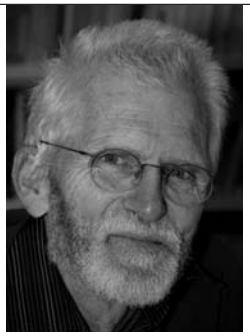
Erik van den Ban
Mathematisch Instituut
Universiteit Utrecht
Postbus 80010
3508 TA Utrecht
e.p.vandenban@uu.nl

Johan Kolk
Mathematisch Instituut
Universiteit Utrecht
Postbus 80010
3508 TA Utrecht
j.a.c.kolk@uu.nl



Hans Duistermaat

In Memoriam Hans Duistermaat (1942–2010)

Grasping the essence

On March 19, 2010, mathematics lost one of its leading geometric analysts, Johannes Jisse Duistermaat. At age 67 he passed away, after a short illness following a renewed bout of lymphoma the doctors thought they had controlled. 'Hans', as Duistermaat was universally known among friends and colleagues, was not only a brilliant research mathematician and inspiring teacher, but also an accomplished chess player, very fond of several physical sports, and a devoted husband and (grand)father. Erik van den Ban and Johan Kolk look back on Duistermaat's life and work in this contribution; it is followed by remembrances and surveys by some of his friends, students, and colleagues.

Hans Duistermaat was born December 20, 1942, in The Hague. After the end of World War II his parents moved to the Dutch East Indies (Indonesia nowadays), where he spent a happy youth. Hans was a student at Utrecht University, where he continued to write his PhD thesis on mathematical structures in thermodynamics. The famous geometer Hans Freudenthal is listed as his advisor, but the topic was suggested and the thesis directed by Günther K. Braun, professor in applied mathematics, who tragically died one year before the defense of the thesis, in 1968.

Since the thesis had led to dissent between mathematicians and physicists at Utrecht, Hans dropped the subject of thermodynamics. Nevertheless, this topic exerted a decisive influence on his further development: in its study Hans had encountered contact transformations. These he studied thoroughly by reading S. Lie, who had initiated their theory. In 1969–70 he spent one year in Lund, where L. Hörmander was developing the theory of Fourier integral operators (FIO's); these are far-reaching generalizations of partial differential operators. Hans's knowledge of the work of Lie turned out to be an important factor in the formulation of this theory. The mathematical reputation of Hans was firmly established by a long joint article with Hörmander concerning applications of the theory to linear partial differential equations. In 1972 Duistermaat was appointed full professor at the Catholic University of Nijmegen, and in 1974 at Utrecht University, as the successor to Freudenthal.

Geometric analysis

In these years, he continued to work on FIO's. At the Courant Institute in New York he wrote a paper on *Oscillatory integrals, Lagrange immersions and unfolding of singularities*, a survey of the subjects in the title that sets the agenda for the study of singularities of smooth functions and their applications to distribution theory. In some sense it is complementary to FIO's and parallel to work of V.I. Arnol'd. Furthermore, together with V.W. Guillemin he composed an article about application of FIO's to the asymptotic behavior of spectra of elliptic operators, and its relation to periodic bicharacteristics; see the article by Guillemin on pp. 238–239 in this issue. In these works one clearly discerns the thread connecting most of Hans's achievements: on the basis of a complete clarification of the underlying geometry deep and powerful results are obtained in the area of geometric analysis.

Moving into new terrain

It is characteristic for the work of Hans that after a period of intense concentration on a particular topic, he would move to a different area of mathematics, bringing thereby acquired insights quite often to new fruition. Usually, this change was triggered by a question of a colleague, but more frequently so, of one of his PhD students. Hans went to great efforts to accommodate the special needs of his students and help them develop in their own way, not in his way. In particular, in several cases Hans has been willing and also able to guide students working on topics initiated by themselves. Examples are the theses of P.H.M. van Mouche and M.V. Ruzhansky.

It was by questions of J.A.C. Kolk and

The Duistermaat–Heckman formula

$$\int_M e^{J_X}\, e^{\sigma} = \sum_j \int_{N_j} \frac{e^{i_j{}^* J_X}\, e^{i_j{}^* \sigma}}{\det \frac{LX+\Omega}{2\pi i}}$$

Hans Duistermaat in 1977

G.J. Heckman that Hans became interested in the theory of semisimple Lie groups. With Kolk and V.S. Varadarajan he published basic papers on harmonic analysis and the geometry of flag manifolds, with the method of stationary phase as the underlying theme. This work also provided an impetus for the groundbreaking work with Heckman that culminated in the Duistermaat–Heckman formula, which will be discussed on pp.240–241 in this issue by Heckman.

In the thesis of E.P. van den Ban one finds the novel idea, suggested by Hans, of taking the integrals representing the spherical eigenfunctions on a semisimple Lie group, which are integrals over a real flag manifold, into integrals on real cycles inside the complex flag manifold. This allowed application of the method of steepest descent in order to study their asymptotics, generalizing the approach known in the theory of hypergeometric functions.

