Ruud Brekelmans
School of Economics and Management
Tilburg University
r.c.m.brekelmans@uvt.nl

Carel Eijgenraam
Centraal Planbureau
Den Haag
c.j.j.eijgenraam@cpb.nl

Dick den Hertog
School of Economics and Management
Tilburg University
d.denHertog@uvt.nl

Kees Roos
Faculty EEMCS
TU Delft
c.roos@tudelft.nl

Research

A mixed integer non-linear optimization approach to optimize dike heights in the Netherlands

In the Netherlands, dike rings protect a large part of the country against flooding. In this article, Ruud Brekelmans, Carel Eijgenraam, Dick den Hertog and Kees Roos describe an optimization model that has been developed to optimize dike heights in the Netherlands. The project led to a saving of €7.8 billion and has won the prestigious international Franz Edelman Award.

This paper is based on the material in two earlier papers, [1] and [5], that were published in the journals *Operations Research* and *Interfaces*, respectively. It describes the optimization model that has been developed to optimize dike heights in the Netherlands. Moreover, it briefly describes the high impact of the results of this project on political decision making in the Netherlands. The project was awarded with the INFORMS Franz Edelman 2013 Award. For more details on the validation of the model, the method used, and the political process and impact, we refer to these papers.

In the Netherlands, dike rings, consisting of dunes, dikes and structures, protect a large part of the country against flooding. After the catastrophic flood in 1953, a cost-benefit model was developed by van Dantzig [2] to determine optimal dike heights. The objective of the cost-benefit analysis (CBA) is to find an optimal balance between investment costs and the benefit of reducing flood damages, both as a result of heightening dikes. The question then becomes when and how much to invest in the dike ring. In [6] we improve and extend van Dantzig's model. In that paper we show how to properly include economic growth in the cost-benefit model, and how to address the question when to invest in dikes. All these models consider dike rings that consist of a homogeneous dike. This means that all parts in the dike ring have the same characteristics with respect to investment costs, flood probabilities, water level rise, et cetera.

Many dike rings in the Netherlands, however, are non-homogeneous, consisting of different segments that each have different characteristics. Differences occur, for instance, if along a dike ring in the delta area a river dominated regime changes into a sea dominated regime, or if a dike ring contains a large sluice complex. Currently, there are dike rings with up to ten segments in the Netherlands. In this non-homogeneous case, it is not necessary and not desirable to enforce that all these segments are heightened simultaneously and by exactly the same amount. Hence, the decision problem for the non-homogeneous case concerns when and how much to invest in each individual dike segment.

In the current paper, we consider the extension of the homogeneous case in [6] to the non-homogeneous case. The research has been carried out as part of a project initiated by the government. The project's main goal is to support decision-making with respect to setting new flood protection standards for the dike rings in the Netherlands. Efficient flood protection standards can be derived from the optimal investment strategy and the resulting flood probabilities. How this can be done is explained in [5]. Here we confine ourselves to a description of the first stage: finding the


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The catastrophic North Sea flood of 1 February 1953

optimal investment strategy. In order to lay a firm base for the new standards, the 53 larger dike rings in the Netherlands need to be analysed thoroughly. This requires that particular scenarios can be analysed within a reasonable amount of time, where each scenario represents a certain instance of the model parameters such as economic growth, interest rate, water level rise, flood characteristics, investment costs and so on.

It is shown in [6] that the homogeneous case can be solved analytically. Unfortunately, we did not succeed in solving the non-homogeneous case analytically. In this paper we show how the non-homogeneous dike height optimization problem can be modelled as a Mixed Integer Non-linear Programming (MINLP) problem.

In addition to the MINLP formulation of the decision problem, we constructed an iterative optimization algorithm that speeds up the solution time considerably. The algorithm has been implemented in AIMMS, which has subsequently been integrated in user-friendly software to perform the dike ring analysis [3–4]. The final results have had a big impact in the political decision making process.

