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Laudatio

Laudatio for Michael Aizenman

Eens per drie jaar reikt het Wiskundig Genootschap in opdracht van de Koninklijke Nederlandse Academie van Wetenschappen de Brouwermedaille uit aan een internationaal toonaangevend onderzoeker. Hij wordt uitgenodigd om een voordracht te geven op het Nederlands Mathematisch Congres, waar na afloop de laureaat de Brouwermedaille wordt uitgereikt. In 2002 werd de medaille toegekend aan Michael Aizenman voor zijn bijdrage aan de mathematische fysica. Op het Nederlands Mathematisch Congres in Eindhoven op 4 april 2002 sprak Frank den Hollander de laudatio uit.

The 2002 Brouwer Medal, in the field of mathematical physics, is awarded to Michael Aizenman of Princeton University. Michael is an outstanding mathematical physicist, who is admired and loved by his colleagues for his many beautiful contributions to the field and for his warm personality. In this laudatio, we give a brief overview of his career and of his scientific contributions.

Michael studied at Hebrew University in Jerusalem. In 1975 he obtained his PhD at Yeshiva University in New York, under the supervision of Joel Lebowitz. From 1975 to 1977 he was a postdoc at Princeton University, with Elliott Lieb. Both Lebowitz and Lieb are stars of mathematical physics, so Michael could hardly have done better in the choice of his teachers! From 1977 to 1982 Michael was assistant professor at Princeton University, from 1982 to 1987 associate and full professor at Rutgers University in New Brunswick, and from 1987 to 1990 full professor at Courant Institute in New York. Since 1990 he is professor of Mathematics and Physics at Princeton University.

Michael is the author of seventy-five research papers in journals of mathematics, physics and mathematical physics. He has collaborated with many co-authors on a broad range of topics. Much of his work is inspired by probability theory and statistical physics, both classical and quantum. In his papers he typically 'rides several horses at the same time', in the sense that cross-fertilization between different areas in physics and mathematics is at the very heart of most of his work.

Among the many honors that Michael has received we mention only three: in 1990 he was awarded the Norbert Wiener Award of the American Mathematical Society; since 1997 he is a member of the National Academy of Sciences of the USA; since 2001 he is Editor-in-Chief of Communications in Mathematical Physics, the leading international journal in mathematical physics.

In his work Michael has solved a number of very hard open problems central to statistical mechanics and field theory. He has repeatedly introduced new, powerful and elegant ideas, concepts and techniques that paved the way for others. Most of his papers are true eye-openers, and contain a high density of truly new ideas. As a mathematical physicist *pur sang* he often opens up new panoramas by combining ideas and techniques from different parts of physics and mathematics.

The two-dimensional Ising model has no interfaces (1980)

The Ising model is the paradigmatic model in statistical physics for the study of phase transitions. It describes a system of two-valued random variables, living on a lattice of a certain dimension. These random variables can be interpreted either as magnetic spins ('up' or 'down') or as particles ('occupied' or 'empty'). Their finite-volume conditional distributions (i.e., the probabilities of events inside a finite volume given the state outside) are prescribed by a nearest-neighbor interaction that tends to 'align spins' or 'glue together particles' and that contains the temperature as a parameter. At low temperature and in two or more dimensions, there exists more than one infinite-volume probability measure having the prescribed finite-volume conditional probabilities. These probability measures are interpreted as the different phases of the system. There is a so-called 'plus-phase', where the majority of spins is up (positive magnetization) or the density of particles is high (liquid), and a 'minus-phase', where the majority of spins is down (negative magnetization) or the density of particles is low (gas). This phenomenon is referred to as a phase transition. The temperature separating the more-phase region from the one-phase region is the 'critical temperature'. Aizenman's result can be interpreted as the statement that below the critical temperature two-dimensional 'crystals' of one phase inside the other phase have no facets. Various other researchers had obtained partial results before him. Aizenman proved the definitive result. R. Dobrushin had earlier shown that in three or more dimensions such facets do exist.

Destructive field theory in high dimensions (1981–1982)

Simple models of quantum field theory can be constructed as mathematically well-defined objects by analytic continuation in imaginary time of statistical-physics-like models, at or near their critical temperature and subject to a scaling limit procedure. Aizenman proved that, in dimension at least four, for a class of scalar field theories, this construction cannot describe systems with interacting particles. Namely, the limiting quantum field theory is trivial in the sense that it is Gaussian, describing only non-interacting particles. Again, although there were partial results before, the deciding step was provided by Aizenman.

Uniqueness of critical points for Ising and percolation models (with D. Barsky and R. Fernández, 1985–1987)
In percolation models, each site (or bond) in a lattice network is empty or occupied in some random manner, governed by a parameter that controls the densities of occupied sites (or bonds). One considers the question whether far apart sites (or bonds) are connected by an occupied cluster and what the geometry of such clusters is. Again, there is a 'critical density' separating a region where clusters are finite from a region where there is an infinite cluster running through the network. The uniqueness result showed that various a priori different notions of critical density coincide. This work widely extended the famous result for two-dimensional independent bond percolation due to H. Kesten (1981 Brouwer Medal in the field of probability theory) and solved a problem that for many years was open. A similar result was proved for the Ising model and various a priori different notions of critical temperature.