One of the basic mathematical interests of Hans, to which he returned throughout his life, was classical mechanics and its relations with differential equations. In this case too, it was often through the work of his students S.J. van Strien, H.E. Nusse, J.C. van der Meer, J. Hermans, B.W. Rink, and A.A.M. Manders, that this topic was taken up again. His activities in this area will be discussed on pp. 242–243 in this issue by his colleague and co-author R.H. Cushman.

F.A. Grünbaum posed a problem that led to the joint article *Differential equations in the spectral parameter*. It classifies second-order ordinary differential operators of which the eigenfunctions also satisfy a differential equation in the spectral parameter. The classification is in terms of rational solutions of the Korteweg–De Vries equation.

Writing a review about the book *Lie's Structural Approach to PDE Systems* by O. Stormark led Hans to further study of that circle of ideas. The result was a paper on the contact geometry of minimal surfaces as well as the thesis of P.T. Eendebak.

Together with A. Pelayo he wrote several papers about symplectic differential geometry; furthermore he directed the thesis of R. Sjamaar. In this part of mathematics Hans was a very influential figure, witness his frequent contacts with other leading investigators, like Guillemin and A. Weinstein.

Mathematics in society

In the later part of his life, Hans had an intense interest in application of mathematics elsewhere in society. For instance, he was a consultant to Royal Dutch Shell, which lead to the thesis of C.C. Stolk on the inversion of seismic data. Interaction with mathematical economists during a conference at Erasmus University in Rotterdam, where Hans had been invited to give an introduction to Riemannian geometry, sparked his interest in barrier functions, used in convex programming. He also collaborated with the geophysicist P. Hoyng modeling the polarity reversals of the earth magnetic field. The lengths of the time intervals between the subsequent reversals form an irregular sequence with a large variation, which make the reversals look like a (Poisson) stochastic process. Within a short period of time he mastered the nontrivial stochastics needed in this problem.

Books

The bibliography of Hans contains eleven books. *Fourier Integral Operators* gives an exposition of seminal results in the area of microlocal analysis. *The Heat Kernel Lefschetz Fixed Point Formula for the Spin-c Dirac Operator* is concerned with a direct analytic proof of the index theorem of Atiyah–Singer in a special case of interest for symplectic differential geometry. *Lie Groups*, jointly with Kolk, contains a new proof of Lie's third theorem on the existence of a Lie group associated to any Lie algebra. The construction of the group as the quotient of a path space in the Lie algebra was the model for many important generalizations, including the integration of Lie groupoids by M. Crainic and R.L. Fernandes.

Analysis of Ordinary Differential Equations (in Dutch), jointly with W. Eckhaus, grew out of a set of lecture notes. Similarly, together with Kolk he authored *Multidimensional Real Analysis I: Differentiation* and *II: Integration* (also published in a China edition), and *Distributions: Theory and Applications*. The last book contains a novel proof of the kernel theorem of L. Schwartz, which in turn is used to efficiently derive numerous important results, and a treatment of theories of integration and of distributions from a unified point of view. The last four books together form a veritable 'cours d'analyse mathématique'.

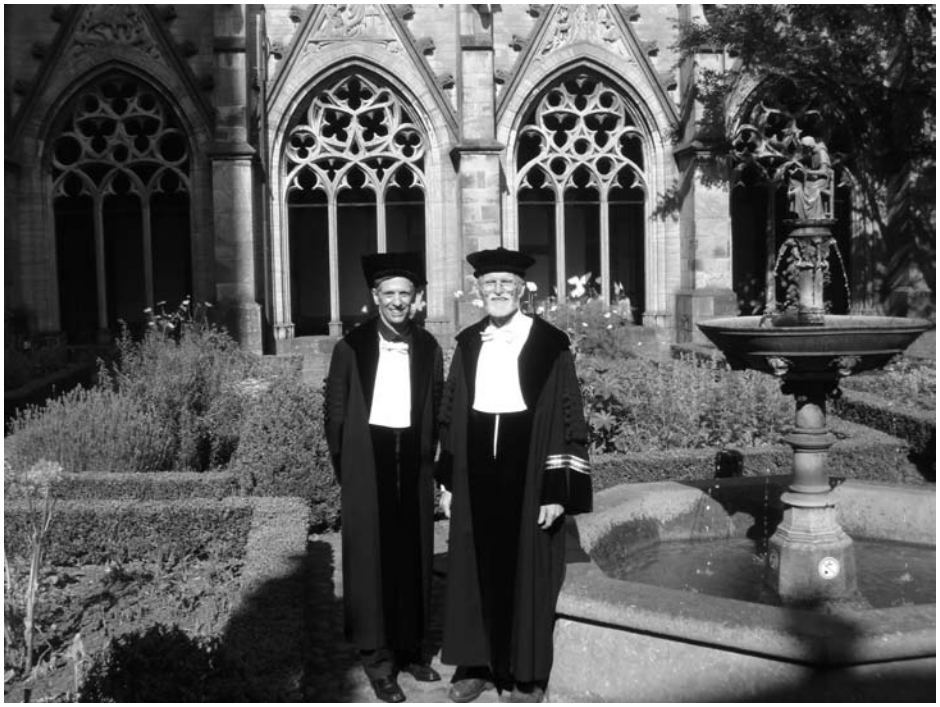
In the book *Discrete Integrable Systems: QRT Maps and Elliptic Surfaces*, QRT (Quispel, Roberts, and Thompson) maps are analyzed using the full strength of Kodaira's theory of elliptic surfaces. A complete and self-contained exposition is given of the latter theory, including all the proofs. Many examples of QRT maps from the literature are analyzed in detail, with explicit formulas and computer pictures. The interest in QRT maps was triggered by interaction with J.M. Tuwankotta. Hans had the idea to use the technique of blowing up, which he had previously encountered in the article *Constant terms in powers of a Laurent polynomial* jointly with Wilberd van der Kallen.