Non-homogeneous Optimization Problem

Problem formulation

In this section we present our model for the non-homogeneous dike height optimization problem. The model is an extension of the homogeneous problem, as introduced in [6]. The reader is referred to [6] for the foundation of the common model parts. A dike ring protects a certain area of land against flooding. The number of segments is denoted as L ($L \geq 1$). A dike ring is said to be non-homogeneous if $L > 1$, and homogeneous otherwise. All segments can be heightened independently of each other. Moreover, each segment has its own properties with respect to investment costs and flood probabilities. To indicate the dependence of a model parameter on a particular dike segment, a subscript l ($l = 1, \ldots, L$) will be added to this parameter. The set of all segments is denoted by $\mathcal{L}$.

The objective is to find an investment plan that minimizes the expected total costs. Only investments in a finite planning horizon $[0, T)$ are considered. An investment plan is represented by a tuple $(\mathbf{U}, \mathbf{t})$, with $\mathbf{U} \in \mathbb{R}_+^{L\times(K+1)}$ and $\mathbf{t} = (t_0, t_1, \ldots, t_K)^T$. The vector $\mathbf{t}$ represents the possible timings of dike segment heightenings, where $t_0 = 0 < t_1 < \cdots < t_K < T$. Hence, $K + 1$ is an upper bound on the number of segment heightenings in the planning horizon. For notational convenience, we denote $t_{K+1} = T$. The matrix $\mathbf{U}$ represents the segment heightenings, where the element $\mathbf{U}_{lk} = u_{lk}$ is the heightening (cm) of segment l at time t_k ($l = 1, \ldots, L,\ k = 0, \ldots, K$). Of course, heightenings are assumed to be nonnegative. If $u_{lk} = 0$, then this means that segment l is not heightened at time t_k. The l-th row of $\mathbf{U}$, with the $K + 1$ heightenings of dike segment l, is denoted by $\mathbf{u}^{(l)}$.

Throughout the remainder of this paper we use the following notation for the cumulative segment heightening and the absolute segment height at time t ($t \geq 0$):

$$h_{lt} = \sum_{k:t_k \leq t} u_{lk} \quad \text{and} \quad H_{lt} = H_{l0}^{-} + h_{lt},$$

where H_{l0}^{-} is the absolute height of segment l immediately prior to a possible heightening at time $t = 0$. For notational convenience, we also use $h_{lk} = h_{lt_k}$ and $H_{lk} = H_{lt_k}$. Note that it follows from this definition that the segment height is a non-decreasing step function. Moreover, this implicitly means that heightenings are measured at the moment that the investment actions are completed. A lead time is not modelled.

The flood probability of segment l at time t is given by

$$P_{lt} = P_{l0}^{-} \exp(\alpha_l(\eta_l t - h_{lt})), \tag{1}$$

with P_{l0}^{-} (1/year) the initial flood probability, α_l (1/cm) the parameter of the exponential distribution for extreme water levels and η_l (cm/year) the structural increase of the water level. Both the hydraulic conditions and the quality of the dike segment are summarized by one indicator: height above the level that corresponds to the flood probability P_{l0}^{-}. The weakest segment fully determines the flood probability of the entire dike ring. Hence, we define the flood probability of the entire dike ring at time t by $P_t = \max_{l\in\mathcal{L}} P_{lt}$.

A property that all segments have in common is that they protect the same area of land. Hence, if there is a flood, the damage does not depend on the segment in which a breach occurs. Furthermore, the potential damage costs increase in time with the economic growth rate γ. The damage costs do, however, also depend on the resulting height of the water level within a dike ring after a flood. In particular, along rivers the damage costs increase by the rise in the height of the lowest segment (in absolute height). Putting all this together yields the following damage costs, at time t, in the case of a non-homogeneous dike ring:

$$V_t = V_0^- \exp\left(\gamma t + \zeta\left(\min_{l\in\mathcal{L}} H_{lt} - \min_{l\in\mathcal{L}} H_{l0}^-\right)\right),$$

with V_0^- the initial damage costs and ζ (1/cm) the parameter that represents the increase in damage costs depending on the height of the lowest dike segment.

