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History

Shreds of memory of Paul Erdős

Paul Erdős has not been among us for more than twenty years. Vilmos Komornik, who had written various papers with Erdős, was asked by several friends to recall some of his memories: maybe they can shed light on his personality from a new perspective.

Zürich, 2–9 August 1994

I first met Paul Erdős personally at the International Congress of Mathematicians (ICM). We already had two joint papers, but they were done by correspondence through our third co-author István Joó. He was sitting in the middle of a large room surrounded by a lot of friends and interested people. I didn't dare to bother him. He suddenly stood up and came straight to me (he probably asked about me, and someone in the company showed him where I was): "Hello! Excuse me for not having greeted you before, but you know my memory of faces is not good. We already have two joint papers, don't we? How are the problems?" I almost sank in shame. We talked for a few minutes and he offered to continue our research collaboration.

Strasbourg, 20–24 October 1994

He sent me a message by e-mail that he would like to visit Strasbourg if I felt like it and had time to work with him. I seized the opportunity. Uncle Paul arrived from Nancy on Thursday afternoon. He waited on the platform of the railway station, smiling, and holding a small suitcase and a nylon bag. I took him home. He quickly got to know my family and wanted to work right away. We went into the room reserved for him. He closed the door, sat in the armchair, and asked me to tell him a new proof I had found.

I was secretly proud of my good teaching skills. He thought differently: "I do not understand. I do not understand a word. Have you ever explained mathematics to someone in your life? Write down your two lemmas without proof and then I will read them."

What could I do? I started to write. Then my 11 year old daughter Tímea appeared, who couldn't bear to be out of the room. She asked me to look at her German homework. Uncle Paul impatiently interrupted, "I'll look, I know German, you just write the two lemmas." See Figure 1. He ran through the two pages in half a minute, then said to Tímea: "Here is an error: not *der*, but *das*. The rest is good, you can leave." My daughter was outraged to be sent away so quickly, surely leaving many other mistakes in her homework (the next day it turned out that there were no more). Meanwhile, I had written down the two lemmas. Uncle Paul read them, they seemed right to him, and he calmed down.

By then dinner was ready. While eating, he asked my wife what her profession was and what was the subject of her master thesis. After her answer — "The press coverage of the Mexican campaign of Emperor Miksa" — she continued to explain what this campaign was, because she herself did not know anything about it before obtaining her university assignment, but Paul Erdős stopped her: "I know, Napoleon III appointed him to be an emperor there and he was executed in Mexico in 1867."

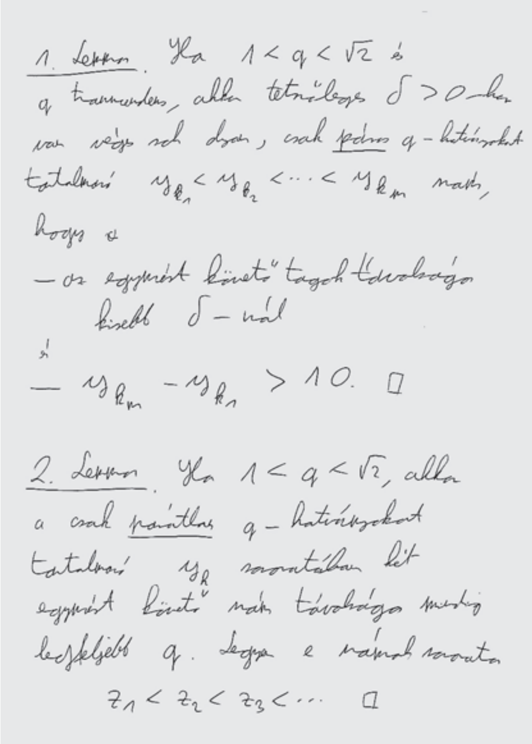
The next morning we worked at the department. I had an idea to improve our results by modifying the 'two lemmas'. "It doesn't work", he said. "But I feel it does", I insisted and began to count. In half an hour, I gave up, "Unfortunately, it doesn't work." "I told you", he replied calmly.

At noon I had to bring my son Zsolt home from school for lunch; Uncle Paul stayed in my office. In front of the school, another dad, a high school math teacher, told me that he and his colleagues had been trying to prove for weeks that all real fractions can be written as the sum of Egyptian fractions. I had never heard about Egyptian fractions before. Returning to the office, I repeated the question to Uncle Paul, who put a piece of paper in front of him, and quickly wrote down the proof: "I wrote an article on the subject, I remember it."

We went home for lunch. Returning to the library, Paul Erdős forgot the name of a mathematician who found a simple proof of one of his theorems with Szekeres. But he remembered that the article was published in 1959 in the *Journal of the London Mathematical Society*, and so he immediately found the name of Seidenberg. I also found his article about the Egyptian fractions in the library: it was written in 1950, and he remembered the proof word by word.

