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## Een eeuw wiskunde en werkelijkheid

# Do mathematical models tell the truth? II

**Dit is het tweede deel van een artikel dat Gerke Nieuwland voor het Nieuw Archief schreef. Het eerste deel begon met een bezoek aan het planetarium van Eise Eisinga in Franeker: een stoffelijk model van het zonnestelsel gebaseerd op de hemelmechanica van zijn tijd. Nieuwland stelde zich naar aanleiding daarvan de vraag: "Kun je over de waarheid van zo'n model spreken?" Op die vraag lijken de traditionele filosofieën van de wiskunde geen bevredigend antwoord te hebben. In dit deel zoekt Nieuwland aansluiting bij de filosofen Quine en Davidson, hetgeen tot een andere invalshoek leidt. Zet men bijvoorbeeld naast elkaar een wiskundige theorie, een daarop gebaseerd stuk genetica en het daaruit voortvloeiend ontwerp van een proefopstelling in het laboratorium, dan is er sprake van drie verschillende talen. De centrale vraag wordt: Hoe wordt de ene taal in de andere taal geinterpreteerd? De vraag naar de waarheid wordt secondair, al ontleent uiteindelijk alle wetenschap haar betekenis en waarheid geheel aan de gewone wereld. Het werk van Davidson biedt zo een uitgangspunt voor een filosofie van de wiskunde in de angelsaksische traditie die tot een interessante discussie kan voeren met sociaal-constructivistische opvattingen en de continentale interpretatieve traditie.**

In a paper published in 1984[1], the American philosopher of science Arthur Fine discusses the failure of the realist wing of the philosophical spectrum to deliver clinching arguments in a longstanding discussion. The stand he proposes in the realism-nonrealism debate has been very influential ever since, but is not of present concern. What is of interest here is his introductory discussion, where he stipulates what arguments can be considered *convincing* in the philosophy of science. He refers to the discussions on the foundations of mathematics in the beginning of the twentieth century, in particular the consistency problem posed by the introduction of Cantor's set theory. A set-theoretic consistency proof being obviously to no avail, Hilbert developed a finite-constructivist metatheory for mathematics. Gödel showed in 1931 the aim to be unattainable — but Hilbert's idea, says Fine, was correct, even though it proved to be unworkable. As he concludes:

*"Metatheoretic arguments must satisfy more stringent requirements than those placed on the arguments used by the theory in question, for otherwise the significance of reasoning about the theory is simply moot."*

In other words: if it is *normative* insights we are after, science journalism and philosophy of science are better kept apart.

So let us return to the question that ended part I of this paper: why do we need a metatheory for science — with or without a special chapter for mathematics? Various answers to this question given in modern philosophy take their starting point in the success of science, perceived as extraordinary in comparison with other historical projects. Such success is felt to be at least in need of explanation and, possibly, emulation in other areas of knowledge. For both purposes authority and expertise are sometimes claimed for the philosopher, to delineate an area of reliability, objectivity or certainty — depending on her philosophical views — as the special province of science.

In part I, we saw Hertz settle for reliability: the progress of science depends on our models predicting reliably the outcome of our empirical observations — and this is all there is to it. Van Fraassen's constructive empiricism is a modern exponent of this view.[2] Van Fraassen is not in any need of metaphysical explanation: the success of science is for him a social phenomenon. However, he has not fully answered the criticism of also inheriting the Hertzian theory-observation distinction. This distinction was, of course, implicit in the phenomenological theories Hertz was concerned with, but became questionable shortly after with the rise of modern physics.

The triumphs of science certainly do motivate realist philosophers: in the words of Putnam[3]:

*"The positive argument for realism is that it is the only philosophy that doesn't make the success of science a miracle."*

Realism demands certainty, it does not settle for less than the truth. We saw Benacerraf give an updated version of the traditional, Platonic idea of truth in the sense of the existence, *out there*, of an ideal original of our mathematical constructions, *independent of human thought or experience*. Fine's objection expresses the reservations held in modern philosophical discourse against Gödel's arguments for Platonism, solely from the awareness of a mathematical experience. However, Maddy's conclusion as to the failure of causal correspondence theories leaves us with little else. Strong intuitions, but not more, also remain the basis for the Platonic metatheory justifying the use of mathematical models across the board of the various sciences — the guarantee that they do in fact represent *the actual way things really are*.[4] Few philosophers are nowadays prepared to foot this bill — and even fewer scientists are at all interested.

