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Daniel Bernoulli and of mechanics in the

During the 18th century three main traditions in mechanics developed, somewhat in competition over reliability and generality: the approach stressing central forces and based upon Newton's principle of inverse-square attraction and the three laws; that seeking to give primacy to energy conservation, and its interchange with work; and an algebraic style, using principles such as those of least action, d'Alembert's, and virtual work. They played differing and often competing roles across the range of mechanics.

Daniel Bernoulli (1700–1782) lived through the bulk of these developments and made major contributions to several principles and topics. But his contributions have not been fully studied, and his place is hard to make precise; he seems to be somewhat overshadowed by his father and uncle, and also by contemporaries such as Euler and d'Alembert. What did he do, and is the appraisal fair or unjust?

1 Beginning and end

When Daniel Bernoulli was born in Groningen in 1700, the Leibnizian differential and integral calculus had been in print for less than 20 years, and its creator and his own father John (1667–1748) and uncle James (1654–1705) were among its leading practitioners; the first textbook, De l'analyse des infiniment petits, written by the Marquis de l'Hôpital with John's hand much upon the pen, had been published just four years earlier. Leibniz's approach to mechanics had been known for a decade, and was also beginning to gain attention, especially in Paris. Newton's very different account of mechanics had appeared as Principia mathematica in 1687, and gaining some discussion; his fluxional calculus was also known but still only in manuscript. A sense of professionalisation of science was growing, especially since the founding of the Royal Society of London and the Paris Academy of Sciences in the 1660s; they were joined in the year of Bernoulli's birth by the Berlin Academy. Among recent deaths, that of Christiaan Huygens in 1695 was the most notable.

When Daniel Bernoulli died in Basel in 1782, Joseph Louis Lagrange (1736–1813) was completing the manuscript of his Méchanique analitique, which was to appear in Paris in 1788, the year after he moved to its Academy from Berlin. He and colleagues such as Pierre Simon Laplace (1749–1827) and Adrien Marie Legendre (1752–1833) were active above all in mechanics and its attendant calculus; the engineering side was in the hands of men such as Gaspard Riche de Prony (1755–1839). Paris was becoming the world centre of mathematics, a status which it would maintain for over 40 years; indeed, the early 1780s were a time of transition, for within 18 months of Bernoulli's demise Leonhard Euler and Jean le Rond d'Alembert also died.

The development of mechanics and the calculus was by far the major concern of mathematicians throughout Bernoulli's career, a situation to which he not only contributed but also helped to form. In this tercentenary tribute I survey the range of his work in mechanics in the context of its general development. First, a short curriculum vitae.

2 Life and career

Daniel's father John was Professor of Mathematics at Groningen University in 1700; but he moved back to his home town of Basel five years later to succeed James, and the son was educated at the university and by members of his talented family. But mathematics was not the only topic; medicine was also on the curriculum, and indeed was to provide him with employment later. He spent several years in Germany and Italy, but nipped home in 1721 to obtain his doctorate. Beginning to publish, in 1725 he was appointed founder Professor of Mechanics at another new academy: that in Saint Petersburg, founded by Peter the Great. Two years later he was joined there by his friend Euler.

The weather was not suitable for Bernoulli's health, however. In 1733 he succeeded in imitating his father by moving back to Basel University as Professor — but of Botany and Anatomy, since John still held the mathematics chair. Never teaching botany, in 1743 he exchanged

the varieties 18th century

it with a colleague for the more amenable subject of physiology. Seven years later he also took the chair in physics, where he alternated semester courses on principles and on practical work. He held both chairs until his death in 1782; however, nephews Daniel II substituted for him from 1776 to 1780, and James II from 1780. (After John's death in 1748 the mathematics chair had been given to his youngest son and Daniel's brother John II (1710–1790).) Daniel also held the rectorship of the university for the academic years 1744–1745 and 1756–1757.[1] He did not continue the dynasty, or at least he was not married.

Physiology and physics seem to have dominated Bernoulli's teaching; but his research work lay largely within mathematics with, as we shall see, a notable medical component. Also unusual for a rational mechanic of his time, quite often he conducted and reported upon experiments carried out to test his theoretical findings. Rather isolated in Basel, he maintained quite a large correspondence; that with Christian Goldbach (1690–1754) and his letters to Euler were published together in (Fuss 1843, 173–655), and were heavily used in a nice non-technical account of his life and work (Wolf 1860), still the closest approximation to a biography.[2] Bernoulli's other good mathematical friends included Alexis Claude Clairaut (1713–1765), whom he met in Paris in 1733 while travelling to Basel from Russia.

Bernoulli published under his own name two books and around 70 papers, which are listed at the end of each volume of the edition in progress of his works. Almost all of the papers appeared with academies, of which he became in due course a (foreign) member.[3] His favourite venues were Saint Petersburg, where he had been resident; and Paris, which offered quite regularly prize problems where he was victorious ten times. (This was two less than Euler; often more than one winner was named for a prize.) Mainly in later life Daniel became unusually active in probability theory and statistics (Huber 1958, ch. 3); throughout it he worked on mechanics and the calculus, like his contemporaries. In mechanics he concentrated upon various problems in fluid flow, elasticity and the vibration of strings, and engineering, but sometimes with long intervals between contributions; for this reason my survey is organised around topic rather than chronology.