The expected damage costs at time t is given by the product of the flood probability and the damage costs:

$$\begin{aligned} S_t &= P_t V_t \\ &= \max_{l\in\mathcal{L}} S_{l0}^- \exp\Big(\\ &\quad \beta_l t - \alpha_l h_{lt} + \zeta\left(\min_{l\in\mathcal{L}} H_{lt} - H_{l_0 0}^-\right)\Big), \end{aligned} \tag{2}$$

where $S_{l0}^- = P_{l0}^- V_0^-$, $\beta_l = \alpha_l\eta_l + \gamma$ and $l_0 = \operatorname{argmin}_l H_{l0}^-$. By using the fact that the segment heights remain unchanged in the interval $[t_k, t_{k+1})$, the total expected damage in this interval can be written as

$$\begin{aligned} &\int_{t_k}^{t_{k+1}} S_t \exp(-\delta t)dt \\ &= \exp(-\zeta H_{l_0}0) \\ &\quad \int_{t_k}^{t_{k+1}} \exp\left(-\delta t + \zeta \min_{l\in\mathcal{L}} H_{lt}\right) \\ &\quad\quad \cdot \max_{l\in\mathcal{L}} \left(S_{l0}^- \exp\left(\beta_l t - \alpha_l h_{lk}\right)\right) dt, \end{aligned} \tag{3}$$

where δ is the discount rate.

From an optimization point of view there are two problems with the integral in (3):

- The minimum absolute segment height $\min_l H_{lt}$ cannot be incorporated in an optimization model as a convex constraint.
- Even though the segment heights do not change during the interval $[t_k, t_{k+1})$, the segment flood probabilities P_{lt} as defined by (1) increase monotonically in time. Hence, the segment l for which the maximum flood probability is obtained may change during the interval $[t_k, t_{k+1})$.

If we want to use (3) in a MINLP model, then we have to make some assumptions about these two issues. The minimum operator in (3) refers to the fact that the size of the damage depends on the segment that is lowest in absolute height. Since in practice it is usually clear which of the segments along rivers is the lowest in absolute height, it is assumed that this segment is known in advance. Let this dike segment be denoted by l^*. It turns out that, for the dike rings in the Netherlands, this assumption is always satisfied.

An obvious approach to deal with the maximum operator in (3) is to interchange the integral and the maximum operator. Note that this yields a lower bound for (3), which introduces an error only if the segment for which the maximum is obtained changes within the interval $[t_k, t_{k+1})$. Clearly, the effect of the error will be more serious if the length of the interval is longer, and consequently this should be taken into account when defining the intervals. In the implementation of the MINLP model to be introduced in the next subsection, we shall make sure that these intervals are small enough to guarantee a sufficiently accurate approximation.

Using the two assumptions from above, (3) can be approximated by

$$\begin{aligned} &\mathcal{E}_k(\mathbf{U},\mathbf{t}) \\ &= \max_{l\in\mathcal{L}} \frac{S_{l0}^-}{\beta_{1l}} \\ &\quad \exp\left(\zeta(H_{l^*t_k} - H_{l_0 0}^-) - \alpha_l h_{lk}\right) \\ &\quad \cdot \Big[\exp(\beta_{1l}t_{k+1}) - \exp(\beta_{1l}t_k)\Big], \end{aligned} \tag{4}$$

with $\beta_{1l} = \beta_l - \delta$. The total expected damage in the planning horizon $[0, T)$ is then approximated by

$$\mathcal{E}(\mathbf{U},\mathbf{t}) = \sum_{k=0}^{K} \mathcal{E}_k(\mathbf{U},\mathbf{t}).$$

Note that for a fixed investment plan, it is possible to evaluate the size of the approximation error, since we can accurately evaluate the minimum and maximum operators in (3). This evaluation can be used to obtain a true comparison between investment plans with different discretization schemes.