To define an area of objectivity without realist commitment is to draw the demarcation line between science and other forms of knowledge on epistemological, e.g. methodological, grounds. This need was strongly felt by the logical positivists and gave rise to Popper's falsificationism. More recently, Quine also wants to separate 'first class science' from 'subjects better dealt with in the literary essay or the sermon'. He opts for *naturalism*, the view that philosophical questions cannot legitimately be asked beyond science: there is no *first philosophy*. But then he has to define what science is about. He proposes to combine *physicalism* — i.e. all scientific concepts are to be be gauged by the concepts of fundamental physics — with the use of a logically purified scientific language, a *canonical notation*.[5] In particular the latter condition prohibits the use of modal logic and probabilistic language. A consequence is that quantum mechanics, and thus most of modern physics, is on the wrong side of the fence. Not surprisingly, he found among scientists many admirers, but few willing to take to this prescription.

And so, if philosophical proposals have turned out less than satisfactory during more than a century, one may well ask what is wrong with the common language or dictionary definition:

*"Science is any system of knowledge that is concerned with the physical world and its phenomena and that entails unbiased observation and systematic experimentation"*

says *Encyclopedia Britannica*. So far so good. It enters dangerous territory where it proceeds to talk of 'covering general truths' and the 'operation of fundamental laws'.[6] But we could take our cue from Quine and suppose that what is meant here is just the logical and mathematical substructure that we need to fill in to make of science a *system of knowledge*. This also suggests an approach to the claims of mathematical realism from a quite different philosophical angle.

**Object, Language and Interpretation**

In part I of this paper we saw mathematical realism claim support from Quine's indispensability argument: if set theory is indispensable for science, sets must be among the ultimate furniture of the universe. However, the specific concepts of 'pure mathematics' are supposedly developed independent of any scientific setting. Quine pictures these as logically 'filling in and rounding out' the conceptual gaps presumably left by the mathematical sciences. Indeed, in the naturalist view the truth of science entails the truth of mathematics — whatever the naturalist idea of truth. The standard, Platonic view usually has it the other way round. I cannot make much of either of these metaphysical theses, but there are two points in favor of Quine's preferred order. First, a strong case can be made for it from the history of mathematics; after all, until around 1900 mathematics was considered the natural history of space and number. The second, more subtle, is that it is part of a folklore, cherished among mathematicians, to arrange order in mathematical thinking as a *spiral*: experience of the world, intuition of structure, finding out truth — and then sensing beauty suffuse experience. Beauty here is not to be taken as the modern, romantic, notion of *what pleases the senses*, but as the much older and firmer philosophical idea of *what accompanies the experience of truth*. This touches on the Pythagoraic tradition, where traditionally mathematics and music, in essence one, provided the key to the world. So let us offer an analogy taken from musical history in support of Quine's point of view.

Johann Sebastian Bach is not known as a innovator, he took musical structure and form as laid down by his predecessors and contemporaries largely for granted. What he did is aptly described as 'filling in and rounding out' the musical conventions of his times. Doing so, he took the challenge of formal exigencies as the path to the heights of invention. Or so we moderns think — at the time of his death his contemporaries were no longer interested.

From time immemorial, mankind has learnt to deploy mathematics in order to cope with the empirical world, finding helpful notions such as quantity, measure, pattern and functional dependence. For many and obvious reasons it proves expedient to order this experience into a *system* of knowledge, invoking notions of coherence, analogy, completion and logic. Doing so, it appears that the margins of interpretation, of whatever it is of our experience that can be organized into mathematical structure, are flexible. Such is the underdetermination of theory by the data. Even the field of mathematical experience can be parcelled out with some latitude, in the course of history adapting to our increasing sensitivity of the world. As in Bach's music, such strict and narrow, but not entirely rigid, boundaries can challenge the imagination to soaring flights of fancy. This gives us pure and applied mathematics, in their continuing historical interplay. Beauty comes in when from time to time one succeeds to organize, within the system of knowledge, a particularly felicitous subsystem. A boon, perhaps in terms of unexpected efficiency, simplicity or applicability, but always with the unmistakable sense of *truth*, of *hitting the nail right on the head*. Indeed, as some would say, with the inevitability of the discourse of a Bach fugue. It is this magical moment that contains the beauty of mathematics.

The last paragraph may have defended Quine's position from the point of view that mathematical experience of the world precedes the formulation of theory, but for Quinians the introduction of words like beauty and magic into the vocabulary is redundant, if not obnoxious. Is not *magic* the very antithesis of *science*? Perhaps so — but let us go on for a moment.

