A special issue for Henri Poincaré

One hundred years ago, on 17 July 1912, Henri Poincaré died at the age of only 58 years. He had been unusually productive with fundamental results in mathematics, physics and astronomy, and he can be regarded as one of the most important mathematical physicists of all times; his work had a major impact throughout the twentieth century. He reshaped several existing fields and created some new ones in pure and applied mathematics. Highlights include his thesis which excels in the qualitative, geometric approach to differential equations, his analysis of automorphic functions that put him in contact with Fuchs and Klein, his creation of the theory of dynamical systems and his major influence on topology (analysis situs).

Poincaré also expressed philosophical ideas, and his philosophical essays made him well-known to the general public. They put him at the heart of the intellectual discourse of those times in France and other European countries. Apart from scientific papers and books, there exist thousands of letter exchanges between Poincaré and other scientists, family and friends. He attended dinner sessions organised by the psychoanalyst Marie Bonaparte where writers, composers and politicians met; Poincaré inspired Friedrich Nietzsche with his recurrence theorem. As chairman of the committee that had to assess the scientific merit of the 'proofs' for the guilt of the unfortunate Alfred Dreyfus, he took a view that was opposed to the ideas of the French reactionary establishment of those times, discarding the so-called evidence.

In this special NAW issue dedicated to Henri Poincaré we have been able to procure an exclusive interview with the eminent scientist, but apart from this, the emphasis is on his scientific work. It is not so well-known that Poincaré made fundamental contributions to the theory of partial differential equations, introducing for instance L^2-convergence (as it is known nowadays) and the idea of a generalised solution. We are happy that Jean Mawhin has written for us about this topic. We all know that Poincaré developed his general theory of dynamical systems with an eye on applications in celestial mechanics, but later he applied this theory to non-linear oscillations with applications in telegraph communication. Self-excited oscillations as studied later by Van der Pol can be found in the papers on this topic; Jean-Marc Ginoux discusses these results. Another aspect is the emergence of algebraic ideas around 1900; Jeremy Gray writes about Poincaré's use of group theory in mathematical physics.

In the nineteenth century the necessity of convergence of series had become a kind of mathematical dogma, so it looked slightly heretic when both Stieltjes and Poincaré introduced around 1885 the notion of approximation by asymptotic- or semi-convergent series; one of the articles describes this development. There was a relation with another famous Dutchman, L.E.J. Brouwer, who was inspired by Poincaré but carried the ideas of intuitionism much further; see the article by Van Dalen. The style of writing of Poincaré was one of permanent discourse with the reader and one cannot think of a style more opposed to this than the writing style of M. Bourbaki. Poincaré's most often used sentence is "ce n'est pas tout" after which he starts elucidating new aspects of the problem at hand. This is also typical for his creation of topology which is inspired by three-dimensional geometry and guided by intuition; see the article by Siersma.

Part of the mathematical ideas of Henri Poincaré are still unexplored, but there are also messages of a broader nature to be obtained from his work. First we note that geometry and analysis are intertwined in his papers. In dynamical systems for instance we note the interplay of geometric ideas, the qualitative aspects that give general insight, and the quantitative analysis resulting in actual formulas and predictions. This combination has been very fruitful. Secondly, we mention the way he presents mathematical and other scientific ideas to the specialist and to the general public. His style of writing produces an efficient transfer of ideas and it helps to communicate to the public the excitement, the interest and the elegance of scientific reasoning. We remind the readers of this journal of the perpetual importance of communicating the excitement of doing mathematics, in particular towards prospective students.

We cannot do justice to all of Poincaré's achievements in a single issue of this journal, but fortunately this is remedied by three biographies that appear in English this year. These are the first modern biographies. The books are so recent that there was no time to have them properly reviewed for this special issue, therefore we asked their authors to highlight the key ideas and contents of each. There is unavoidably some overlap between these biographies but as a whole they supplement each other nicely. With a scientist of the stature of Henri Poincaré, there is room for even more biographies.

Ferdinand Verhulst and Steven Wepster (eds.)
Mathematical Institute, Utrecht University