Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome.
For each problem, the most elegant correct solution will be rewarded with a book token worth 20 euro. At times there will be a Star Problem, to which the proposer does not know any solution. For the first correct solution sent in within one year there is a prize of 100 euro.
When proposing a problem, please either include a complete solution or indicate that it is intended as a Star Problem. Electronic submissions of problems and solutions are preferred (problems@nieuwarchief.nl).
The deadline for solutions to the problems in this edition is June 1st, 2011.

Problem A (proposed by Hendrik Lenstra)

Prove that every commutative ring with identity having at most five ideals is a principal ideal ring.

Problem B (proposed by Gabriele Dalla Torre, inspired by the Dutch Mathematical Olympiad 2010)

Let $(a_n)_{n\geq 1}$ be a sequence of integers that satisfies

$$a_n = a_{n-1} - \min(a_{n-2}, a_{n-3})$$

for all $n \geq 4$. Prove that for every positive integer k there is an n such that a_n is divisible by 3^k.

Problem C (proposed by Rik Bos)

Find all positive integers N, r, p with p prime that satisfy

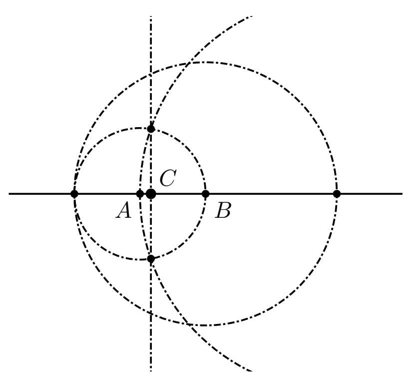
$$\prod_{\ell \leq N} \ell = p(p^r + 1),$$

where the product runs over all primes ℓ not exceeding N.

Edition 2010-3 We received solutions from: Maurice Bos (Delft), Pieter de Groen (Brussel), Thijmen Krebs (Nootdorp), Tejaswi Navilarekallu (Amsterdam), José H. Nieto (Maracaibo), and Sep Thijssen (Nijmegen).
All three problems asked for a construction with ruler and compass in a limited number of steps. More precisely, given a collection of points, lines, and circles in the plane, a *move* consists of adding to the collection either a line through two of the points, or a circle centered at one of them and passing through another. At any time one is allowed to freely add any intersection point among the lines and circles, as well as any sufficiently general point, either in the plane, or on any of the lines or circles.

Problem 2010-3/A Given a line and distinct points A and B on it, construct in at most four moves the point C between A and B that satisfies $6|AC| = |AB|$.

Solution



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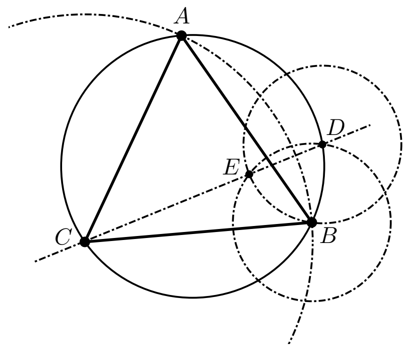
This problem was solved by Pieter de Groen, Thijmen Krebs, Tejaswi Navilarekallu, José H. Nieto, and Sep Thijssen. All found the construction above. Pieter de Groen receives the book token (by random draw).

Problem 2010-3/B Given a circle, but not its center, construct in at most seven moves an equilateral triangle whose vertices lie on the circle.

Solution This problem was solved by Maurice Bos, Pieter de Groen, Thijmen Krebs, Tejaswi Navilarekallu, and José H. Nieto. All found the following construction. Thijmen Krebs receives the book token (by random draw).
Let B and D be two distinct points on the given circle. We start with two circles, one centered in B and going through D, the other centered in D and going through B. We then proceed as follows.



To see the correctness of this construction, note $\angle CDB = \angle EDB = \pi/3$, and as the angles CDB and CAB stand on the same chord, $\angle CAB$ equals $\pi/3$ as well.

Problem 2010-3/C Construct a square in at most eight moves.

Solution This problem was solved by Pieter de Groen, Thijmen Krebs, Tejaswi Navilarekallu, and José H. Nieto. All found the following construction. Tejaswi Navilarekallu receives the book token (by random draw).