In a conversation with a logician I once asked whether in his opinion a majority of mathematicians shared *the standard view* — were Platonists. "Yes," he said, "but I do not think the commitment is very deep." So, before either accepting or rejecting the traditional dogma, let us join the mathematicians and listen to what they say. In their home language, mathematicians often talk as if they can manipulate mathematical entities, or observe while walking around them. In public, they also like to advertise mathematics as one of the fine arts — this is obviously very bad public relations, but the fact is there. Now, the idea of beauty is not at all specific for the practice of mathematics, it happily accompanies every instance of successful creative endeavor, be it growing particularly lush carrots, constructing bridges, or writing string quartets.[7] Objectual language, however, is an unmistakable defining characteristic of mathematical Platonism. Therefore, let us return to Benacerraf's examples (a) and (b) in part I. Mathematician's certainly talk that way, but do they really believe that perfect numbers must exist as cities do?

Maddy says such belief is prior to embarkment on Benacerraf's project. Belief is one's trust in a proposition, but it comes in degrees. So let us take it as the disposition to defend the thesis *Cities exist, so do perfect numbers*, in relevant company. The logician's comment can then be taken in the sense that a majority of mathematicians would undertake to defend the proposition in ordinary language, say, a dinner-table conversation (this is a place where much important business is transacted). However, they would refuse the task in the philosopher's construction called 'ordinary language', say, as in Benacerraf's sentences that must display identical 'logico-grammatical form'. Mathematicians do not naturally speak this language, nor are they sufficiently interested to learn it. Finally, in the professional mathematical language the case of a defense does not arise, just because the thesis is not part of the lingo. Now how does a mathematician, in conversation, defend the thesis? She first describes the fact already noted, that the geometer bends, cuts up, glues and generally disfigures his manifolds; that the analytician lets flow, throws back, pushes forward, lifts up, is in the habit of knocking about his stock-in-trade; that the number theoretician can tell endearing stories about the very special characteristics of his particular pets. All this with appropriate gestures and body language. That is: mathematicians live in a world furnished with items that *present themselves as existing to them*. If the objection against the above thesis is raised that one can visit Benacerraf's three cities, but one cannot actually visit the three perfect numbers following 17, the mathematician's answer might well be *why not?* Of course, this poses the demand to *engage in* this universe.[8] It is at this point that the time-honored existential analogy of mathematics and music comes in, because without engagement music also remains an empty shell. So, if we can think of mathematics and music as made of the same stuff, let us substitute 'the Chaconne from J.S.Bach's second Partita for solo violin' for 'perfect numbers'. This legerdemain leaves us with the thesis 'Cities exist, so does Bach's Chaconne'. The switch immediately saddles the audience with an awkward burden of negation: one has either to admit in public never to have fallen under the spell of this cultural monument, or else to develop an impromptu philosophical theory of different modes of existence. The objection, that there is a difference in so far as the sound of the violin piece is physically different in every new performance, is countered by the observation that a *complete* protocol of a perfect number's material footprint — in terms of printed symbols, registration on brain cells, and so forth — becomes also different with every new theoretical interpretation of the integers. And so, with some luck, the point is carried, by exhaustion of the audience.

Now, what has all this to do with the mathematical models we were supposed to discuss? Mathematical entities can be modeled in other areas of mathematics, notably in set theory. If asked — again in ordinary language — why one does so, it is natural to say: to *reveal other aspects*. And if so, of what else than their existence? Today's ultimate ordinary language argument may well be that such models exist in virtual reality, as soon as someone takes the trouble to develop the appropriate computer language and graphics.

More often, of course, mathematical models are intended to represent aspects of empirical reality. In the Platonic everyday language, they exist 'here' as mathematical systems, picturing something 'out there', embedded in a material system. The situation takes on a new dimension, but does not appear as fundamentally different from the one described. In a famous study, Pickering has written the history of particle physics.[9] In the years 1964–1980, in which quarks developed from a convenient mathematical tool to organize certain clusters of experimental data, to items awarded by the physicists their own identity — although they never were experimentally isolated. What happened was that, while theoretical and experimental evidence came in, quarks gradually became recognized as a particle *in laboratory practice* — they could be manipulated, visualized, tricked into behavior. The vocabulary change took place in no more than three years after 1972.