To take into account the period after the planning horizon, it is assumed that there are no changes to the expected damage after T, and hence no more investments are required. Thus, the discounted expected damage after the planning horizon is $S_T \int_T^\infty \exp(-\delta t)dt$, which can be approximated analogously to (4), i.e.,

$$\begin{aligned} \mathcal{R}(\mathbf{U},\mathbf{t}) = \max_{l\in\mathcal{L}} \frac{S_{l0}^-}{\delta} \exp\big(&\beta_{1l}T - \alpha_l h_{lK} \\ &+ \zeta(H_{l^*t_K} - H_{l_0 0}^-)\big). \end{aligned} \tag{5}$$

The investment costs associated with the heightening of segment l at time t_k depend, of course, on the actual amount of the heightening. The costs, however, are assumed to be independent of the heightening of other segments, regardless of the moments of these heightenings. We use the same investment cost function as introduced by [1], and refer to it as *exponential* investment costs. For any *positive* heightening u_{lk}, the exponential investment costs are given by

$$\begin{aligned} &I_{lk}(\mathbf{u}^{(l)}) = (c_l + b_l u_l k)\exp\left(-\lambda_l \textstyle\sum_{i=0}^k u_{li}\right), \\ &\quad \mathbf{u}^{(l)} \in \mathbb{R}_+^{K+1}. \end{aligned} \tag{6}$$

Hence, the investment costs depend on the amount of the heightening and the amount of the total heightening up to time t_k. Since there are no investment costs when there is no heightening, the investment cost function is discontinuous at zero, i.e.,

$$\mathcal{I}_{lk}(\mathbf{u}^{(l)}) = \begin{cases} I_{lk}(\mathbf{u}^{(l)}) & \text{if } u_{lk} > 0, \\ 0 & \text{if } u_{lk} = 0. \end{cases}$$

The total discounted investment costs in the planning horizon are then given by

$$\mathcal{I}(\mathbf{U},\mathbf{t}) = \sum_{l=1}^{L}\sum_{k=0}^{K} \mathcal{I}_{lk}(\mathbf{u}^{(l)})\exp(-\delta t_k).$$

Since the objective is to minimize the sum of the investment costs and expected damage costs, the resulting optimization model can now be formulated as

$$\begin{aligned} &\min I(\mathbf{U},\mathbf{t}) + \mathcal{E}(\mathbf{U},\mathbf{t}) + \mathcal{R}(\mathbf{U},\mathbf{t}) \\ &\text{s.t. } \mathbf{U} \in \mathbb{R}_+^{L\times(K+1)}, \\ &\quad t_0 = 0 < t_1 < \cdots < t_K < T. \end{aligned} \tag{7}$$

MINLP model

This section discusses how the general dike height optimization problem (7) can be transformed into a mathematical optimization model that can be solved using optimization solvers. The problem as stated by (7) can be considered as a Non-Linear Programming (NLP) model since the decision variables **U** and **t** are continuous and the objective function's components are clearly non-linear.

Dike ring	Segments	MINLP objective (M€)	True objective (M€)	Solution time (min)
10	4	107.51	107.51	0.52
13	4	10.38	10.38	0.07
14	2	94.04	94.04	0.54
16	8	1044.45	1046.08	6.24
17	6	377.05	377.37	3.33
21	10	217.40	217.71	2.23
22	5	373.98	374.08	7.62
36	6	395.65	395.65	60.19
38	3	136.26	136.29	59.33
43	8	486.72	488.10	1.65
47	2	16.57	16.57	8.54
48	3	42.92	42.92	2.77

Table 1 Results optimization algorithm for a selection of dike rings.

From an optimization point of view, however, there are some issues that prevent us from actually solving the problem as stated by (7): the discontinuity of the investment cost functions at zero, and the approximation error of the expected damage in (4). The latter issue forces us to discretize the planning horizon, since continuous time variables could result in large intervals and consequently serious approximation errors. The discontinuity of the investment cost function can be resolved by discretization of the heightenings as well, or by adding binary decision variables that indicate whether a heightening is actually greater than zero or not. If both the moments and the amounts of the heightenings are discretized, then, theoretically, the problem can be solved using a dynamic programming approach. Unfortunately, the state space grows too large if multiple segments are considered, which implies that a dynamic programming approach is not applicable. Therefore, we consider a MINLP approach with discretization of the planning horizon only.

