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History

Ancient and modern secrets of Isfahan

The art of medieval Islamic symmetry is well known throughout the world, albeit for a large part only from pictures. Recently, students from Leiden and Utrecht had the opportunity to see some extraordinary tilings through their own eyes on a study trip to Isfahan in Iran, under the guidance of Jan Hogendijk. As a result of this trip, a lasting contact was formed between the students here and those in Iran. Jan Hogendijk is professor in the history of mathematics in Utrecht and Leiden, and is a specialist on islamic mathematics.

On 27 December 2007, 39 mathematics and physics students from the Utrecht and Leiden Universities went with me on a study visit to Iran. The program had to include an extensive tour of Isfahan with its many spectacular Islamic tilings. Most of the tilings are based on pentagons and decagons but some involve heptagons and nonagons and so they cannot be constructed, at least not exactly, by ruler and compass.

An 11th century example is the tiling in figure 1, which is high on a wall of the North Cupola of the Friday Mosque. The tiling is also drawn in a medieval Persian manuscript in Paris. To illustrate the relationship, the drawing in the manuscript and three mirror images are superimposed on the tiling. The tiling arises from the tessellation of the plane by means of equilateral hexagons of two types (see the continuous lines in figure 2): type A with angles $4\alpha, 5\alpha, 5\alpha, 4\alpha, 5\alpha$ and 5α and type B with angles $4\alpha, 4\alpha, 6\alpha, 4\alpha, 4\alpha$ and 6α, where $\alpha = \frac{1}{7} \times 180^o$. The 'stars' inscribed in A and B have angular points at the midpoints of the sides of the hexagons and their angles at these points are $3\alpha, 2\alpha, 3\alpha, 3\alpha, 2\alpha, 3\alpha$. The figures in bold broken lines in figure 2 are regular heptagons. When the North Cupola was constructed (in 1080 A.D.), the famous mathematician and poet Omar Khayyam lived in Isfahan, and some of his work is known to be related to other diagrams in the Persian manuscript. He may have invented this heptagonal tiling as well.

A more modern secret in Isfahan is its *House of Mathematics*, (www.mathhouse.org), which encourages mathematics awareness among highschool students. Teams of volunteers (consisting of highschool teachers and university students) work together with highschool students in groundbreaking educational projects. The circumstances are sometimes difficult but this only seems to make the staff more enthusiastic and more inventive. Dutch mathematics educators can learn a lot in Iran and formal cooperation agreements have been made between the House of Mathematics, the Freudenthal Institute at the University of Utrecht and Fontys teacher training college in Eindhoven. Our group of students was received warmly by the staff, who organized most of our scientific program in Iran.

Travelling to Iran is risky because of the danger of losing one's heart. Some students, who are showing the symptoms, are planning to organize a summer school on Islamic tilings in Isfahan in 2009, of course in cooperation with the House of Mathematics.   ⬅······
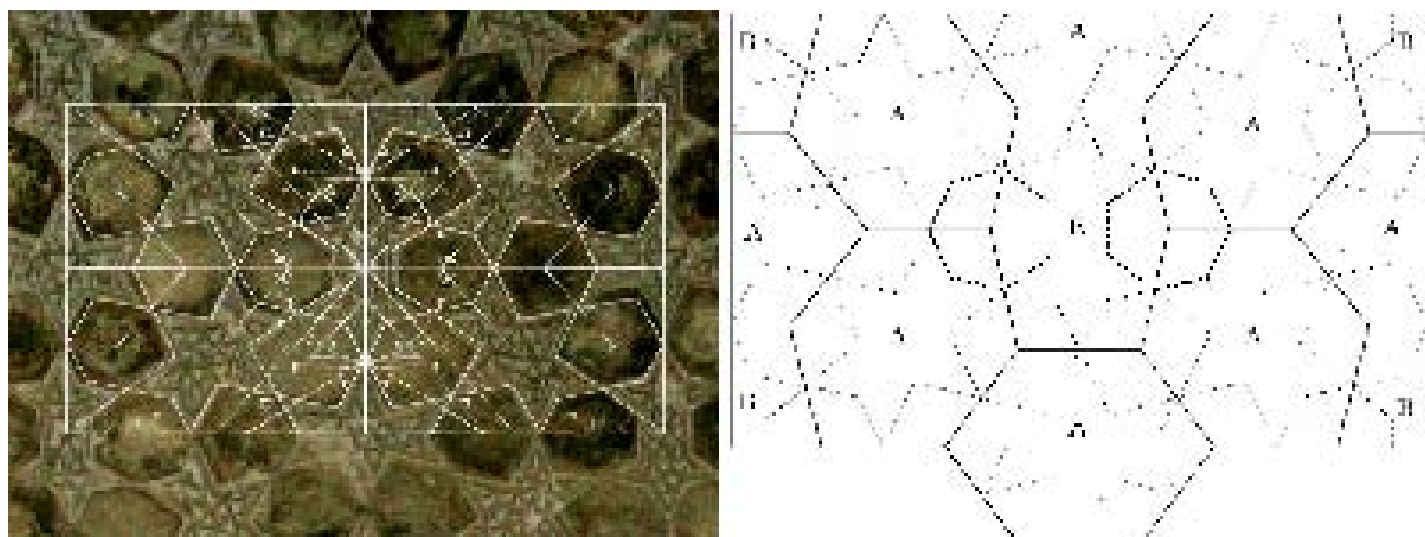

Figure 1   Left figure: Tiling in the North Cupola of the Friday Mosque. In white lines: drawing in ms. Paris, Bibliothèque Nationale, Persian Manuscript Ancien Fonds 169, f. 192a, with mirror images. Right figure: Drawing of the tiling. The left figure was produced by Mr Tom Goris, Eindhoven.