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Book Review Michèle Audin: Remembering Sofya Kovalevskaya

A great role model

Sofya Kovalevskaya (Sophie Kowalevski), who lived in the nineteenth century, was the first major Russian female mathematician. Gert Heckman describes the life of this interesting woman and reviews the biography by Michèle Audin. This review was submitted a while ago, but not yet published due to an editorial mistake.

Sophie Kowalevski was born in 1850 in an aristocratic family in Moscow as Sophia Vasilevna Krukovski. Her name has about 25 different spellings, and the reviewer will stick to the one she used herself in her rigid body paper. Her first interest in real mathematics might have been aroused by the lecture notes of Ostrogradsky on calculus, bought by her father in his youth. By lack of enough wall paper for the country house the walls of her room were covered with these lecture notes, and as a child she used to spend hours before the mysterious wall trying to decipher the content of the incomprehensible formulae.

In 1868 she married Vladimir Kowaleski. The main purpose of this marriage was that it allowed her to go abroad and study mathematics, a thing that would have been utterly inappropriate for a single woman in those days. Sophie did her undergraduate work in Heidelberg, and in 1870 began her graduate study in Berlin with Karl Weierstraß. In 1874 her thesis passed *in absentia* in Göttingen, and she returned to Russia. The most important result of her thesis is the Cauchy–Kowalevski theorem on the uniqueness of the analytic Cauchy problem.

As a woman it was impossible for her to find a position, and she more or less abandoned mathematics for several years. In 1876 she met the four years older Gösta Mittag-Leffler, also a student of Weierstraß, and the two became close friends. She did some traveling and also visited mathematics conferences, but not to the extent she would have liked. In 1878 her daughter Fufa was born. In 1883 Vladimir committed suicide, an act that led to the insight that “a very important obstacle in my eyes has disappeared” in a letter to Mittag-Leffler from Weierstraß.


Photo: Wikimedia Commons

Sophie Kowalevski

In the next year Mittag-Leffler was able to arrange for Kowalevski a five years teaching position at the University of Stockholm. She became editor of *Acta Mathematica*, a prestigious journal founded two years before by Mittag-Leffler. Finally she was able to live the life as a professional mathematician. In 1889 she got a permanent position as full professor of mathematics in Stockholm to become the first female university professor in Europe. It is hard to overestimate the political skills and powers of Mittag-Leffler to establish such position for Kowalevski. About thirty years later the great David Hilbert was not able to repeat this move in Göttingen for Emmy Noether: “Gentlemen, I do not see that the sex of the candidate is an argument. After all, the senate is not a bathhouse.”

Sophie Kowalevski continued to work in Stockholm for two more years, and died in 1891 of pneumonia just 41 years old.

The Kowalevski top

In the Stockholm years she did her most important work on the ‘Kowalevski top’. The two classical integrable tops are the Euler top (no gravity) and the Lagrange top (symmetric top with center of gravity on the axis of symmetry), and she discovered an additional integrable top (symmetric top with $I_1 = I_2 = 2I_3$ for the principle moments of inertia and the center of gravity on the plane of symmetry). Her exact solution in terms of genus 2 theta functions is a tour de force. The work was highly appreciated at the time. She won the Prix Bordin, and because her contribution was found extraordinary the prize was elevated from 3000 to 5000 francs. Of course, all this must have helped Mittag-Leffler to create her position.

In 1905 Husson proved that the Euler, Lagrange and Kowalevski tops are the only three tops which are completely integrable with algebraic integrals. This result was extended by Ziglin in 1981 by replacing algebraic integrals by analytic integrals. These results are parallel to the results of Bruns and Poincaré that the three body problem has no more algebraic (Bruns in 1887) or analytic (Poincaré in 1889) integrals than the usual ones, both results also published in *Acta Mathematica*. Notably, the work by Poincaré had a tremendous influence on the development of the qualitative theory of dynamical systems. The Kowalevski top was for many years the example par excellence of what exact solution could do. Integrable systems went into hybernation for a long period, and renewed interest only started in the nineteen sixties with the treatment by Peter Lax of the Korteweg–de Vries equation. Another highlight was the discovery of the Hitchin system as an outstanding example of an algebraically integrable system.

The book by Michèle Audin

The book by Audin is a pleasure to read. There are the many Kowalevski stories like the above wall paper story. There are two separate chapters on the Cauchy–Kowalevski theorem and the Kowalevski top. On the last topic Audin is a specialist and has written a textbook: *Spinning Tops. A Course on Integrable Systems* (Cambridge University Press). Audin clarifies the crucial role in the life of Kowalevski played by her companion Mittag-Leffler and her teacher Weierstraß.

Audin also discusses the opinions held by other mathematicians on the reputation of Kowalevski. First and foremost there is the not quite favorable opinion by Eric Temple Bell on Kowalevski in his *Men of Mathematics*, which is discussed and analyzed paragraph after paragraph in a full chapter. In the next chapter, entitled ‘I too remember Sofya’, we can read opinions of many mathematicians on Kowalevski, most for the good, but some for the bad. Examples of the latter are Lars Gårding and Lars Hörmander, who opposed the idea that *Acta Mathematica* should commemorate in 1984 the fact the Kowalevski started her teaching position as first female professor in Europe a century ago. Their argument for rejection was that in that same year 1884 Kowalevski had written a paper on wave propagation in crystals that turned out to be wrong. Mistakes are not allowed, period.

It is clear that Audin has a passionate stand on the case of Kowalevski. Sophie is her great hero. Still now in 2016 the reviewer is working at a mathematics department without women on its faculty, and we are in this respect a representative of the other mathematics faculties in the Netherlands. This is a missed opportunity. At this moment our female students perform on the average better than their male fellows. It is to be hoped that some day women and men alike enter university faculties of mathematics. Until that day Sophie Kowalevski, as well as Emmy Noether or Maryam Mirzakhani and others, remain the great role models!

Michèle Audin, *Remembering Sofya Kovalevskaya*, Springer, London, 2011, 284 p., ISBN 9780857299291, € 39.95.