Erdős–Szekeres

Erdős and Szekeres proved that every sequence $x_1, x_2, \dots, x_{n+1}$ of distinct real numbers contains either an increasing or a decreasing subsequence of length $> \sqrt{n}$. Seidenberg's proof associates with each x_k an integral vector (k^+, k^-) recording the lengths of the longest increasing and decreasing subsequence ending at x_k. If $j \neq k$, then $(j^+, j^-) \neq (k^+, k^-)$. Since the box $[1, \sqrt{n}] \times [1, \sqrt{n}]$ contains at most n integral vectors, one of the (k^+, k^-) has to be out of the box.



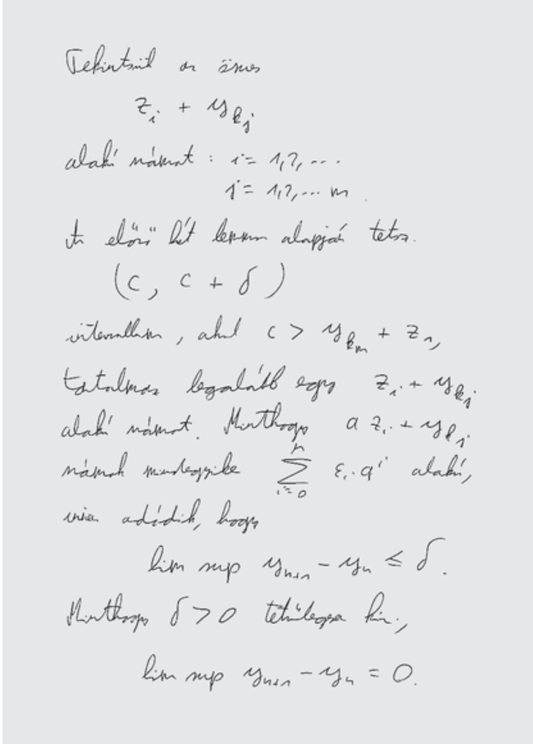


Figure 1 The two lemmas.

Lemma 1. If $1 < q < \sqrt{2}$ and q is transcendental, then for each $\delta > 0$ there exist finitely many $y_{k_1} < y_{k_2} < \cdots < y_{k_m}$ numbers, containing only *even* powers of q [i.e., polynomials of q^2 with coefficients 0 and 1], such that the distances of consecutive terms are smaller than δ, and $y_{k_m} - y_{k_1} > 10$.

Lemma 2. If $1 < q < \sqrt{2}$, then in the sequence of numbers y_k containing only *odd* powers of q the distances of consecutive terms are at most q. Let us denote this sequence by $z_1 < z_2 < z_3 < \cdots$. Consider all numbers of the form $z_i + y_{k_j}$: $i = 1,2,\dots,$ $j = 1,2,\dots,m$. By the preceding two lemmas every interval $(c, c+\delta)$ where $c > y_{k_m} + z_1$ contains at least one number of the form $z_i + y_{k_j}$. Since each number $z_i + y_{k_j}$ is of the form $\sum_{i=0}^{n} \varepsilon_i q^i$, it follows that $\limsup y_{n+1} - y_n \le \delta$. Since $\delta > 0$ is arbitrarily small, $\limsup y_{n+1} - y_n = 0$.

These lemmas were used to prove Theorem 5 in our paper in collaboration with I. Joó: On the sequence of numbers of the form $\varepsilon_0 + \varepsilon_1 q + \cdots + \varepsilon_n q^n$, $\varepsilon_i \in \{0,1\}$, *Acta Arithmetica* 83(3) (1998), 201–210; see also Lemmas 2.2 and 3.2 in Developments in non-integer bases, *Acta Math. Hungar.* 79(1–2) (1998), 57–83.

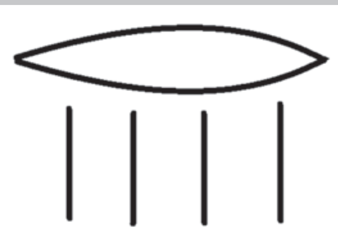
Egyptian Fractions

During the middle kingdom, around 1800 BC, the Egyptians developed notation for the unit fractions $\frac{1}{k}$. In number theory, an Egyptian fraction is defined as a finite sum of distinct unit fractions such as

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{16}.$$

Every vulgar fraction (positive rational) can be represented as an Egyptian fraction. In his *Liber Abaci*, Fibonacci includes several methods to convert vulgar fractions to Egyptian fractions. The challenge is to find a representation with as few unit fractions as possible. The Erdős–Strauss conjecture predicts that every fraction $\frac{4}{n}$ can be represented as a sum of three.