Next, the reformulation of problem (7) into a MINLP model is discussed. We assume that a *discretization scheme* $\mathbf{t} = (t_0, \ldots, t_{K+1})$ with $t_0 = 0 < t_1 < \cdots < t_K < t_{K+1} = T$ has been prefixed. The MINLP model then becomes:

$$\min \sum_{l=1}^{L} \sum_{k=0}^{K} \exp(-\delta t_k)(c_l y_{lk} + b_l u_{lk}) \cdot \exp\Big(-\lambda_l \sum_{i=0}^{k} u_{li}\Big) + \sum_{k=0}^{K} E_k + R \tag{8}$$

$$\text{s.t. } E_k \geq \frac{S_{l0}^-}{\beta_{1l}} \exp\Big(\zeta(H_{l^*k} - H_{l_0 0}^-) - \alpha_l h_{lk}\Big) \cdot \Big[\exp(\beta_{1l} t_{k+1}) - \exp(\beta_{1l} t_k)\Big], \quad l = 1, \ldots, L,\ k = 0, \ldots, K, \tag{9}$$

$$R \geq \frac{S_{l0}^-}{\delta} \exp\Big(\beta_{1l} T - \alpha_l h_{lK} + \zeta(H_{l^*K} - H_{l_0 0}^-)\Big), \quad l = 1, \ldots, L, \tag{10}$$

$$h_{lk} = \sum_{i=0}^{k} u_{li}, \quad l = 1, \ldots, L,\ k = 0, \ldots, K, \tag{11}$$

$$H_{lk} = H_{l0}^- + h_{lk}, \quad l = 1, \ldots, L,\ k = 0, \ldots, K, \tag{12}$$

$$0 \leq u_{lk} \leq y_{lk} M, \quad y_{lk} \in \{0,1\}, \quad l = 1, \ldots, L,\ k = 0, \ldots, K, \tag{13}$$

$$h_{lk}, H_{lk}, E_k, R \in \mathbb{R}, \quad l = 1, \ldots, L,\ k = 0, \ldots, K. \tag{14}$$

The objective function (8) includes the exponential investment costs with the fixed cost component c_l multiplied by y_{lk}. The binary variables y_{lk} combined with (13) are required to ensure that either $u_{lk} = 0$ and the investment costs in the objective function are zero, or $u_{lk} > 0$ and the investment costs are equal to $I_{lk}(\mathbf{u}^{(l)})$. In (13), M denotes an upper bound of the highest possible dike heightening. The auxiliary variables E_k and R represent the expected damage costs in $[t_k, t_{k+1})$ and $[T, \infty)$ respectively. Constraints (9) and (10) are used to model the damage costs as convex constraints without using the maximum operator, as occurs in (4).

It is clear that the optimal solution to problem (8)–(14) is fully determined by the decision variables u_{lk} ($l = 1, \ldots, L$, $k = 0, \ldots, K,$). These decision variables could be considered the 'pure' decision variables of problem (8)–(14), which, together with the discretization scheme $\mathbf{t}$, represent the investment plan $(\mathbf{U}, \mathbf{t})$ that answers the fundamental questions of *when* and *how much* should be invested in dike heightening.

Implementation issues

One of the project goals, set by Deltares, was that the model (8)–(14) could be solved for all major dike rings in a reasonable amount of time without the necessity to tune the algorithm's settings for specific dike rings. We were able to design a generic solution method that can solve any particular instance of the model without any fine-tuning. The model (8)–(14) has been implemented in AIMMS. Moreover, the software company HKV has integrated this model in the software package OptimaliseRing [3–4], used by the actual performers of the cost-benefit analysis. We used the AIMMS Outer Approximation (AOA) method that is implemented in AIMMS to solve the MINLP problems.

A heuristic algorithm is needed because MINLP (8)–(14) cannot be solved in reasonable time for dike rings with more than six segments. For example, we used a dedicated discretization scheme to reduce the number

