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In Memoriam Hans Duistermaat (1942–2010)

Recollections of a godsend talent

In 1982 Hans Duistermaat and Gert Heckman wrote the article 'On the variation in the cohomology of the symplectic form of the reduced phase space', introducing the well-known Duistermaat–Heckman formula. Gert Heckman describes the becoming of this article, and the important role that Duistermaat played in the early days of his career.

I would like to share with you some recollections of Hans Duistermaat from the period 1978–1981, during which he played a crucial role in my mathematical development. In 1976 I had started my dissertation work under the guidance of Gerrit van Dijk. In his thesis of 1962, the Russian mathematician Alexander Kirillov had developed a very elegant geometric method, the so-called orbit method, for understanding the representation theory of connected nilpotent Lie groups. In this method the branching rule for understanding how an irreducible representation decomposes under restriction to a subgroup has a very simple and elegant answer.

Gerrit suggested to me that I try to understand to what extent this orbit method could shed new light on the representation theory of semisimple Lie groups, in particular for the discrete series representations. In my first short paper from the summer of 1978 I worked out a particular example for compact Lie groups. According to the customs of those days I sent it around to several potentially-interested people, and in return quickly received a reaction of Hans. My main result turned out to be already in the literature, and in addition Hans sketched an alternative and more elegant geometric proof. Aware of the fact that his letter might be intimidating for me he wrote at the end: "It is maybe superfluous to emphasize that I do not write you this proof out of pedantry, but rather as a sign of interest for your work and hopefully also leading to a still better understanding of the whole situation." So my first little paper went into the wastebasket, but I was to receive something more valuable in return. I visited Hans regularly in Utrecht, and in June 1980 I defended my dissertation in Leiden with both Gerrit and Hans as thesis advisors. I realize now how lucky I was to have these two complementary teachers: Gerrit with his extensive knowledge of the work of Harish-Chandra, and Hans as the eminent analyst and geometer.

Symplectic Geometry

In August 1980 I went to Boston, to spend two years as a postdoc at MIT, and in September I lectured in the Lie groups seminar about my thesis work: how the orbit method for compact Lie groups describes the branching rules in an asymptotic way, and how this leads to a convex polytope in which the multiplicities of the branching rule had their support. The talk was received well, most notably by Victor Guillemin. Victor knew Hans well, and had great admiration for him. In 1975 they had written a beautiful article on the spectrum of elliptic operators on compact manifolds. Looking back at my time at MIT I realize again how lucky I was to be there during that period with Victor around.

That fall a number of new insights were unveiled regarding continuous symmetry reduction in symplectic geometry through the work of Guillemin–Sternberg, Atiyah–Bott and Mumford. In these at first sight rather different contexts, namely quantum mechanics, quantum field theory and algebraic

Hans Duistermaat in 1981

Photograph: Archives of the MFO

In 1982 Hans Duistermaat and Gert Heckman published the article 'On the variation in the cohomology of the symplectic form of the reduced phase space'. The following theorem is the main result of this article.

Theorem. Let a torus T act effectively on a symplectic manifold (M, σ) in a Hamiltonian way with a proper momentum map $J : M \to \mathfrak{t}^*$. Let (M_ζ, σ_ζ) be the reduced phase space over a general point ζ in a connected component C of the set of regular values of J. Then

$$[\sigma_\xi] = [\sigma_{\xi_0}] + \langle c, \xi - \xi_0 \rangle$$

varies linearly with ξ for $\xi, \xi_0 \in C$, with $c \in H^2(M_\zeta, \mathfrak{t})$ the (common) Chern class of the principal T-fibration $J^{-1}(\zeta) \to M_\zeta$.