The result on Egyptian fractions that Erdős remembered and quickly wrote down was: if $b > a$, then there exist a presentation of $\frac{a}{b}$ by at most a distinct unit fractions. A solution may be



A hieroglyph for the fraction $1/4$. The denominator is represented by vertical bars.

constructed by the greedy algorithm, by observing that if x is the smallest natural number satisfying $\frac{1}{x} \le \frac{a}{b}$, then $\frac{a}{b} - \frac{1}{x} = \frac{a_1}{b_1}$ with $a_1 < a$ and $b_1 > b$. The Erdős–Strauss conjecture thus says that one can do better than the greedy algorithm if $a = 4$. It is contained in the 1950s article by Paul Erdős: *On the solutions of the equation* $\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n} = \frac{a}{b}$, which is written in Hungarian, but has an English abstract: "Denote by $N(a,b)$ the smallest integer n so that

$$\frac{a}{b} = \frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}, \quad x_1 < x_2 < \cdots < x_n,$$

is solvable in positive integers x_i. Sharpening previous results of Nakayama, Strauss and de Bruijn, the author proves that for $1 \le a < b$,

$$N(a,b) < c_1 \frac{\log b}{\log\log b}$$

and

$$\sum_{a=1}^{b-2} N(a,b) > c_2 b \log\log b.$$

It seems likely that for $1 \le a \le b$,

$$N(a,b) < c_3 \log\log b.$$

Nakayama proved that $N(3,b) = 3$ if and only if all prime factors of b are of the form $6k+1$. Strauss and the author conjecture that $N(4,b) < 4$ for every $b \ge 4$, Strauss proved this for $b < 5000$."

In the afternoon, he gave a lecture at our department on “some of my favorite problems in number theory, in combinatorics, and in geometry”. So many people came to listen to him that we had to change the room. Luckily, the big lecture hall was free, although it was unheated. Paul Erdős held his presentation in a coat. He spoke in English, but he repeated the statements in French. It had a big impact on the audience, who until then knew almost only the radically different Bourbaki style.

We went to the city on Saturday. I showed him a memorial plate that the French national anthem, the *Marseillaise*, was first sung here. “Rouget de l’Isle had a difficult life: I read about it from Stefan Zweig”, he replied. I knew nothing about Rouget de l’Isle’s life until then.

When we arrived at the Cathedral he did not like going up the tower: “I’ve been there twenty years ago, I remember it.”

There were mendicants in the square. He gave money to everyone, then they returned to him, and he gave again. I had to take him away before he distributed all his money.

I suggested to look at the organ of the St. Thomas Church on which Mozart played two centuries ago. He was interested. We sat in a car and I maneuvered for twenty minutes before I finally found a nearby parking place. But we didn’t spend half a minute in the temple. He went straight to the organ, looked up, read the brochure, then turned around and went out: “Let’s go back to work.” I was sure he learned the text of the tutorial by heart.

In the evenings, he liked to listen to music, usually the Brandenburg concertos from Bach. When he traveled to Lyon on Monday, he borrowed from us István Diószegi’s book on the history of diplomacy: *In the Shadow of Two World Wars*, that he found on our bookshelf. We gave it to him a little bit afraid, because it would have been difficult to get another copy, but in two weeks he sent it back to us from Sao Paulo. (This was his only visit to South America, at the age of 81.)

Budapest, 3 August 1995

Paul Erdős gave a lecture at SZTAKI (Institute for computer science and control of the Hungarian Academy of Sciences). The small room was full of people but, despite the heat wave, all windows were shut so that the noise of the outside road repair did not interfere with the talk. He talked about one of his results with Zsolt Tuza, when Zsolt interrupted him: “This is the proof of the other direction.” “Yeah, of course”, he replied and wanted to start the other proof, but he suddenly stopped and began to feel dizzy. The organizers emptied the room, opened the windows, laid Uncle Paul on four chairs and called an ambulance. As a result of the fresh air, he came to himself, “Is Tuza here?” he asked in a dying voice. They called Zsolt. “How was the proof?” he asked. Zsolt began to talk to him, and Uncle Paul, lying on the chairs, became more and more revitalized. He told the arriving rescuers that he was all right now, and it was very difficult to convince him to go with them for a medical check.

Strasbourg, 23–26 September 1995

At the beginning of September, he phoned me at home while I was at the university. My wife wanted to give him my office phone number. “Thank you, but I remember the phone number”, he replied, and then he called me. He hadn’t forgotten the phone number since last year.

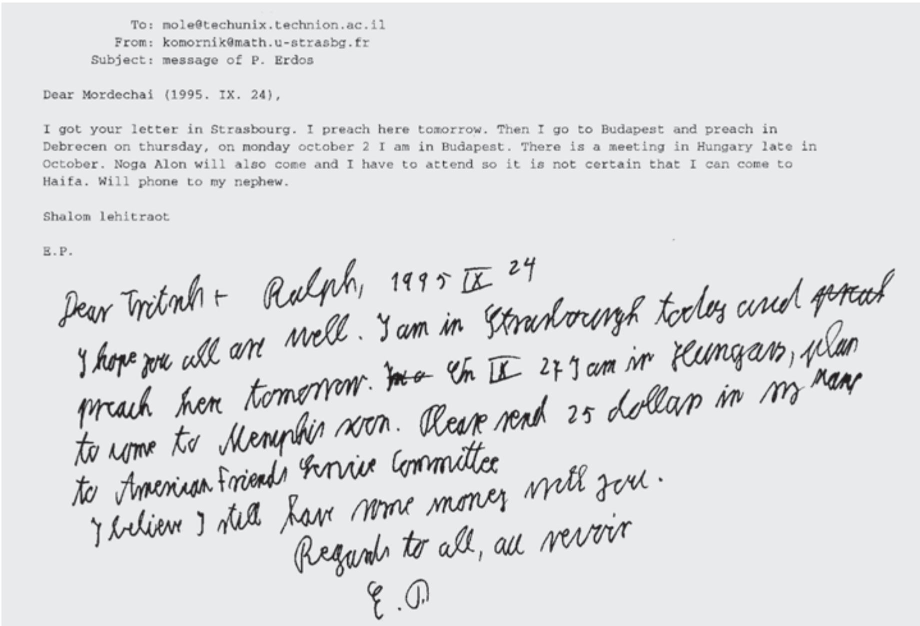
He arrived on Saturday afternoon. In the apartment, my daughter received him saying that she already knows German. Uncle Paul immediately switched to German, talking to her. Timi had a hard time to understand him, and he was wondering: “Why do you say you know German if you don’t know?”