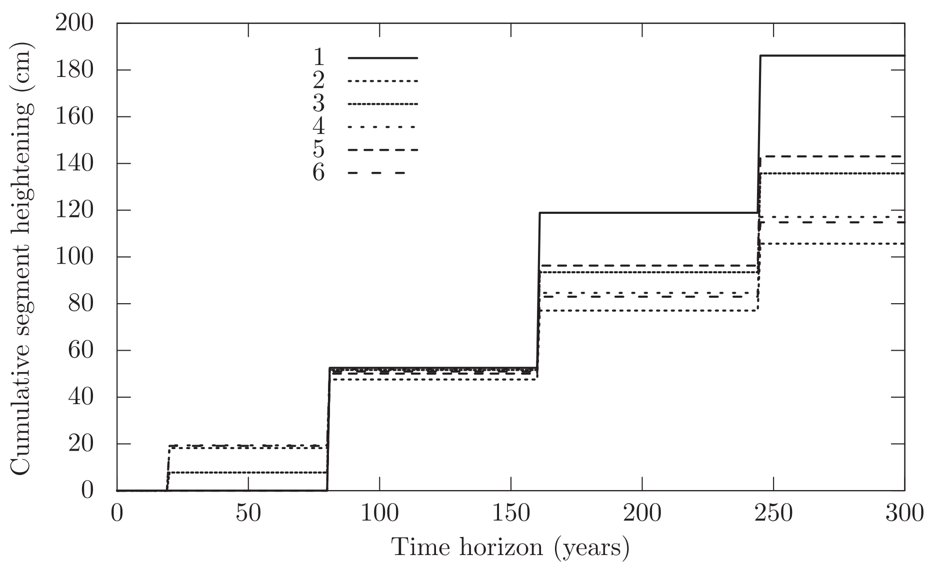


Figure 1 Cumulative segment heightenings for a dike ring with six segments.

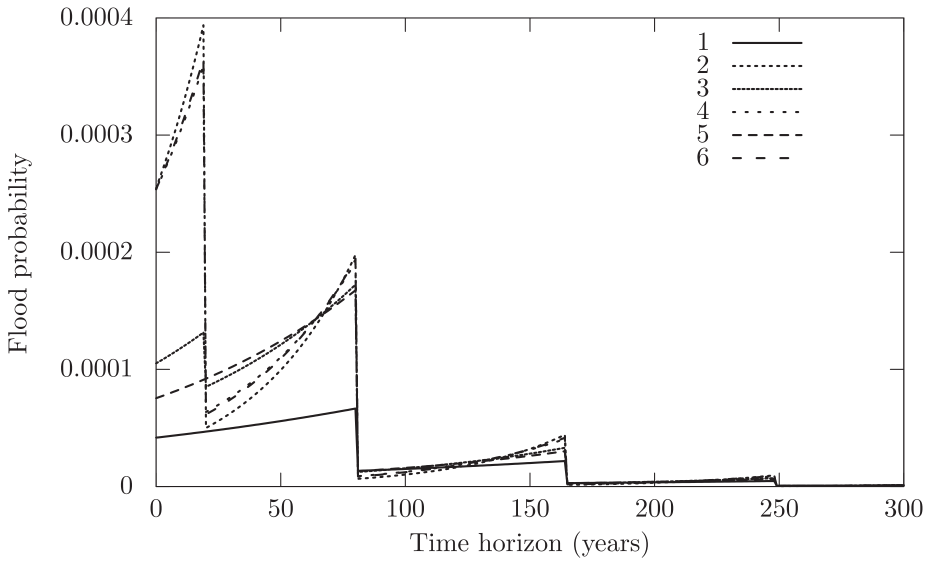


Figure 2 Segment flood probabilities for the same dike ring as in Figure 1.

of variables, and we added (nearly) redundant constraints to reduce the search space. Moreover, since it is clear that model (8)–(14) requires the input of several parameters, which in practice are often uncertain, we also developed a regret approach to obtain a solution that is robust with respect to these uncertainties. For more details we refer to [1].

Numerical results

As discussed in the previous section, the optimization algorithm has been implemented in AIMMS using the AOA solver. All numerical results in this section were obtained using AIMMS 3.8.5 with CPLEX 11.2 and CONOPT 3.14G on a PC with an Intel Core 2 CPU processor.

A database with data about the dike rings in the Netherlands was provided by Deltares. This database contains all relevant parameters for the non-homogeneous dike height optimization problem.