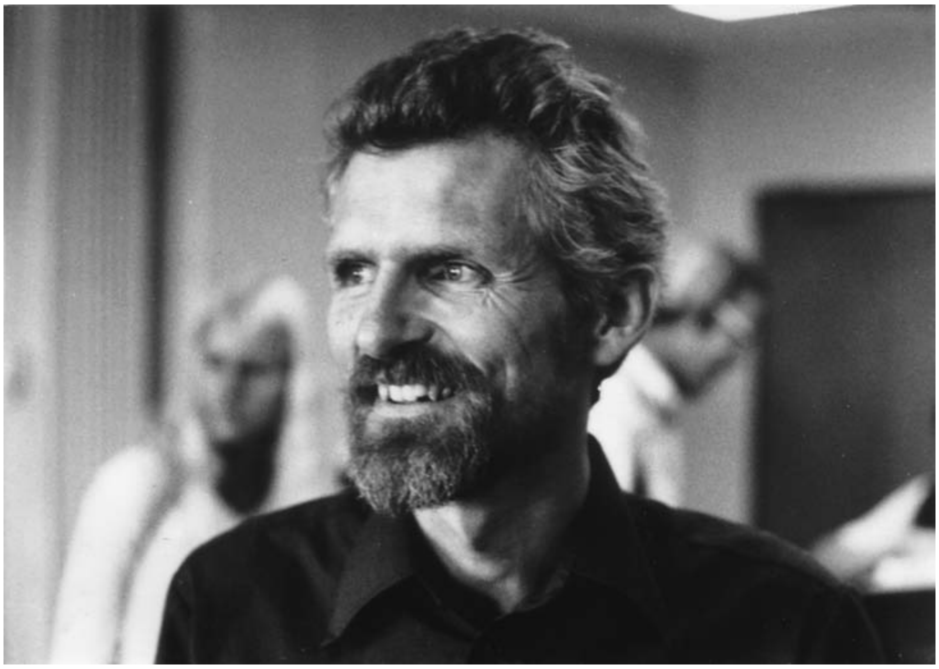
geometry there was a single fundamental underlying concept for the description of symmetry, namely that of the geometry of the moment map (or momentum map as Hans preferred to call it). I quote from a survey article of Bott from 1988: "In fact, it is quite depressing to see how long it is taking us collectively to truly sort out symplectic geometry. I became aware of this especially when one fine afternoon in 1980, Michael Atiyah and I were trying to work in my office at Harvard. I say trying, because the noise in the neighboring office made by Sternberg and Guillemin made it difficult. So we went next door to arrange a truce and in the process discovered that we were grosso modo doing the same thing. Later Mumford joined us, and before the afternoon was over we saw how Mumford's stability theory fitted with the Morse theory. The important link here is the concept of a moment map, which in turn is the mathematical expression of the relation between symmetries of Lagrangians and conserved quantities; in short, what the physicists call Noether's theorem and which is one of their great paradigms." In this quote Bott refers to the results of fundamental publications by Guillemin–Sternberg, Mumford, Ness, Atiyah–Bott and Kirwan. Since then, symplectic geometry has become a truly independent field in its own right.

The becoming of our joint paper

In the spring of 1981 Victor gave a course on symplectic geometry, with special emphasis on the geometry of the moment map, and I learned the subject well. During the month of August I went back to the Netherlands to visit family and friends. The day before my return I was doing some last-minute work at MIT, when it occurred to me that the rather complicated locally-polynomial formulas for the multiplicities could be explained by a linear variation of the symplectic form in the cohomology of the reduced phase space, at least over the generic fiber. A nice idea, but I had no clue how to prove it. A few days after my return I visited Hans, and we spent a whole afternoon talking about symplectic geometry. I told him about my question, and he listened attentively. That same evening he called me up at my parents' house, and with a piece of scratch paper on my lap I got an exposition of what later would become our joint paper.

An important role in my career

Our work was well-received. Independently of one another, Berline–Vergne and Atiyah–Bott placed it in the more general framework of equivariant cohomology. Our article was later used by Ed Witten in his work on two dimensional Yang–Mills theory. More recently our theorem was used again by Mariyam Mirzakhani in her computation of the Weil–Petersson volumes of the moduli space of curves. In September 1982 I got a permanent position in Leiden as assistant to Gerrit van Dijk: a solid base from which to pursue mathematical work professionally. I now appreciate very well the important role played by Hans during the early stages of my career. It is not inconceivable that without him I would have become a high school teacher rather than a university professor of mathematics.



Photograph: Archives of the MFO

Hans Duistermaat in 1981

A great emptiness

After this period of intensive contact from 1978 to 1981 our mathematical roads diverged. Our personal relationship remained however, and I cherish the memories of the parties held for his 60th birthday and on the occasion of his royal decoration.

The sudden passing of Hans leaves behind a great emptiness, in the first place for his wife Saskia, his daughters Kim and Maaike and his relatives, but also for the many mathematicians with whom he worked together. During the cremation ceremony many affectionate words were spoken about Hans. His sister Dineke told the story of how, when she asked him as a student why he had chosen for mathematics, Hans replied that he had no other option, because his talent for mathematics was such a godsend. I realize how very lucky I am that Hans shared this talent with me so generously.