Uncle Paul didn’t find his sandals: either left in Nancy or left on the train. He called Tenenbaum, who found them under a chair. “Shall I send them to you?” he asked. “No, I just wanted to know where I lost them. Throw them away, they were already old”, he replied. We worked after dinner, and after 11 pm he asked, “Is it true that you have a key to the library?” True, I replied and we were there 15 minutes before midnight. Paul was browsing the freshly-received journals with great attention. He stepped out of the library at 1 am with satisfaction.

We spent the Sunday morning in my office. He wrote several letters that I forwarded by fax or e-mail to the addresses provided. He asked one of his colleagues to send 25 dollars in his name to the ‘American Friends Service Committee’ (see Figure 2). Almost every day, he made small donations for humanitarian purposes or to promote mathematics.

In the afternoon he found *Set Theory by Hajnal and Hamburger* on my bookshelf and read it carefully. Suddenly he exclaimed, “Yeah, there’s a mistake!” I was incredulous: remembering András Hajnal’s precise and elegant university lectures, I couldn’t imagine that there might be a mistake in the book. But he persisted: “They write that Gödel proved in 1939 that the continuum hypothesis is consistent with the usual set theory axioms. But it was in 1938, not in 1939. I remember because I was sitting in a café with him in Vienna when he told me and it was in 1938.”

In the evening we celebrated my son’s birthday together.



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To: mole@techunix.technion.ac.il
From: komornik@math.u-strasbg.fr
Subject: message of P. Erdos

Dear Mordechai (1995. IX. 24),

I got your letter in Strasbourg. I preach here tomorrow. Then I go to Budapest and preach in
Debrecen on thursday, on monday october 2 I am in Budapest. There is a meeting in Hungary late in
October. Noga Alon will also come and I have to attend so it is not certain that I can come to
Haifa. Will phone to my nephew.

Shalom lehitraot

E.P.
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Dear Tritsch + Ralph, 1995 IX 24
I hope you all are well. I am in Strasbourgh today and ~~preach~~
preach here tomorrow. ~~Tue~~ On IX 27 I am in Hungary, plan
to come to Memphis soon. Please send 25 dollar in my name
to American Friends Service Committee
I believe I still have some money with you.
Regards to all, au revoir
E.P.

Figure 2 Letter from Paul to Patricia and Ralph Faudree outlining his itinerary and asking to donate 25 dollars on his behalf.

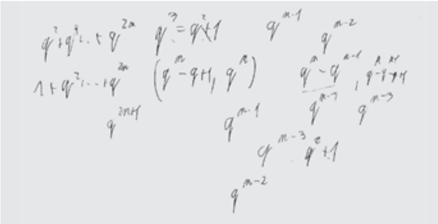


Figure 3 Scraps of work done in Strasbourg on expansions in base q. For $1 < q < 2$, we investigated $\liminf y_{n+1} - y_n$ and $\limsup y_{n+1} - y_n$ where (y_n) is the increasing sequence of the polynomials of q with coefficients 0 and 1. For example, $y_0 = 0$, $y_1 = 1$, $y_2 = q$ and $y_3 = q^2$ or $y_3 = q + 1$ depending on whether $q \leq (1+\sqrt{5})/2$ or $q \geq (1+\sqrt{5})/2$.

We had lunch together with my German colleague Petra Wittbold on Monday. After Uncle Paul learned that she was from Essen, he switched from English to German (which I don't speak) for a few minutes. Later Petra said that Paul Erdős had told her several interesting stories about medieval Essen in perfect German. And she was very ashamed because she knew almost nothing about the history of her hometown. In the afternoon, Uncle Paul gave another talk with great success at the department. The royalties were just enough to buy his flight ticket to Budapest the next day. In the evening, he escorted us to the ping pong club of my son Zsolt, and he played with him.

On Tuesday morning we went to Germany to buy him new Birkenstock sandals. He found a pair: they weren't really comfortable, but he bought them anyway. I paid for him in marks, and he immediately gave me the amount in francs. Later, I checked that it was correct to a penny. Meanwhile, he told me that András Hajnal once reacted to his constant complaints: "If we were able to send people to the Moon, we can make comfortable shoes for you!" "Now in this, Hajnal was wrong", Uncle Paul commented.