Overview dike rings

A selection of the dike rings from Deltares' database was optimized by our optimization algorithm. For all experiments we used common values for the discount rate per year ($\delta = 0.0247$) and the economic growth rate per year ($\gamma = 0.019$). A summary of the results for the exponential investment costs is shown in Table 1. The first two columns give the dike ring number along with the number of segments in the dike ring. The third column gives the MINLP model's objective value of the algorithm's final iteration. The fourth column gives a true evaluation of this objective value that does not suffer from an approximation error in the expected damage. It can be seen that the MINLP's objective is indeed a lower bound and that the approximation error is very modest, which indicates that the approximation of the expected damage is suitable for our MINLP model.

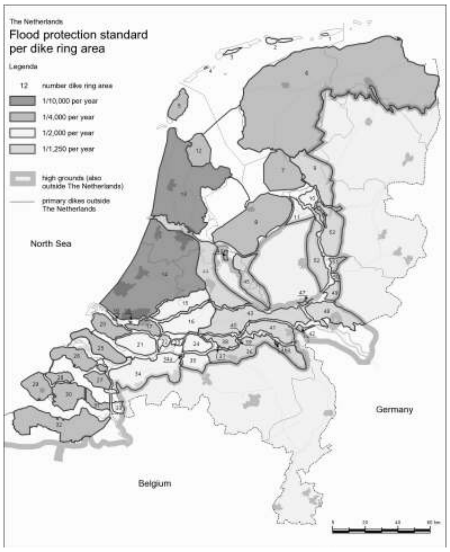
The fifth column in Table 1 gives the solution time in minutes. There does not appear to be a clear relationship between the number of segments and the solution time. This is mainly due to the fact that the discretization scheme is created in such a way that the number of resulting decision variables does not depend on the number of segments. In other words, a dike ring with more segments has a rougher discretization scheme than a dike ring with less segments, as explained in [1].

For the same set of experiments, Table 2 shows the moments of the first three updates of the dike rings, which correspond to one or more segment heightenings taking place at the same point in time. In addition, the table shows the effect the heightenings have on the dike ring's flood probability, i.e., the flood probabilities just before and just after the updates are listed. For the new safety standards in this example, there are five out of twelve dike rings that require immediate segment heightenings at $t = 0$. The results also clearly indicate that the flood probabilities just prior to a heightening decrease over time. This is a result of the economic growth, which increases the damage costs if a flood occurs, and therefore it is beneficial to let the flood probabilities decrease over time.

Let us take a closer look at the resulting solution for a dike ring with six segments. Figures 1 and 2 give a graphical overview of the final solution obtained with the iterative algorithm. Figure 1 shows the cumulative heightenings of the six segments during the 300-year planning horizon. Figure 2 shows the resulting segment flood probabilities. It can be seen that the two segments 1 and 5 are not heightened together with the other segments at $t = 20$. Figure 2 also shows why it is not necessary to heighten these two segments: their flood probabilities are still very low compared to the other segments. Although in this particular example there is a moment at which not all segments are heightened simultaneously, the figure clearly demonstrates why simultaneity very frequently leads to very good, or even optimal, results. Recall that a dike ring's flood probability is determined by the maximum segment flood probability. Hence,

	First heightening			Second heightening			Third heightening		
Dike ring	t	$P^-(t)$	$P^+(t)$	t	$P^-(t)$	$P^+(t)$	t	$P^-(t)$	$P^+(t)$
10	68	6.6×10^{-4}	6.7×10^{-5}	156	1.2×10^{-4}	1.4×10^{-5}	244	2.5×10^{-5}	2.9×10^{-6}
13	140	1.8×10^{-4}	1.6×10^{-5}	244	3.7×10^{-5}	2.5×10^{-6}	-	-	-
14	36	1.5×10^{-4}	2.3×10^{-5}	104	4.6×10^{-5}	6.5×10^{-6}	168	1.3×10^{-5}	1.8×10^{-6}
16	0	5.0×10^{-4}	2.8×10^{-4}	40	3.7×10^{-4}	7.7×10^{-5}	105	1.2×10^{-4}	2.5×10^{-5}
17	20	3.8×10^{-4}	9.1×10^{-5}	81	1.9×10^{-4}	1.3×10^{-5}	165	4.3×10^{-5}	2.9×10^{-6}
21	0	5.0×10^{-4}	2.5×10^{-4}	45	5.2×10^{-4}	5.3×10^{-5}	120	1.5×10^{-4}	1.4×10^{-5}
22	7	5.2×10^{-4}	4.5×10^{-5}	100	1.1×10^{-4}	8.5×10^{-6}	200	2.3×10^{-5}	1.2×10^{-6}
36	36	1.1×10^{-3}	1.7×10^{-4}	102	4.1×10^{-4}	6.3×10^{-5}	165	1.5×10^{-4}	2.4×10^{-5}
38	0	6.7×10^{-4}	2.7×10^{-4}	28	4.6×10^{-4}	1.9×10^{-5}	126	8.6×10^{-5}	3.2×10^{-6}
43	0	2.7×10^{-4}	2.7×10^{-4}	30	4.6×10^{-4}	3.9×10^{-5}	120	9.7×10^{-5}	7.3×10^{-6}
47	30	2.5×10^{-4}	1.2×10^{-5}	120	4.0×10^{-5}	1.2×10^{-5}	200	1.6×10^{-5}	5.8×10^{-7}
48	0	2.8×10^{-4}	1.2×10^{-5}	77	3.0×10^{-5}	2.9×10^{-6}	154	7.1×10^{-6}	6.6×10^{-7}