So why is there Platonic talk and why is it seemingly ineradicable? Why talk of pushing around mathematical entities that surely could better have been left to their Platonic immutable rest? Why this pretense of a confident and clear vision of mathematical systems as the inner workings of nature, while if challenged we know full well that in physicalistic terms this must be an illusion? And if this talk is not even part of the formal language of science, why has more than a century of philosophical analysis failed to wipe it out altogether?

The question where such talk comes from, of course, has since long been answered in the humanities and the social sciences. These take as their basic data the pictures, sound and narrative that human imagination continually produces to sort out the welter of experience. Some such — call a registration of any of them a *text* — are found particularly worthwhile or convincing, and are retained in the traditions of religion, music, the arts, literature, scholarship and science. But also the access to texts is only through experience, and so their transmission is mediated by an imaginative process: interpretation. The Continental philosophical traditions since 1920 have all made interpretation one of their keywords, by contrast the Anglo-American heirs of the tradition of scientific philosophy pronounced their anathema at precisely this point. So, for better or worse, the source of the Platonic talk we encountered must be identified as the interpretive element in science.

Why cannot we get rid of it? Of course, if the foregoing analysis is in the right direction, attempts at a total elimination would be not less than self-destructive, so let us rather settle for control. I size down the discussion to one, but crucial, aspect: control of mathematical language. In mathematical practice this control is effected through an interpretive tradition.

Tradition is what gives professional identity to a mathematician: the school in which she is raised, the people she relates with, the publications she reads, the language she speaks — what makes her an algebraic geometer, a probabilist. For the professional, this tradition places any relevant mathematical text immediately and ineluctably into an interpretive framework: a problem history; a network of relations with other work, in particular one's own; a judgment on perspectives and prospects. This framework is shared with colleagues, it can be communicated and discussed. Tradition stands for continuous development over a lifespan, and this certainly remains a defining characteristic in the present context. However, what catches the eye in the modern era is the extent and depth of its periodic upheavals. What is meant are no revolutionary paradigm changes, but the rearrangements within the same paradigm necessitated by the growth of amassed knowledge, expressed as it is in already very compact symbolic language. In the established subdisciplines of mathematics the selection of paths over the mountains of information that must be climbed to enter into new territories is a major part of the agenda. Every new generation must spend a considerable effort to find the concepts that most clearly and efficiently organize their field of knowledge, in view of what became recently known and in view of the perspective this opens. There is a considerable tension of this fact of mathematical language with what is usually advertised as the backbone of mathematical discourse: logical syntax. The disturbing fact is that if mathematical theory is elaborated in such detail that all logical steps are displayed, the resulting text is computer-readable, no longer readable. This effectively kills all interpretive processes as were just described: they reduce at most to an intuitive interpretation of what is accepted as axiomatic. By contrast, the mathematical texts of everyday life are abbreviations, and what is left out is a matter of convention. In mathematical practice these gaps in understanding are bridged by an educational tradition: there is an area of standard concepts, methods, tricks and theorems everybody is supposed to have ready. Even so, this practice makes the divisions between the subdisciplines nearly inpenetrable; there is an impression that the relative size of the territory common to all qualified practitioners may have become dangerously small. Be this as it may, among the experts of one subdiscipline and their students, the contents of a formal mathematical text is usually much too long to convey in spoken language. The medium then used is an informal mathematical language, for the outsider characterized by its strongly graphical, 'picturesque', elements. The contents of these pictures, again, is a matter of convention, of educational tradition and of both-sided imaginative creativity in the master-student relation. Their use is not surprising, because well-chosen pictures can very efficiently convey an enormous amount of information. This is the reason for the cropping up of geometrical imagery in the formal language of all mathematical disciplines. And it is, in a nutshell, the reason why in informal discourse we cannot get rid of Platonic language. There remains the question whether, nevertheless, we ought to keep trying.

This is, of course, a philosophical question, and in the last section I will try and follow the American philosopher Donald Davidson some steps in the direction of an answer. Let us here suppose we are in the possession of the following texts:

1. The article *On a Semi-Algebra of AB-operators*, by X, recently published in J. of Very Fundamental Math.
2. A transcript of an informal colloquium talk by X, on 1.
3. A rewrite of 1 in New Automath, that has been proof-checked by computer.
4. An article in Biology Tomorrow, by $Y_1, \ldots, Y_{47}$, containing the news that semi-algebras of AB-operators very accurately model the occurrence of faults in the copying of DNA-strings, with experimental evidence.
5. The actual experimental setup from 4, inputs DNA, outputs faulted copies.