Strasbourg, 5–8 April 1996

On 3 April, he suddenly called me to ask whether he could spend the Easter weekend with us and arrived two days later. I showed him a new proof I had found for the Erdős–Mordell inequality: it seemed to him new and simpler than the earlier ones. In his book *Geometric Inequalities*, written in 1961, Kazarinoff meditated on how Paul Erdős discovered this theorem: "One possibility is that he generalized Euler's Inequality $R \geq 2r$." To my question, Uncle Paul answered that he had discovered the inequality in 1932, after having drawn a lot of triangles to find new laws. Interestingly, neither Kazarinoff, nor anyone else asked him about it during more than sixty years. How much more could we have learned from him if we only had asked!

"Did Einstein believe in God?" I asked once. "No," he replied categorically. "How do you know?" I asked again. "I asked him once."

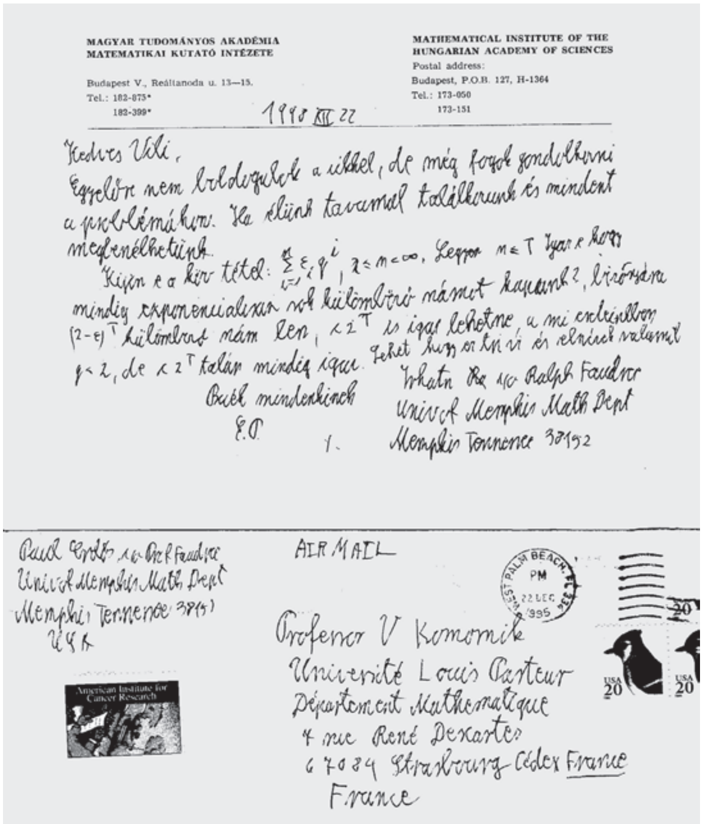
He had to take his train to Marseille on Sunday morning. But when I went to his room in the morning, he said he didn't feel good, he couldn't go: "It's not a serious thing, just lie back and continue to sleep." Half an hour later, when it was too late to reach the train, he suddenly got better and got up.

But after breakfast, he said there was a problem: "The train will arrive in Lyon in the afternoon and András Sárközi will join the train as planned. If he doesn't find me on the train, he'll panic." Somehow Sárközi should be notified that Erdős was unable to travel, but there was nothing

Essen

The town of Essen grew around an abbey for wealthy women that was founded around 845. Interestingly, celibacy was not mandatory. In 1216, Holy Roman Emperor Friedrich II declared the abbess to be a Reichsfürstin, which led to a power struggle between abbess and city council that lasted for centuries. Coal mines opened up in 1450 and the town quickly became a centre of the weapons industry, ruled by the Krupp family dynasty since the end of the sixteenth century.

Essen coat of arms



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AIR MAIL

Figure 4 Letter from Paul dated Dec. 22, 1995. It says: "Dear Vili, For the time being I do not make progress with the paper but I will think more about the problems. If we will still be alive in Spring, we may meet and talk about everything. Is the following theorem true: $\sum_{i=1}^{n} \varepsilon_i q^i$, $1 \leq n < \infty$. Let $n \leq T$. Is it true that we always get exponentially many different numbers? We should have $(2-\varepsilon)^T$ different numbers, maybe even $c2^T$, in our case $p < 2$, but maybe $c2^T$ is always true. Maybe it is trivial and I overlook something. Happy New Year to everybody, E.P. You can write c/o Ralph Faudree, Univ. of Memphis Math. Dept., Memphis Tennessee 38152."

Erdős–Mordell

The Erdős–Mordell inequality says that for any internal point P of a triangle, the sum of its distances to the sides is at most half the sum of its distances to the vertices.