Uniqueness of infinite percolation clusters (with H. Kesten and C. Newman, 1987)
This result, which applies to independent percolation on regular lattices with a certain growth restriction, states that no two infinite percolation clusters can coexist, i.e., above the critical density there is one and only one infinite cluster. The proof was afterwards beautifully extended and simplified by R. Burton and M. Keane.

Revival of the Fortuin-Kasteleyn representation (with J. Chayes, L. Chayes and C. Newman, 1986–1988)
The random cluster model, introduced in the late 1960's by Dutch physicists C. Fortuin and P. Kasteleyn, relates in a precise manner a correlated percolation model to various magnetic models, including the Ising model and its generalization to random variables taking more than two values (called the Potts model). The relation is achieved in such a way that magnetic correlations are mapped to probabilities of connections, and vice versa. This relation was used to study phase transitions in

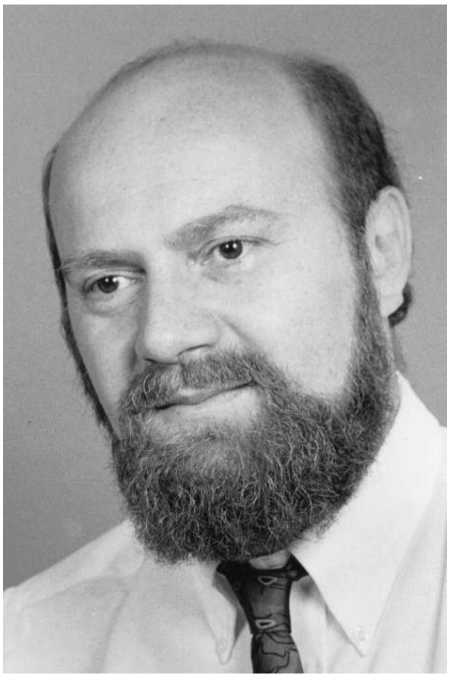

Photo: Pryde Brown

long-range Ising, Potts and percolation models. Since this revival, a plethora of applications has come up, including applications in numerical simulations.

The two-dimensional random field Ising model has no phase transition (with J. Wehr, 1990–1991)
If one considers the Ising model in a weak symmetric random external magnetic field, then the minimum dimension in which coexistence of different phases can occur becomes three instead of two. This result was predicted in the physics literature on the basis of a non-rigorous argument due to Y. Imry and S.-k. Ma, but it took many years to be mathematically confirmed.

Fractional moment estimates for the establishment of localization in disordered quantum systems (with S. Molchanov, 1993)
The problem of localization — "Why are certain materials insulating?" — was analysed in physics, among others by Nobel Prize winners N. Mott and P. Anderson. Their ideas — "It is due to impurities in these materials and these impurities can be modelled by introducing a random term in the appropriate Schrödinger equations!" — found mathematical confirmation in the KAM-type analysis of J. Fröhlich and T. Spencer. KAM-theory — after Kolmogorov, Arnol'd and Moser — is a kind of perturbation theory for dynamical systems that works even if the perturbation series diverges for a dense set of parameter values. For disordered quantum systems, this dense set is the set of eigenvalues of the random Schrödinger operator, describing the energy levels of the system. Localization is modelled by the existence of a complete set of eigenfunctions that decay exponentially fast in space. Localization is expected to occur for all energies in the case of sufficiently strong disorder, and for a restricted range of energies in the case of weak disorder (in three or more dimensions). The work of Fröhlich and Spencer confirmed part of this picture. Aizenman and Molchanov gave a different, very elegant and much simplified proof of localization, based on estimating the expectation of fractional moments of resolvents of random operators with a dense point spectrum.

Conformal invariance in percolation, existence and properties of scaling limits (1995–1998)
If one takes the scaling limit of a lattice theory in which the lattice distance approaches zero, then one obtains a theory that lives in the continuum. For many two-dimensional statistical-physics-like models 'at criticality', the limit is predicted to be non-trivial and to be invariant under conformal transformations. The existence of such limits and their properties are mathematically puzzling. Aizenman has provided various important ideas, which have developed further in the hands of other researchers, in particular, G. Lawler, O. Schramm, S. Smirnov and W. Werner, leading to great leaps forward in our understanding of critical phenomena in two dimensions. For these developments we refer to Aizenman's Brouwer Lecture.

The above summary gives an impression (not a full one) of the wide scope and broad interests of Aizenman's research. His work has been influential both among mathematicians and among physicists, which makes him a true representative of the field of mathematical physics. He has a deep intuition as well as an impressive technical mastery. He enjoys scientific interactions, whether in the form of collaborations, discussions or otherwise.

Based on these considerations, the selection committee for the 2002 Brouwer Medal, consisting of Robbert Dijkgraaf, Aernout van Enter, Frank den Hollander (chair) and Floris Takens, unanimously decided to honor Michael Aizenman and congratulates him wholeheartedly with the award.