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In Memoriam Hans Duistermaat (1942–2010)

Working single-mindedly on a project

In 1975 Hans Duistermaat and Victor Guillemin published the article 'The spectrum of positive elliptic operators and periodic bicharacteristics' in Inventiones Mathematicae. Victor Guillemin describes his reminiscences of the period that he and Duistermaat worked together on this article.

These are a few brief recollections of mine from the period 1973–1974, the two years in which Hans Duistermaat and I worked together on our article 'The spectrum of positive elliptic operators and periodic bicharacteristics'. For me the two most memorable and exciting years of that decade. In the summer of 1973, Hans and I met for the first time at an AMS sponsored conference on differential geometry at Stanford and began to formulate the ideas that became the wave trace part of our paper. Then in the fall of 1974 he made a long visit to MIT during which we firmed up these ideas and also proved the periodic bicharacteristic results that became the second main part of our paper.

The wave trace part

A little pre-history: In the early 1970's, Bob Seeley, David Schaeffer, Shlomo Sternberg and I ran a seminar at Harvard which was largely devoted to Hörmander's papers [1] and [2] and Duistermaat and Hörmander [3]. In particular we spent a lot of time going through [3], which was the first systematic application of microlocal techniques to the problem of propagation of singularities. (Like analysts the world over, we were amazed at how simple this subject becomes when viewed from the perspective of the cotangent bundle.) Therefore, when I met Hans that summer I was well primed to discuss with him the contents of these papers. However, what initiated our collaboration was another memorable event from that conference: the announcement by Marcel Berger of Yves Colin de Verdière's result on the spectral determinability of the period spectrum of a Riemannian manifold. I vividly remember sitting next to Hans at Berger's lecture and our exchanging whispered comments as it became more and more evident that what Yves had done was intimately related to the things the two of us were currently thinking about. By the time the conference ended we had formulated a trace theorem for Fourier integral operators which asserted that the singularities of the wave trace are supported on the period spectrum of P (and hence that the wave trace gave one a simple means of accessing these data). As I mentioned above, this result became the first part of our paper.

The periodic bicharacteristic results

The second part of our paper was based on an observation that Hans and I had made (each independently) that spring apropos of a result of Hörmander in [1]. One of the most quoted results of Hörmander's paper is a generalization of a theorem of Avakumovic in which he obtains an 'optimal' error term in the Weyl law for an elliptic pseudodifferential operator, P, and shows that this error term *is* optimal by showing that this is the case if P is the Laplace operator on the standard round sphere. I noticed that this can be related to the fact that for the n-sphere the bicharacteristic flow associated with P is periodic. (More explicitly, I noticed that if the bicharacteristic flow of an elliptic operator P is periodic (i.e., P is Zoll) there has to be a clustering of eigenvalues about a lattice which prevents a sharpening of the Weyl law and vice versa.) In proving this result I made essential use of techniques developed in [3], so it was not surprising that when I described it to Hans at Stanford, I found that he had been thinking along similar lines. Moreover, it slowly began to dawn on us that the Hörmander example was just the tip of the iceberg. Among other things we noticed that his optimal error term could be replaced by a slightly better optimal error term (a big 'O' could be converted into a little 'o') if P was not Zoll, and also noticed that in this case the Weyl law could be differentiated to give an equidistribution result for eigenvalues. We also obtained a much sharper version of my clustering result: we showed that the clusters are clearly demarcated eigenbands of fixed width. Subsequently, Alan Weinstein and Yves Colin de Verdière added a further dimension to this story by discovering that when Zoll phenomena are present, these clusters satisfy their own beautiful distribution law. Furthermore, Bill Helton discovered an extremely clean and economical version of our result: Let A be the set of numbers obtained by taking all differences of

Hans Duistermaat in 1976

pairs of eigenvalues, and let B be the cluster set of A. Then, if the bicharacteristic flow is periodic, B is an integer lattice, and if not, it is the whole real line.

Finishing the paper

To conclude these reminiscences: by the spring of 1974, most of the conjectures we had made at the Stanford meeting had been supplied with rigorous proofs, although Hans continued, as was his wont, to tinker with them for the next several months just to make sure that they were 'best possible'. (No one was going to be able to achieve instant immortality by slightly improving them.) When Hans visited me in the fall, the only unfinished piece of business was the Zoll part of the paper, and that consumed all our energy for four intense weeks. One typical 'Hans' memory from that time: One evening I return home late, exhausted in mind and spirit following a frustrating day in which the two of us struggled without success to settle a delicate point about how large sets of periodic bicharacteristics have to be for clustering to occur. At 2 o'clock in the morning I get jarred awake by a phone call from Hans letting me know that he had settled it. I remember the aftermath of Hans's visit as a period of a slow, painful decompression. Never before had I worked so intensely and so single-mindedly on a project (and, for better or for worse, was destined never to do so again).

References

1 L. Hörmander, The spectral function of an elliptic operator, *Acta Mathematica* **121**, 193–218 (1968).

2 L. Hörmander, Fourier integral operators I, *Acta Mathematica* **127**, 79–183 (1971).

3 J.J. Duistermaat and L. Hörmander, Fourier integral operators II. *Acta Mathematica* **128**, 184–269 (1972).