Komornik's proof of the inequality, which can be found in issue 104 of the *American Mathematical Monthly* (1997), is as follows. The areas of the colored triangles sum up to

$$\frac{r_a a + r_b b + r_c c}{2}.$$

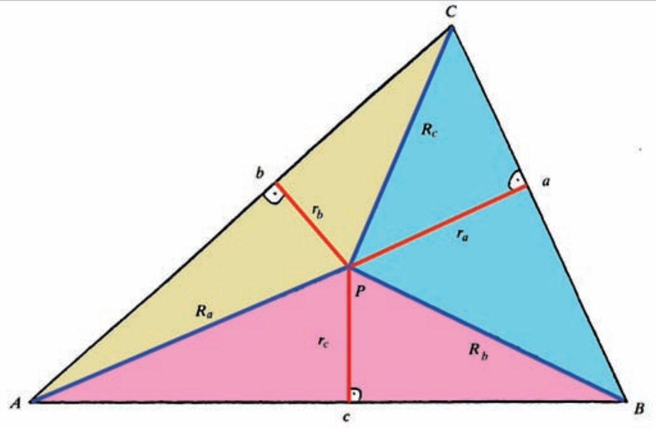
The distance between vertex A and side a is bounded by $R_a + r_a$. Therefore the area of the entire triangle is bounded by $(R_a + r_a)\,a/2$ and we get that

$$R_a a \geq r_b b + r_c c.$$

Here's the trick. We leave it to the reader to verify that this inequality remains valid for any P that is inside the sector $\angle(BAC)$, even if P is outside the triangle. If we reflect P in the bisector of $\angle(BAC)$ then r_b and r_c are switched while all other quantities remain the same. We get that

$$R_a a \geq r_b c + r_c b.$$

By symmetry, this holds if we permute a, b, c and we find



Distances r of P to the sides are red. Distances R to the vertices are blue.

$$R_a + R_b + R_c \geq r_a\Big(\frac{b}{c} + \frac{c}{b}\Big) + r_b\Big(\frac{a}{c} + \frac{c}{a}\Big) + r_c\Big(\frac{a}{b} + \frac{b}{a}\Big) \geq 2r_a + 2r_b + 2r_c,$$

where we have used that $x + \frac{1}{x} \geq 2$. Which solves the problem on the triangle but leaves us with the question: what is the Erdős–Mordell inequality for the simplex?

serious. "He may travel to Marseille without feeling anxious, and then I call him by phone in the evening." There were no cell phones yet, so I called a friend in Lyon: "You should go to the Lyon Perrache railway station at noon, find a mathematician named András Sárközi and pass him on the following message..." I didn't know how he did it, but my friend solved it. (I learned it from Sárközi twenty years later: he went to the math department in the morning, and my friend found him there.)

Meanwhile, we had lunch at home and Uncle Paul looked out of the window. "What are those things there? Mountains? Are they far away?" "Yes, the Black Forest in Germany, at about 20–30 kilometers from here." "Could we go there?" We sat in the car and went on a trip.

During this time the train arrived in Marseille. Sárközi, having traveled very nervously for five hours, ran to the first phone and called us. "Uncle Paul went on a trip with my husband to the Black Forest", my wife replied. In the evening, after we got back, the phone rang again. Vera T. Sós, Sárközi and Hajnal cut into each other's words to tell Uncle Paul to stop kidding: everybody was waiting for him at the conference, he should take the first flight the next morning. I escorted him to the airport on Monday. He was cheerful after a good weekend break with my family and the Sunday excursion. He checked his suitcase containing all his material possessions (he did not have an apartment), including a pair of boots: it weighed 9.40 kilograms.

Strasbourg, 27 April to 2 May 1996.

Two weeks later he called from Paris and he arrived to Strasbourg the following Saturday afternoon, after a stop in Nancy. He wore the sandals that he lost a year before. Tenenbaum kept them for him.

Uncle Paul was intransigent on moral issues. He sent a letter to the University of Waterloo for a colleague who he thought was disproportionately punished for a small mistake. He suggested that if they do not find a suitable solution, he would resign his honorary degree. (He did this after the university's response, see Figure 5.)

He complained about his heart on Monday. His usual medication had not arrived from the USA. I called our doctor. Fortunately, the drug was also available in France. The doctor gave him a prescription, but also advised him to stay and relax for at least one week because of his heart. "It's impossible!" Uncle Paul exclaimed, "people are waiting for me everywhere. Thanks for the medicines, I'll take them and everything will be fine." Then I talked to the doctor in private. "If he were not allowed to continue traveling, his condition would only deteriorate further. We have to let him travel, though he'll die one day because of it", he said to me.

On Tuesday, he gave another lecture on "my favorite problems in number theory, combinatorics, geometry and analysis". In the evening, during the dinner, my daughter told us that her French teacher was trying

Erdős pairs

Suppose $n > 11$. If $\{1, 2, \ldots, n-1\}$ is partitioned into two disjoint subsets, then one of these contains distinct elements that sum up to n. There exists a maximum number m such that $\{m, m+1, \ldots, n-1\}$ still has this property. This maximum is denoted by $l_2(n)$ and if $m \leq l_2(n)$ then (m, n) is called an *Erdős pair*. In 1992, Erdős posed the problem of determining $l_2 = \liminf l_2(n)/n$. It is quite easy to convince oneself that $l_2 \leq \frac{1}{4}$. In 1993, Bollobás and Jin proved that $l_2 = \frac{1}{4}$. The general problem of determining l_p, for partitions into p parts, remains open.