Writing

The mode of writing preferred by Hans was top-down exposition: starting from the general, coming down to the more concrete. Yet, hidden under the façade of a polished and sometimes quite abstract exposition, there usually was a detailed knowledge of explicit and representative examples. Many of the notebooks he left are filled with intricate calculations, which he performed with great precision and unflagging concentration. Not surprisingly, he greeted the advent of formula manipulation programs like *Mathematica* with great enthusiasm. Furthermore, Hans put a high value on correct illustrations; in private, he could express annoyance about misleading or ugly pictures. In the days of the programming language Pascal and matrix printers, he spent a substantial amount of time in order to put a dot exactly at the

position he wanted: one of his favorite techniques for creating complicated illustrations was by printing just a huge amount of dots.

In addition to his patience and powers of concentration, he was capable of grasping the essence of a problem and its solution with lightning speed. When this happened during someone's lecture, he usually mentioned this not critically, but kindly and supportively.
While Hans had clearly exerted a substantial influence on mathematics through his own research and that of his many PhD students, the books written by him alone or jointly traverse a wide spectrum of mathematical exposition, both in topic or level of sophistication. But in this case, again, there is a common characteristic: every result, how hackneyed it may be, had to be fully understood and explained in its proper context. In addition to this, when writing, he insisted that the original works of the masters be studied. Frequently he expressed his admiration for the depth of their treatment, but he could also be quite upset about incomplete proofs that had survived decades of careless inspection. The last project that he was involved in exemplifies this: in joint work with Nalini Joshi reliable proofs are provided of old but also many new results concerning Painlevé functions.



Hans Duistermaat (right) and Alan Weinstein at a thesis defense in Utrecht in July 2009

Teaching and administration

As a teacher, Hans was quite aware that not every student was as gifted as he. Despite the fact that he could ignore all restrictions of time and demanded serious work from the students, he was very popular among them. Repeatedly he gave non-scheduled courses on their request. He was an honorary member of A-Eskwadraat, the Utrecht Science Students Society. He shared this honor with Nobel laureate G. 't Hooft and with J.C. Terlouw, a nuclear physicist who pursued a successful career in Dutch politics.

As an administrator, however, he was less successful. Although he served our institute, the mathematical community, and the Royal Netherlands Academy of Arts and Sciences in many different qualities, he was at his best with concrete issues that could be solved rationally, not with situations that required intricate political maneuvering. For instance, he was very actively involved with the Scientific Programme Indonesia–Netherlands, which was an initiative of the Academy, aimed at the selection and training of new researchers, the improvement of the supervising infrastructure at Indonesian institutes, and the conduct of joint research activities. In addition, the task of refereeing manuscripts was taken very seriously by Hans: many authors greatly benefited from his long e-mails. He was a member of a substantial number of selection committees, devoting a lot of energy to evaluating the candidates' achievements and potential.

Honored by the Royal Academy

In 2004, Hans was honored with a special professorship at Utrecht University endowed by the Royal Netherlands Academy of Arts and Sciences. This position allowed him to exclusively focus on his research, without being distracted by administrative obligations. The five years that followed were a happy period in which his mathematics blossomed. Hans demonstrated by the breadth and depth of his accomplishments that his chair was aptly named 'pure and applied mathematics'.

Persistence, power and success

His mood was almost invariably one of equanimity; even in difficult situations, he always tended to look for positive aspects. Immense concentration on a topic of momentary interest was natural for him. In fact, at several occasions he confessed he had a 'one-track mind', which made it necessary to mentally exclude disturbances. At times, however, this trait of character could be infuriating for his colleagues.

Very remarkably, Hans had no personal vanity, neither in human nor in professional relations. About his own work he once expressed to consider himself lucky for having become well-known for results he considered to be relatively simple. Most of his more difficult work, which had been far more difficult to achieve, had not received similar recognition. Honors did not mean much to Hans, although he was at first surprised and then gratified by them. He gave himself without any reservation to his friends and colleagues, always illuminating whatever was under discussion with characteristic insights based on his wide knowledge of mathematical and other topics.

In mathematics, Hans's life was a search for exhaustive solutions to important problems. This quest he pursued with impressive single-mindedness, persistence, power and success. We know this is a very sketchy attempt to bring him to life. In our minds, however, he is very vivid, one of the most striking among the mathematicians we have met. We deeply mourn his loss; yet we can take comfort in memories of many years of true and inspiring friendship.