

In de verdediging

| In defence



Multi-scale Riemann-Finsler geometry

Laura Astola

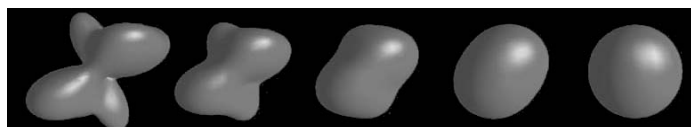
The cover of Laura Astola's thesis shows the everyday life that helped to inspire her research on medical image analysis to study the brain white matter. It is an interpretation of Astola's work by her 15-year-old daughter Mandi. In reality, her mathematical work has applications to Diffusion Tensor Imaging and to High Angular Resolution Diffusion Imaging. These rather novel, noninvasive medical imaging methods are used to discover the anatomical structure of inhomogeneous tissue such as brain white matter or muscle.

Diffusion tensor imaging

On January 27, 2010, Laura Astola successfully defended her Ph.D. thesis *Multi-scale Riemann-Finsler geometry, applications to diffusion tensor imaging and high angular resolution diffusion imaging*, written under supervision of Luc Florack. Her research mainly concerns some basic methods in differential geometry, namely in Riemann- and Finsler-geometry. These may be used in the development of robust ways to reveal the architecture of the neural fibers in brain white matter.

White matter is composed of bundles of neural axons, which connect various grey matter areas (the locations of nerve cell bodies) of the brain to each other, and carry nerve impulses between neurons. Astola was interested in the structure of this tissue, and specifically in its directions of elongation and the connectivity of the subregions in the tissue. She aimed to recover this information from noisy, diffusion weighted magnetic resonance images (MRI) of the brain. Such images roughly measure the displacements of water molecules (undergoing random thermal motion) in several spatial directions. The water displacements differ from place to place according to the structure of the surrounding fibrous tissue of the axons.

In standard diffusion tensor imaging (DTI), the data is described by a second order positive tensor. More specifically, the tensor represents a level set of water molecules, and the simplest model to describe an anisotropic diffusion profile that is symmetric with respect to the origin, is a second order tensor. The level sets are level sets of diffusion time; particles that have diffused for an equal time interval. This time interval is typically around 30 milliseconds, during which the average displacements of the molecules in the white matter depend on the surrounding tissue and therefore also on the spatial direction. An illustration that is often used, is to compare the motion of water molecules in a glass of water with those in a glass of water filled with a



A higher order tensor level set evolves under Laplace-Beltrami regularization; this can be useful in noise-reduction.

Pas gepromoveerden brengen hun werk onder de aandacht.

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These curves that model the real neural fibers were found using streamline tracking and Finsler-metric. This allows a voxel to contain also fiber bundles with different orientations, which was not possible in the standard DTI streamline method.

bunch of spaghetti. Clearly the elongated structures affect the movement of molecules, favouring migration along the spaghetti and hindering the motion across them.

Riemann and Finsler geometry

Being a mathematician, Astola of course focused on the mathematical side of the imaging methods. The standard modelling with a second order tensor naturally leads to a Riemannian framework. Hence local properties of a diffusion tensor image, such as curvatures, and the behaviour of geodesics, can be studied with Riemann geometry.

If the average displacements are measured only in, say, 6-30 spatial directions, the angular resolution is rather low. Measuring in 80-130 directions is still in the range that is comfortable for the patient, but gives a much higher angular resolution. Such a large amount of spherical data can be modelled more accurately using higher order tensors (homogeneous polynomials on the sphere). But such models no longer allow a Riemannian approach. By making the higher order tensors properly homogeneous and convex, they may be seen as defining Finsler norms, so that Finsler geometry provides the natural geometric generalisation. This way, Astola obtained a consistently generalised framework to analyse diffusion weighted images, that exploits the well-established and rich set of tools from differential geometry. According to her, the derivation of a systematic way to apply Finsler geometry using the tensor notation is the most important result in her thesis.

The geometric viewpoint indeed proved fruitful. By attaching geometric meaning to the physical properties represented by data, measures and algorithms to extract information from images could be de-

rived. This resulted, among others, in a novel, yet simple algorithm for tracking neuronal fibers.

Happy moments with pencil and paper

On the one hand, Astola's research was very applied. The applications in medical image analysis played an important role, and methods she developed were tested and demonstrated on both analytic and real High Angular Resolution Diffusion Imaging (HARDI) data. She joined a couple of medically inclined conferences and also visited a hospital, to get an idea of the very many practical challenges. The main challenges she encountered were to deal with the huge amount of medical data and in circumventing the problems due to the discrepancy between required and available memory capacity.

However, in her daily life she only had rather loose ties with the medical community, via colleagues at the biomedical engineering department. She herself is first and foremost an applied mathematician, who particularly liked the hours spent doing calculations by pencil and paper and who produced quite a pile of full notebooks in the past four years. Other happy moments were the moments of succeeding in programming new procedures.

A Finnish woman in the Netherlands

At the TU/e, Astola was member of a very active research group. They had at least two regular meetings per week, one with a group of people working on a similar subject and a weekly seminar combined with a coffee break for the whole research group/laboratory. Astola really enjoyed being a student in the Netherlands. One big difference she observed between this country and Finland, her home-country, is that the Dutch have many more social occasions that are also genuinely valued. She felt that it is, much more than in Finland, essential to be an active member of the community. And even though her life differed a lot from that of most Ph.D. students – with three children she had to go home early on two to three days per week and continue her work at night – she managed to really participate in the group's activities and to finish her research within four years. So it is not surprising that she chose to continue her academic career as a post-doc at the Scientific Computing group of the Center of Analysis, Scientific Computing and Applications (CASA) at the TU/e. ←



Image courtesy Koninklijke Philips Electronics N.V.

Diffusion weighted image acquisition is done with an MRI-scanner.