Question: How must the relation of mind and world be defined, if the semi-algebra is to come out as *real*, or as may be, *operative*, or *structuring*? In the past various philosophical schools have argued for a metatheory that regarded contents as typified by exactly one of the above papers as foundational. Presumably, at the time, such studies answered important and relevant questions. The discussions we saw Benacerraf and Maddy engage in suggested that at the present time this seems no longer to be the case — either by lack of answers or of relevance of such essentialist discussions to today's problems. Still, what we can say is that here are five texts, each produced by an author-speaker possessing an expertise not shared by the philosopher. Each is presumably discussing the same subject, although using a different vocabulary. If, in the philosophical discourse of an entire century, we have failed to locate a privileged position — empiricist, logicist, realist, whatever — for the philosopher, there is also no reason to reject any of these texts out of hand. So what remains for the philosopher are questions of translation, then interpretation — and finally, perhaps, truth.

**The Truth of Models**

Every visitor to Eisinga's Planetarium can jump up a chair and tamper with the model — not that such action is encouraged by the management. But an *Archimedean position* is easily found: the point needed to lever the earth out of orbit. In the philosophical parlance the term has come to indicate the need for the philosopher to define a position with respect to her object, the world — of which she is obviously part both as a physical being and as a speaker. This is the subject of, arguably, Quine's greatest single contribution to philosophy, chapter 2 of *Word & Object.*[10] His metaphor is the position of the anthropologist, out in the field to study the linguistic performance of a native community. The field linguist's objective is a manual enabling sentence to sentence translation, itself to be produced by *radical translation*, i.e. without previous exposure to the native language. In the final analysis, the metaphor suggests, what philosophy is about is *translation* of alien language.

To this end, the linguist observes the response of the natives, in marks and noises, to changes in their environment, as are plausibly and publicly available. Such a stimulus is the advent of a small, furry, long-eared animal passing by at high speed through the jungle — on the way to everlasting philosophical fame. The linguist registers the sound 'gavagai' that Quine's scenario lets the members of the community emit, takes all available data into account and decides on translation as 'a rabbit!'. In this way, beginning by ostentation of everyday objects, the manual is to be interactively built up. So far, so good, according to the strictly behavioristic investigative protocols fashionable at the time. Philosophical worries begin with Quine putting a second translator in place, working independently to exactly the same protocol. She eventually comes up with an entirely different translation of this episode — 'an undetached rabbit part!' or 'the universe minus a rabbit!' — in an otherwise perfectly consistent manual. Quine concludes from this thought experiment to his doctrine of the *inscrutability of translation*. If only physicalist criteria are valid, there is no fact of the matter as to the difference of the manuals — all must be considered equally acceptable. And indeed, considerations of plausibility have no place in a strictly logical procedure that only accepts physical response to physical stimuli in evidence.