Table 2 Moments (in years measured from the start of the planning horizon) of the first three dike ring updates and the flood probabilities just before ($P^-(t)$) and after ($P^+(t)$) the updates.



if a single segment is not heightened simultaneously with the other segments, then it is likely that this segment's flood probability will become, or even remain, the dike ring's maximum flood probability. The benefit of heightening the other segments, in terms of decreasing the expected damage, is therefore usually smaller than the incurred investment costs. Finally, we remark that this is an example of a dike ring where, in the optimal solution, not all segments are always heightened simultaneously. Very often this is the case, however.

Practical impact

The ultimate goal of our project was to give recommendations for new flood protection standards in the Netherlands. In [5] it is described how to construct flood protection standards based on the optimal investment strategy resulting from our MINLP model. Based on the final results, published in [7–8], we concluded that increasing the legal protection standards of all dike-ring areas tenfold, as the Second Delta Committee recommended, is unnecessary. The current protection standards are (more than) appropriate, except for three regions: a part of the dike rings along the Rhine and Meuse Rivers (i.e., part of the areas that now have a standard of $1/2000$ or $1/1250$ per year), the southern part of dike ring 8 Flevoland (comprising the large, rapidly growing city of Almere), and some dike rings (e.g., 20) near Rotterdam.

The Water Advisory Committee, chaired by Crown Prince (currently the King) Willem-Alexander, discussed the final report of the CBA WV21 [7] and endorsed our results in a letter dated 9 March 2012. The House of Parliament discussed the report on 5 December 2011 and 4 April 2012. In an unanimous motion on 17 April 2012, the parliament asked the government to present a concrete proposal for new legal standards in 2014, explicitly referring to the three regions named in [7–8] and under the condition that improvements are justified by a CBA. The state secretary of the Ministry of Infrastructure and the Environment (I&M) followed the report's results and recommendation: A tenfold increase in protection standards for all dike-ring areas is not needed and only the protection standards in the three regions named in the report need improvement.

The state secretary therefore instructed the Delta Commissioner to adapt, as necessary, the protection standards derived for these areas according to local situations, and to ensure that a minimal protection level is guaranteed everywhere in a dike ring area. On 26 April 2013, the Minister of I&M, Melanie Schultz van Haegen, confirmed these conclusions in a policy letter to the parliament.

The Delta Commissioner has announced that his proposal for new flood protection standards will closely follow the main conclusions of this project, which have already been recognized in discussions with the water boards and the provinces. In 2014, the cabinet will take a decision on these proposals. In 2015, the final decision on the improvement of these protection standards will be taken in parliament, such that new standards — after approval of the law in parliament — will be legally effective by 2017. Finally, in a letter dated 27 November 2012, the chairman of the renowned Second Delta Committee agreed with these conclusions, which clearly deviate from the committee's earlier advice. Compared to this earlier recommendation, this successful application of operations research yields both a highly significant increase in protection for these regions (in which two-thirds of the benefits of the proposed improvements accrue) and approximately 7.8 billion Euro in cost savings.

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