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Research

Computational challenges in electrical power networks

Networks for the high-voltage distribution of electrical energy are currently undergoing far-reaching developments. National power grids are evolving from static entities, producing mainly a uni-directional flow from generation to loads, to more dynamic and decentralized structures. These emerging power systems should accommodate the local generation by renewable sources and peak demands of electrical vehicle charging. The cross-border interconnection of power grids further imposes new challenges in the design, planning and daily operation of these networks. In this paper Domenico Lahaye and Kees Vuik describe recent work by the chair of numerical analysis at the TU Delft on the numerical simulation of electrical power networks.

This work is intended to guide the transition towards nextgeneration of the power systems. We subdivided this paper into three parts.

We first describe the development of a dedicated load flow solver for the state in a network or in a collection of closely resembling networks. The networks we consider model the interconnection of a set of loads and generators through power lines. The term *state* refers to the set of voltage phasors associated with each node of the network. The ability to compute efficiently the nodal voltages is fundamental in any modeling and optimization of the power network. Our algorithm allows to solve large scale problems [3] faster than more commonly used techniques.

We subsequently outline a screening technique we developed to monitor the power system subject to critical events. We analyze how the system state alters after the removal of major interconnections and identify the need for measures to bring the network back to a save operation point. These measures include the redistribution of generation and the removal of loads in more severe cases. Unlike more commonly used techniques, our method accounts for both overloads of the interconnecting lines and the violation of the nodal voltage lower and upper bounds [4].

In the final part of the paper we describe a novel approach to compute transients caused by switching actions. Unlike other approaches in literature, our approach avoids the reassembly of the state-space representation at each switching event.

Noot van de redactie
Dit artikel is geschreven als bijdrage voor het themanummer over Netwerken, maar door een samenloop van toevalligheden heeft het de eindredactie helaas net te laat bereikt. Daarom publiceren we het alsnog in het voorliggende nummer.

Newton–Krylov load flow solver

Let us consider a network consisting of n nodes and m edges. Each of the nodes represents either a generator or a load. In a generator and load node electrical power is produced and consumed, respectively. An edge represents a transmission line that transports power from the generators to the loads. The electrical power can be expressed in terms of the voltage at the nodes and the current flowing through the edges. We consider a formulation in which the electromagnetic quantities are assumed to vary sinusoidally at a frequency of $50\,$Hz. Ohm's Law then allows to eliminate the current in such a way that the power can be expressed in terms of solely the complex-valued voltage phasors.

Let $\mathbf{x}$ denote the n-dimensional vector that holds the voltage phasor at each node. The constraint that the total amount of power consumed should balance the total amount of power generated can be expressed as a system of n non-linear algebraic equations that we can express as

$$F(\mathbf{x}) = \mathbf{0} \in \mathbb{C}^n \,. \tag{1}$$

Let $J_i(\mathbf{x}_i)$ denote the Jacobian of F with respect to $\mathbf{x}_i$ at the i-th Newton iteration. Solving the system (1) using Newton's method requires solving a sequence of linear systems of the form

$$J_i(\mathbf{x}_i)\,\mathbf{s}_i = -F(\mathbf{x}_i) \qquad (2)$$

for the step length $\mathbf{s}_i$. In literature this linear system is typically solved by a direct solution method.

Currently, however, there exists a pressing need to compute the power flow in networks that are large in size. Such networks originate for instance in the modeling of the interconnection of the network in Europe with that in Russia or in Northern Africa. It is well known that direct solution methods are not suited for such large scale problems [5]. We therefore developed a new generation of load flow solvers in which the LU-factorization of Jacobian $J_i(\mathbf{x_i})$ is replaced by a preconditioned Krylov subspace iteration. The term Newton–Krylov for these new solvers derives from the fact the accuracy of the inner linear solver is linked to the outer non-linear iteration residual.

Numerical results confirm that the Newton–Krylov solver is computationally more efficient than previously existing direct solution methods. The observed linear convergence of the outer GMRES iteration resulted in a convergence theory that provides insight in how to set the stopping criterion for the Krylov iteration. We also demonstrated that the flexibility of Newton–Krylov methods allows to efficiently solve a collection of closely resembling network models.

Monte Carlo security assessment

We extended previous work to the security assessment of power networks [4]. The network is said to be in a secure operating mode if the generator and load settings are such that the voltage at the nodes and the currents through the edges are within a priori defined ranges. In a security assessment one seeks to identify perturbations in the operating mode that bring the network away from being secure. Such perturbations might include changes in the generators and loads as well as changes in the topology of the network. In a subsequent stage one seeks actions that brings the network back to a secure state. Such actions include the redistribution of the total amount of power generated across the different generators, the installation of new interconnections and the removal of part of the load.

In our approach, the generated power and demanded load are stochastic variables. The load flow equations are solved within a Monte Carlo procedure. For each sample the safety of the operating point is evaluated after solving the load flow equations (1). Subsequently a tentative least invasive remedial action that brings the network operating back into save operation is proposed. The load flow equation are again solved for to validate action proposed. The procedure is repeated until the network is brought back to save operation or until the network is deemed to be collapsed. Data on the safety of the power network is gathered in the post-processing stage of the Monte Carlo simulation.

Switching induced transients

In a parallel project [8], we studied time integration methods to compute transients caused by switching actions in power systems [1, 7]. We investigate Runge–Kutta methods with adaptive time step to solve

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t) \quad \text{given } \mathbf{x}(t = 0) = \mathbf{x}_0\,, \qquad (3)$$

for the current through the inductors and the voltage across the capacitors. A switching action induces a change of topology of the network. Existing time integration methods require the reconstruction of the state-space matrices after each switching event. We developed a new modeling method that avoids this need to reassemble the matrices. Computation time is therefore saved. We are currently looking into the possibility to replace the direct linear system solve at each time step by an approximate iterative solver.

Conclusions

In this paper we described recent work by the chair in numerical analysis of the TU Delft on the modeling of electrical power networks. We briefly outlined the development of a fast and versatile load flow solver, of a Monte Carlo based simulation tool for the security assessment of a power system and of a new modeling approach for the switching-induced transient.

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