Konstantinos Efstathiou
Johann Bernoulli Institute for Mathematics and Computer Science
University of Groningen
k.efstathiou@rug.nl

Column Tenure-tracker

Symmetries in Hamiltonian systems

In this column holders of a tenure track position introduce themselves. The tenure track positions in mathematics became available in 2013. Excellent researchers could apply in several expertise areas of mathematics. Konstantinos Efstathiou has a tenure track position at the University of Groningen.

Since April 2014 I am Tenure Track Assistant Professor in the group Mathematical Physics, Geometry, and Dynamical Systems at the Johann Bernoulli Institute of the University of Groningen. The road to here was sinuous but with great views along the way.

I got a BSc and then MSc in Physics at the University of Athens in Greece. After a little bit of soul-searching ("should I work on galaxy formation or dynamical systems?") I decided to do a PhD on Mechanics and Symmetry. I moved to the Université du Littoral at the north of France, a region whose charms were depicted in the French comedy *Bienvenue chez les Ch'tis*. My PhD supervisors were Boris Zhilinskiĭ and Dmitriĭ Sadovskiĭ. They, and Richard Cushman in Utrecht with whom I was collaborating, played a fundamental role during these first research-oriented steps, offering their guidance and support.

The main theme of my research, forged during these PhD years, is properties of Hamiltonian systems with symmetries. This is a vast field but one can start getting a feeling for what is going on by looking at a very simple system: two uncoupled identical harmonic oscillators. The system is completely integrable: one can write down expressions for the orbits; they all turn out to be periodic with the same period (unless both oscillators stand still). Nevertheless, the geometry of how all orbits fit together on an energy level set is rather intricate and gives rise to what is known as the *Hopf fibration* (Figure 1). For more details see Richard Cushman's lecture notes [1] — writing these notes was how I started learning this topic.

After finishing my PhD and a compulsory stint in the Greek army, I joined the University of Groningen as a post-doc in the group of Henk Broer. During the time in Groningen I continued my work on integrable Hamiltonian systems and also started working in new projects, out of which I would like to highlight the study of dynamics on networks. Many of these projects were in collaboration with Henk Broer whose broad view of mathematics significantly influenced my own thinking and scientific development.

The time as a post-doc in Groningen was very good. Not only in terms of work and research but also because I met my wife and we had our son. Nevertheless, every post-doc must end, and at the beginning of 2013 I moved to Suzhou, China, as Lecturer at the Xi'an

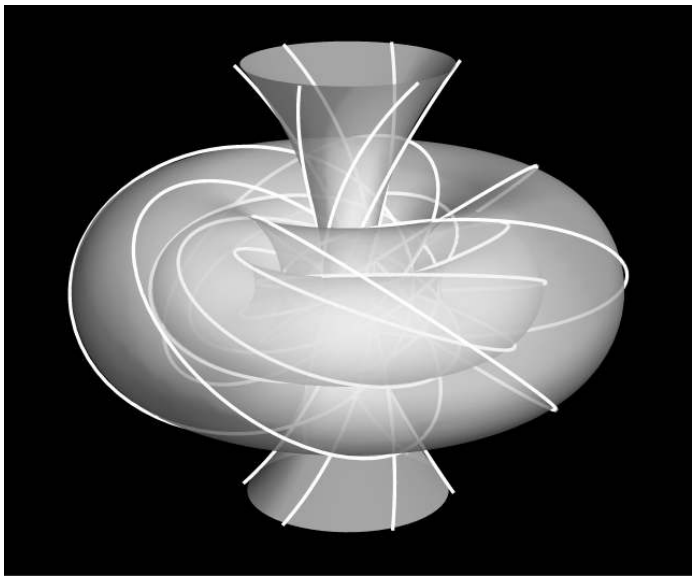

Figure 1 The Hopf fibration.

Jiaotong-Liverpool University. I stayed there for slightly more than a year, teaching, doing research, and even serving as Interim Head of the Department of Mathematical Sciences for a few months. I also had the chance to learn a completely different culture — an amalgam of British academic culture and Chinese life. And just as things were settling down in Suzhou I learned that NWO was funding a number of tenure track mathematics positions in the Netherlands; one of them was in Groningen. Certain opportunities are impossible to let pass by.