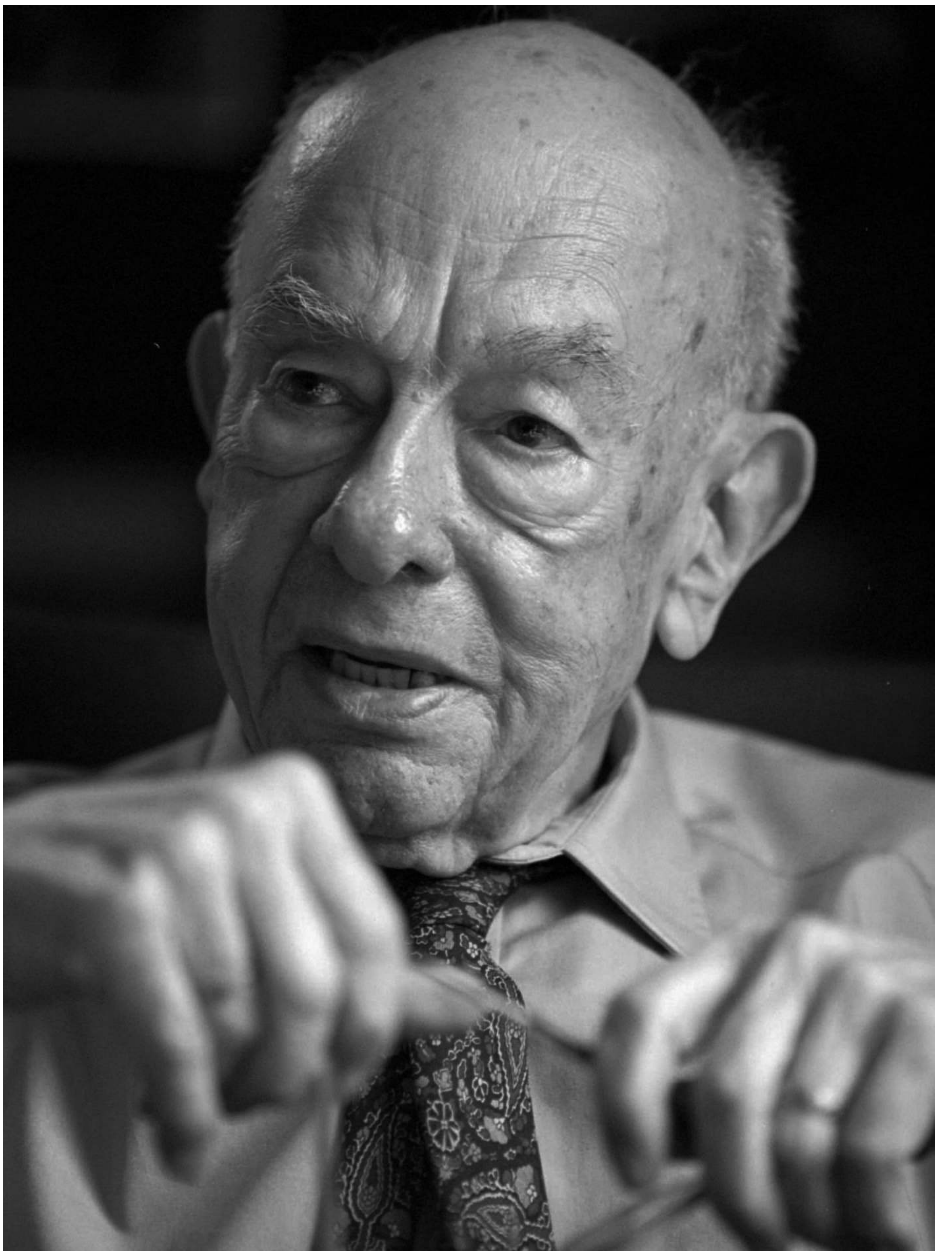
The consequences of this position were shattering, at least in the 1960 philosophical world. It meant a rejection — in a linguistic philosophy! — of the accepted notions of translation, sense, synonimy, meaning and reference, in so far these notions could not behavioristically be certified in specific cases. Under this ban fell most of the work in the semantics of natural language, and in modal logic, of the second half of the twentieth century. Philosophy, for Quine, elaborates scientific notions of logic, ontology, linguistics — it is 'continuous with science'. It may call on cognitive science to chart the broad area between the input of the empiricist 'irradiation of our nerve endings' and our linguistic output, in order to get a firmer tie of theory to evidence. But the traditional moral and educational concerns have no proper place in philosophy — not that they are unimportant, but such discussions simply belong elsewhere.

Quine's normative philosophical view on language entails that for 'first class science' only the use of a canonical language is acceptable. This should support the quantification and predication of first order logic — we saw Benacerraf already use this criterion of an underlying 'logico-grammatical form'. Moreover, it should be *extensional*.[11]

The 'truth of a mathematical model' in the sense of some notion of correspondence to 'reality' cannot be intelligibly discussed within Quine's philosophy. In model theory in mathematics and logic both object language and metalanguage are formalized structures, and truth can be defined through Tarski's satisfaction relation. But in Quine's view, in natural languages a semantics is only to be had in the form of a translation manual between languages. Consequently, the ontology, the 'ultimate furniture' of the universe spanned by the language, must be relative to such a theory. Maddy's shock at finding the ontology of the continuum theories of classical physics in conflict with quantum physics is lost on Quine — ontology is relative to a theory anyway, and even if quantum theory is the best we have today, its conceptual basis remains shaky, pending an extensional rewrite. Of course, everybody is equipped with a 'home language' that is a proper subset of an indicative metalanguage, in this case the translation manual reduces to the identity map. Then, the 'truth of science' means precisely that one has absorbed 'at face value' the tenets of science, as part of the metalanguage, in the home language. Presumably, this is what every civilized person ought to do; although, as Quine's jungle metaphor teaches us, *she may not be able to communicate her insights to a speaker of another language*.

