaie naam Radix, dat wij delen met de plantenwetenschappers, bijna klaar was won ik de naamgeefwedstrijd. Wat ik nou zo jammer vind is dat zo veel mensen de diepere laag in die naam niet meer beseffen. Want Radix betekent niet alleen wortel in de plantenzin, maar ook wortel in wiskundige zin (Figure 19). Bedenk dus, als u langs gebouw Radix fietst, dat het zowel plantenals wiskundegeleerden herbergt.

Aan het begin van mijn wetenschappelijke loopbaan schafte ik mijzelf een poster aan om mijn werkkamer op te fleuren. Die poster is alle verhuizingen meegegaan. Vooral vanwege de tekst: 'Wie in Mij blijft, en Ik in hem, die draagt veel vrucht'. Jezus zegt dat in het kader van de bekende gelijkenis van de wijnstok en de ranken in Johannes 15. De poster geeft precies aan hoe

ik mijn werk altijd heb willen doen: in afhankelijkheid van God, onze Schepper. En u begrijpt hopelijk wel dat met die vrucht niet bedoeld is het produceren van zo veel mogelijk wetenschappelijke artikelen. Maar het impliceert wel de vraag: heb ik werkelijk iets betekend voor God en mijn collega's? Zo'n simpele poster heeft al die tijd een prachtige belofte gevormd, maar roept me bij zo'n afscheid als vandaag ook ter verantwoording.

Hoewel mijn vrouw Gerda en ik erg gelukkig zijn met elkaar en elkaar doorgaans goed aanvoelen, vraag ik me de laatste maanden serieus af of ze wel de juiste verwachtingen heeft van mijn tijdsbesteding in de pensioenjaren. Ze schonk me ter voorbereiding het boek *Hoe houd ik huis* (Figure 20a), waaruit ik zou moeten leren hoe een moderne man, al of niet met pensioen, gras maait (Figure 20b) of knijpers aanreikt (Figure 20c). Dat grasmaaien zal nog wel gaan, maar die knijpers...