Let me now say a few more words about my research. As I wrote above, the main theme of my research is symmetries in Hamiltonian systems. I combine ideas from the theory of integrable Hamiltonian systems, group theory, and algebraic topology to understand the dynamics and geometry of Hamiltonian systems with continuous or discrete symmetries. I like to see my research in the more general context of dynamical systems, a topic with a huge tradition in Dutch mathematics (one only has to recall the contributions of, among others, Floris Takens and Hans Duistermaat). A lot of my work has focused on the notion of generalized monodromy in Hamiltonian systems. Hamiltonian monodromy usually refers to the monodromy of torus bundles — natural objects of study in integrable Hamiltonian systems. The role of the monodromy of such bundles for the global existence of nice (action-angle) coordinates was uncovered in the late seventies / early eighties (the protagonists here are Nikolaiĭ Nekhoroshev, Hans Duistermaat, and Richard Cushman). But monodromy is more than a purely geometric concept. It also explains the structure of the joint spectrum of integrable quantum systems such as molecules and atoms. Generalized Hamiltonian monodromy refers to the properties of integrable Hamiltonian fibrations where some of the fibres are singular, for example, curled tori — cylinders over a figure-8 where the two ends of the cylinder are glued after a half-twist (Figure 2). The article [2] with Henk Broer presents the latest results in this direction.

Another significant part of my research has to do with the dynamics of pulse-coupled oscillator networks. Such networks model, among other things, the interaction between neurons. The pulse-based coupling causes dynamical behavior that cannot appear in systems described by smooth differential equations, for example, unstable attractors. Therefore standard methods for the analysis of smooth systems are not always suitable for the study of pulse-coupled networks and we have to come up with new techniques to understand what are the types of typical dynamics in such systems and the transitions between them.

Besides mathematics and physics I enjoy programming, not only as a means for gaining insight to mathematical questions but also for its own intellectual rewards since it exercises different thinking modes compared to mathematics. This interest (and a collaboration with my wife) led to my only, until now, Scientific Visualization paper [3]. Actually, one thing that I particularly enjoy in research is to combine very different approaches: purely theoretical approaches on the one hand, but also numerical simulations and visualizations on the other. Furthermore, I like to understand a subject from different points of view, often returning back to the same problem to examine it from a different perspective.

In the coming years I plan to further develop my research on Hamiltonian systems with symmetries on the one hand and dynamics on networks on the other. I am very excited that I will be able to do this as part of the Dutch mathematical community.

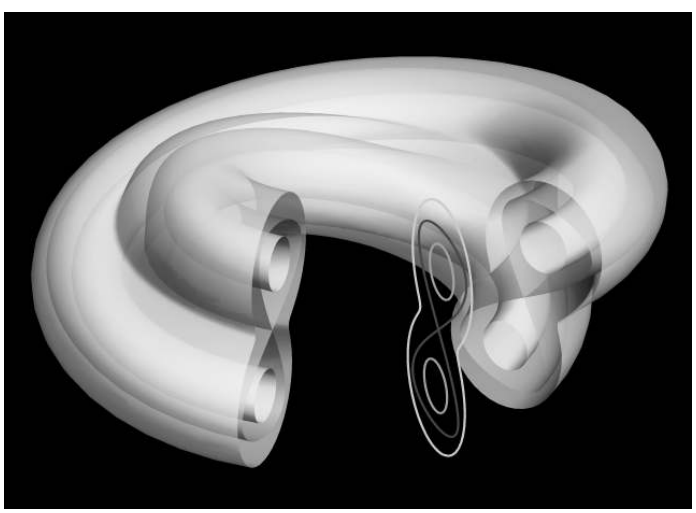

Figure 2 Smooth tori near a curled torus.

References

1 K. Efstathiou and D.A. Sadovskiĭ, No polar coordinates. Lectures by Richard Cushman, *Geometric Mechanics and Symmetry. The Peyresq Lectures*, J.A. Montaldi and T.S. Ratiu, eds., London Mathematical Society Lecture Notes Series No. 306, Cambridge University Press, 2005, pp. 211–302.

2 K. Efstathiou and W.H. Broer, Uncovering fractional monodromy, *Communications in Mathematical Physics* 324(2) (2013), 549–588.

3 L. Yu, K. Efstathiou, P. Isenberg and T. Isenberg, Efficient Structure-Aware Selection Techniques for 3D Point Cloud Visualizations with 2DOF Input, *IEEE Transactions on Visualizations and Computer Graphics* 18(12) (2012), 2245–2254.