Marie-Paule Muller proved that $l_2(n)$ is equal to the integer part of $(n-3)/4$ for every $n \geq 56$. She also proved that $l_p \leq \frac{1}{2p}$. Her results appeared in 1998 in the *Annales Universitatis Scientiarium Budapestinensis*.

to persuade her pupils to take her ancient Greek class next year. “The famous mathematicians of ancient times were Greeks. If you do not study Greek, you cannot become a good mathematician”, she told the best mathematician in the class. “What a stupid thing”, Uncle Paul commented, and then quietly stirred his soup.

He continued his travel to Berlin on Thursday.

Several months later, a colleague of mine from Strasbourg, Marie-Paule Muller, contacted me. She said that she had practically stopped research more than five years ago, but Paul Erdős gave such an appealing presentation that she began thinking about one of the unsolved questions that he mentioned and she had managed to solve it partially. Her paper was published in 1998 in Hungary.

During his visits to Strasbourg, we talked a lot during walks or trips. Any topic could be raised: family, politics, history.

On a weekend I took him to the Sainte-Odile Mountain, with a beautiful view of the Alsatian plain from the old monastery there. On the road, we met surprisingly few cars. On the hilltop, however, many hundreds of cars were parked on each other’s back, it was very difficult to find a parking place. “The whole Alsace came here”, Uncle Paul said laconically.

According to his long-time acquaintances, Paul Erdős was much faster at his youth than in his last years. But I still had to well prepare myself for his visits. Sometimes he was wrong. “The old man is stupid,” he said.

Once we were walking on the riverbank of the Rhine, and I asked him about old Hungarian mathematicians. “What kind of a man was Leopold Fejér?” I asked. “He was… slow”, he replied. It was a joke, of course: Uncle Paul was very fond of joking but never at the expense of others. I heard that Fejér found it hard to follow the quick explanations by Paul Erdős, who on his turn had no patience to listen to the slowly talking Fejér. Often, Paul’s friend Turán interpreted between them. (Later, as a first-year undergraduate in 1973, I learned number theory from the wonderful lectures of Paul Turán.) “But speed is not important, the important thing is to prove a big theorem”, he added. “It does not matter whether it takes a month or ten years, the main thing is to find an important result. Fejér was an excellent mathematician and a wonderful person”, he continued. (Erdős, Turán and Alfréd Rényi were very close friends. Rényi is also known for his definition of a mathematician as “a machine for turning coffee into theorems”. Once all three were together at a conference in the USA, where coffee is known to be much weaker than in Europe. As Turán remarked: “With this coffee we can only prove lemmas.”) “And what kind of a man was Riesz?”, I continued to ask. “Oh,” Uncle Paul replied, “Riesz was… less slow.”

“Didn’t it hurt you that you didn’t get a Fields Medal?” I once asked him. “No. I have received many other awards. But the prizes are not important: what is important is to try to prove a result that will be remembered 500 years from now.”

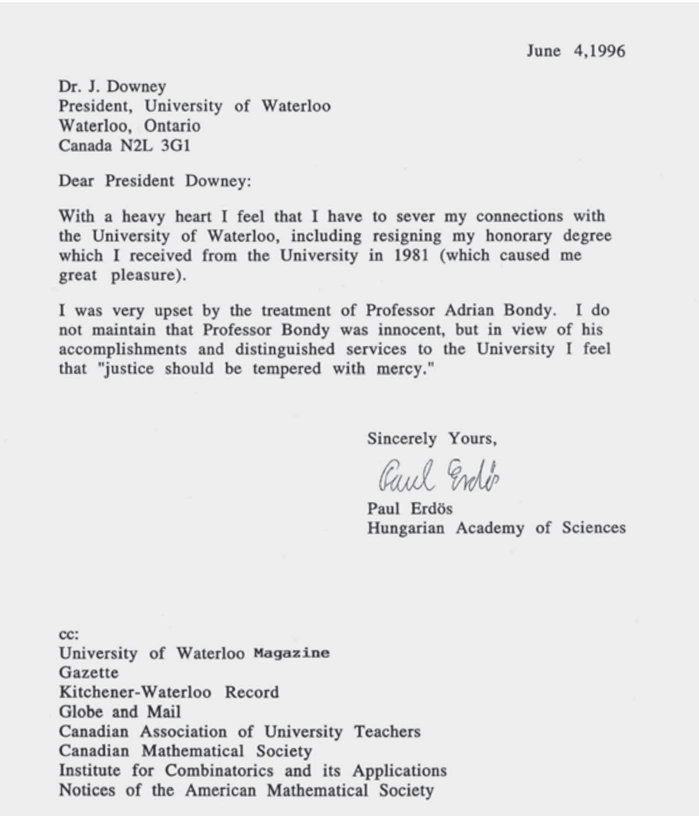
“Did you physically suffer during the war?” I asked him another time. “Of course”, he replied instantly. “Once I was walking on a beach in America in 1942. Because of the blackout, I walked into a lamp post, and I had a pain in my knee even the next day.”