It is this relativism, reaching right down into the experience of the everyday world, that motivated Donald Davidson to propose a major reform of what might well be, once granted its fundamental views, the most rigorously argued and rational philosophy of the twentieth century. Davidson, Quine's Ph.D.-student and lifelong friend, is the author of two volumes of essays[12], that made him another cornerstone of American philosophy. Quine's consistent empiricist and physicalist *a priori* lets him draw a skeptical demarcation line between science and the rest of human experience. Davidson considers this arbitrary; why refuse the philosopher the right to think of the mind rather than the brain — of music rather than sound? And if this means leaving the confines of materialism and empiricism, so be it.

Davidson's move is to put one other item on Quine's radical translator's agenda: interpretation. That is: if the translator records the natives' reaction on an event, it is also of his concern *whether what they say is true*. For those familiar with Tarski's theory, Davidson has some surprises up his sleeve: truth is, in Davidson's vocabulary, a primitive, undefined notion[13], and it is empirical. If our use of a sentence is effective, if it *copes* with the world, there is not much more to say of it than that it is true — and Davidson agrees with Dewey: *we all know what truth is*. And so




Willard Van Orman Quine (1908-2000)

the interpreter can only weigh, according to his own lights, what a native says against what together they observe in the world.

Davidson's solution is highly ingenious, deeply philosophical — and remains controversial. He develops a theory of truth[14] that generates Tarski-style T-sentences for all declarative sentences in the native language. Prototypically, given a sentence about the properties of snow, a T-sentence for it is:

"*'Snow is white' is true-in-English if and only if snow is white.*"

However, Davidson's T-sentences have on the left-hand side a canonical description of the sentence containing all semantically relevant syntactical structure, on the right-hand side a sentence that is true if the mentioned sentence is true. This modification makes the build-up of such a theory testable in an interactive process with the natives, when and where feasible letting them decide on the truth of T-sentences, by assertion or negation. The idea is of a stupendous technical difficulty — if only one thinks of the presupposition that the language is initially unknown to the interpreter. Moreover, the theory generating the truth-conditions of the left-hand side requires an extensionalization of the language; in this area important progress has been made by Davidson, but its problems are far from solved. So, opinions vary as to how far Davidson has advanced in his project.

However, his work lies at the root of what has become the foremost school in contemporary linguistics.[15]

The philosophical bone of contention lies in the fact that in the sentences as spoken by the natives, meaning and belief are inextricably mixed. In particular in the first stages of the translation process, it is often impossible to decide what is true, and what is opinion. To separate these is where interpretation must come in, and the interpretive process is ruled by Davidson's famous *principle of charity*. The basic idea is that if the translation would consistently yield a weird or awkward picture of the natives' perception of what goes on in the world, it is the translation that must be in doubt, not the common sense of the speakers. The principle of charity therefore states that, in the course of the interpretive process, the interpreter cannot but *initially* assume that the native attribution of sentential truth is the same as what would follow from the interpreter's own assumptions about (1) her causal relations with the world, and (2) the coherence of the patterns of her own rationally held beliefs. This principle should guide the interactive interpretive process,[16] at least until a satisfactory translation of the native rendering of the world of everyday experience has been obtained. The interpreter can, of course, dissent from the truth values attributed by the native speakers, and will in general do so in various instances in the later, less mundane stages of the interpretive process. However, the principle of charity also implies that the extent of his agreement remains to be taken as a measure for the dependability of his interpretation.

In the end a complete truth-conditional analysis of the native language is supposed to result. *Truth* in the language, then, is the collection of all its T-sentences, and *meaning* the network of relations between these truth-conditions. An almost-standard Quinian manual for translation into the interpreter's home language falls out as a by-product: the interpretive process cannot be expected to iron out all differences, but should reduce these to the level of footnotes to the main text.

Evidently, there is a very long way to go before we can apply a Davidsonian analysis to the study of translation, and truth, in something like the example concocted at the end of the last section[17] . However, whatever the cost, it seems worth the attempt, for several reasons.

As in every human enterprise, there is an interpretive side to science. In the philosophical history of the last century, the logico-positivist denial of its interest has led to historicist and constructivist interventions that sometimes, in the absence of a central philosophical perspective, emphasized again the sectarian attitude. Davidson's views, if only because they are *not* primarily offered as a philosophy of science, provide an at least interesting focus. The corrective empirical element in his theory of truth, considered idiosyncratic by philosophers for whom presumably the armchair remains the principal tool of their trade, has meanwhile shown conclusively its worth in the hands of the social constructivists, notably Pickering.[18]

The principle of charity offers a particular challenge for a philosophy of mathematics. The principle expresses Davidson's deeply rooted conviction of the fundamental interdependence of the knowledge of oneself, the other, and the world. This interdependence is not constructed as a metaphysical idea, but situated in the world of everyday experience, where ordinary people speak ordinary language — and where they can communicate and interact only in so far as there is a common basis of rationality, understanding, and trust. Benacerraf suggested that the truth of mathematics should be established before the tribunal of science; the prospects for this still do not look particularly good. The value of Davidson's perspective is the demand that meaning and truth of mathematics, as indeed of all science, should ultimately be shown in the common world where all of us transact.

I like to think that this is, in fact, what motivated Eise Eisinga long ago to build his model. One might call the attempt in the proposed direction an answer on what some philosophers ask for in the Babel of the post-post-modern era: a *second naivete*. Perhaps so — but first there is certainly a lot of work to be done.

**Notes and references**

1 A. Fine, *The Natural Ontological Attitude*, in: J. Leplin (ed.), *Scientific Realism*, Berkeley (1984), reprinted in: D. Papineau (ed.), *The Philosophy of Science*, Oxford (1996).

2 B.C. van Fraassen, *The Scientific Image*, Oxford (1980) and *Laws and Symmetry*, Oxford (1998).

3 H. Putnam, *Mathematics, Matter and Method*, Cambridge (MA) (1975).

4 This is the closing phrase of R. Klee's excellent book *Introduction to the Philosophy of Science*, Oxford (1997). Klee argues for realism from the 'overwhelming evidence' as presented by the modern biological sciences, in particular immunology. Like many realists, he seems to consider the quantum mechanical picture of the physical world too far out to be of philosophical import..

5 W.V.O. Quine, *Word & Object*, Cambridge (MA), 1960.

6 Cf. B.C. van Fraassen, *Laws and Symmetry*, ibid.

7 Opinions differ as to its origin: some think of it as an acquired habit in the evolutionary struggle for life, others as a moment of God's grace..

8 I must confess perfect numbers leave me perfectly cold, but one can easily image a person whose pride and joy they are. It is intriguing that it is still unknown whether there is a largest perfect number. And recently, there is a renewed interest in the subject in connection with cryptography..

9 A. Pickering, *Constructing Quarks*, Chicago, 1984.

10 The best introduction to Quine's work is L.E. Hahn and P.A. Schilpp (eds.), *The Philosophy of W.V. Quine*, The Library of Living Philosophers, vol. XVIII, 2nd ed., Chicago, 1998, in particular the two surveys in it by R.F. Gibson, Jr., and J. Woods, with Quine's comments..

11 As opposed to intensional or modal. Extensional language is *truth-functional*, that is: the truth value of a sentence is invariant under substitution of sentences with the same truth value, and of predicates and singular terms, with the same denotata and designata, respectively. Example: (1) John went *and* Mary came, is extensional, i.e. true if both component sentences are true. (2) John went *because* Mary came, is not. Extensional language allows a theory to be developed *more geometrico*..

12 *Essays on Actions and Events*, Oxford, 1980 and *Inquiries into Truth & Interpretation*, Oxford, 1984. The best introduction to Davidson are his 1989 Dewey lectures, *The Structure and Content of Truth*, J. of Phil., vol. 87, 1990, p.153–66.

13 He has written on *The Folly of Trying to Define Truth*, J. of Phil., vol. 93, no. 6, 1996.

14 More precisely: "We may think of a theory of truth for a language L simply as a sentence T containing a predicate $t$ such that T has as logical consequences all sentences of the form '$s$ is true if and only if $p$' with '$s$' replaced by a canonical description of a sentence of L, '$p$' replaced by that sentence (or its translation), and 'is true' replaced, if necessary, by $t$." in *Inquiries into Truth & Interpretation*, p. 66.

15 Cf. the evaluation in W.G. Lycan, *Philosophy of Language*, London, 1999, where also some more recent developments and prospects are discussed..

16 Including an analysis by Bayesian decision theory of the numerical values for probabilities and preferences awarded to the propositions contained in the T-sentences..

17 Except for (3). Davidson's truth-conditional analysis of a formal mathematical text should come out remarkably like De Bruijn's Automath. Cf. R.P. Nederpelt et al., *Selected Papers on Automath*, Amsterdam, 1994.

18 Ibid.