Budapest, 21–27 July 1996

I last met him during the European Mathematical Congress. He invited me to lunch at the Hotel Astoria. I asked him what had happened in the last 2–3 months. “Nothing worthwhile”, he replied, and then turned to discuss mathematics. “Don’t wait any longer, let us submit our paper for publication.” I wanted to solve a pending issue first, but I promised to finish the article during his next visit of Strasbourg in October. Only a long time later I heard about his sickness in June and the implantation of a pacemaker.

The next day or the third day my parents invited us to lunch. At noon, we were waiting at the bus stop next to the conference site, but Uncle Paul lost his patience after

Figure 6 Paul is proving two theorems.



June 4,1996

Dr. J. Downey
President, University of Waterloo
Waterloo, Ontario
Canada N2L 3G1

Dear President Downey:

With a heavy heart I feel that I have to sever my connections with the University of Waterloo, including resigning my honorary degree which I received from the University in 1981 (which caused me great pleasure).

I was very upset by the treatment of Professor Adrian Bondy. I do not maintain that Professor Bondy was innocent, but in view of his accomplishments and distinguished services to the University I feel that "justice should be tempered with mercy."

Sincerely Yours,

Paul Erdős

Paul Erdös
Hungarian Academy of Sciences

cc:
University of Waterloo Magazine
Gazette
Kitchener-Waterloo Record
Globe and Mail
Canadian Association of University Teachers
Canadian Mathematical Society
Institute for Combinatorics and its Applications
Notices of the American Mathematical Society

Figure 5 In 1996 Erdős famously resigned his honorary degree of the university of Waterloo after an infamous dispute between the university and one of its employees.

Figure 7 At the congress in Budapest.

two minutes: "It doesn't seem like this bus goes. Let's go by taxi!" We only had to take five or six stops by bus, and I didn't like to pay for a taxi. "It will come very soon", I answered. Erdős went to ask two other men waiting at the stop. "Is this bus going?" After finding that they were not Hungarians, he repeated the question in other languages, but they still did not understand. Uncle Paul came back annoyed: "They don't speak any language." I saw on their badge that they were Russian conference participants, so I replied that he didn't speak Russian either. "That's true", he replied, and calmed down as the bus appeared at that moment. We got in, we sat down on a double seat, the bus started. Then he turned to me, and began to speak in Russian. It was a long text, though I did not understand its relevance. "Do you know Russian?" I asked quite surprised. "No, but in 1966, when the ICM was organized in Moscow, I thought I should learn it a little bit, and then I dealt with it a little." Thirty years later, he still remembered the first pages of the textbook.

We ate goulash soup and pancakes for lunch, and he was pleasantly talking to everyone. He asked my parents how they had survived the war.

There was the conference banquet in the evening. He noticed a baby girl in a pram with their parents, and he had been trying for a long time until she accepted without crying that Paul stood near her. (Norbert A'Campo, the baby's father, recently told me: I came across Erdős by chance near the congress building. He told me not to have children and he was visibly happy to push the pram. We looked at Clara — the baby — and I did not really look at that very friendly man in that bit of a worn suit. He asked me what I was doing and I quickly understood that he was very comfortable with mathematics. To learn his name was a huge surprise to me.)

The last afternoon, a film about him was screened at the congress. "I have seen the movie, and you too, let's stay outside and talk", he suggested. We walked up and down the empty corridor and talked about everything. Suddenly he stopped and looked at me seriously. "You know, I think I made a mistake by not having set up a family", he said. Then we stood there silently for a while. That was the last time I saw him. The next morning he traveled to Sárospatak, a town about 250 km from Budapest, and he called me three times from there in the afternoon. He felt alone.

I received his last greetings via e-mail at the end of August, when he met the Rudins in Budapest. He would have come to Strasbourg in early October. But his heart couldn't stand it.

After all newspapers announced the news in the headlines, one of the secretaries asked me at the department: "Was he really that kind old man who always walked so modestly in the corridor? I would have never thought that he was famous."

His funeral was on 21 October 1996. Very moving farewell talks were made by his colleagues and friends, and mathematical societies from distant countries were also represented by a wreath, sometimes even by a person. I later mentioned to Ákos Császár my surprise over the Hungarian state's absence. "I don't think it was disturbing," he replied, "he did not consider these things important. What was really painful for me is that the Presidence of the Hungarian Academy of Sciences was not represented. I asked them to do it, but they replied that Paul Erdős was not a member of the Presidency, so it was not necessary by the protocol."

Acknowledgement

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Figure 8 Paul Erdős (left) at the conference banquet in Budapest with Norbert A'Campo, baby Clara A'Campo, François and Monique Sigrist and on the right Vilmos Komornik with behind him Annette A'Campo-Neue, the mother of the baby.