Limitations of space and occasion prevent the citation of all pertinent publications; the years used are those of publication, which were often some years after composition. Similarly, I shall not discuss available manuscripts or correspondence; often his work interacted with that of colleagues and contemporaries, especially his father and Euler, but the full story is far too complicated for description here.

Among the leading historical sources, many details are contained in (Truesdell 1954, 1955 and 1960), within the second series of the edition of Euler's works; prefaces to other volumes are also relevant. The editorial material in the available volumes of the edition of the works of the Bernoulli family are also valuable, especially for volume 3 on mechanics for Daniel (1987) in regards to the review to follow.

1 Much of the information in this paragraph draws *passim* on (Staehelin 1957), an informative history of the university. In the 1870s a building for the teaching and practise of the physical sciences, named the 'Bernoullianum' in honour of the family, was built in the city near the University Library and the Botanical Garden (Doublet 1914). It now houses the Geographical Institute of the university.

2 This article is one of dozens on Swiss scientists which Rudolf Wolf (1816–1893), Professor of Astronomy at Zürich University, produced in four volumes between 1858 and 1862. Other figures so treated included James and John Bernoulli, and Euler.

3 These academies included the Royal Society of London, to which Bernoulli did not send any paper; but he was successfully proposed as a foreign member on 3 May 1750 (Royal Society Archives). His supporters were Martin Folkes (antiquarian — and President), H.S. Stevens ('gentleman') and Cromwell Mortimer (physician).

Daniel Bernoulli

3 Three traditions in mechanics

The general situation in mechanics prior to Bernoulli's entrance and some features of its development during his lifetime will now be reviewed. By the time of his debut in the early 1720s, both mechanics and the calculus had received much consideration. Unfortunately the aged Newton had badly soured the atmosphere by initiating in the 1710s a quite unfounded charge against Leibniz of plagiarism over the invention of the calculus (Newton 1981). As a result, two camps were in rivalry in the practise of the calculus itself, with John Bernoulli a prominent campaigner for Leibniz's version; however, a few members of each camp had good contacts and made some use of the other's theory.

The clash lay more in the calculus itself rather than mechanics; as (Bertoloni Meli 1993) has shown, had Newton attacked Leibniz on mechanics then his charge would have been much stronger! In any case there was some competition between the approaches adopted by the two giants, both over principles as such and their generality of use (Grattan-Guinness 1990b). With Newton central attracting forces were held to obtain between any two objects or particles, and their motion was controlled by three laws: 1) uniform rest or motion of a body if left undisturbed; 2) force as the product of mass and acceleration, as it came to be understood (Maltese 1992) although Newton had formulated it in terms of micro-impulses affecting micro-changes in momentum; and 3) action balanced by reaction. In Britain all these components were usually adopted; on the content the notion of central forces and the three laws were used, though the inverse-square underlay was treated much more sceptically (Guicciardini 1999). Indeed, Newton himself and some followers examined situations in which laws of attraction other than the inverse-square obtained, partly for their mathematical interest but also for their suspected utility in other sciences such as electricity and chemistry.

In competition was the energy approach, partly fostered by Leibniz's efforts to refine the vortex theory by positing that bodes moved from some sphere of aether to neighbouring concentric ones at nearby levels (lesser or greater) of vis viva (the forerunner of a theory of kinetic and potential energy). Some overall total content was held to obtain, although impact mechanics (particularly important in engineering) posed special difficulties about the nature of the category whither the lost energy went; much later a general concept of work as the algebraic product of force and distance was to fulfil this role, with Lazare Carnot (1753–1823) playing a major role from the 1780s onwards (Scott 1970, bk. 2).

Overlying both of these traditions was much concern with the various forms in which extended bodies and fluid masses came: hard, elastic and inelastic in the first case, (in)compressible and (non-)viscous in the other. The laws of conservation which may obtain needed careful consideration: of momentum, or energy or vis viva, or of no category at all? These preoccupations arose as part of the widening scope of phenomena which fell under the attention of mechanics; from the actions between the supposed constituent 'molecules' of bodies and fluids through 'ordinary' extended bodies to heavenly ones and indeed the entire planetary system. Figure 1 shows this range, divided into five departments. The distinction made between celestial and planetary mechanics rests upon treating a planet as a mass-point in the former department whereas in the latter it is extended; indeed, the shape of the Earth itself constituted a major problem.

Figure 2 shows a companion layout for the calculus, as it expanded in both the Newtonian and the Leibnizian forms until mid century; after then Britain fell badly behind in all areas of mathematics and the Continent of Europe took centre stage, with Bernoulli as a major player. I cannot describe the development of the calculus here, but mechanics was its greatest single inspiration. A striking feature is that from early on isoperimetric problems were held in high esteem, and the methods of their solution expanded especially from the 1730s onwards into the calculus of variations, which became a major sub-branch of the calculus with Euler and especially Lagrange (Engelsmann 1984). A decade later the multi-variate calculus was developed, with partial differential equations vastly supplementing ordinary ones; and many

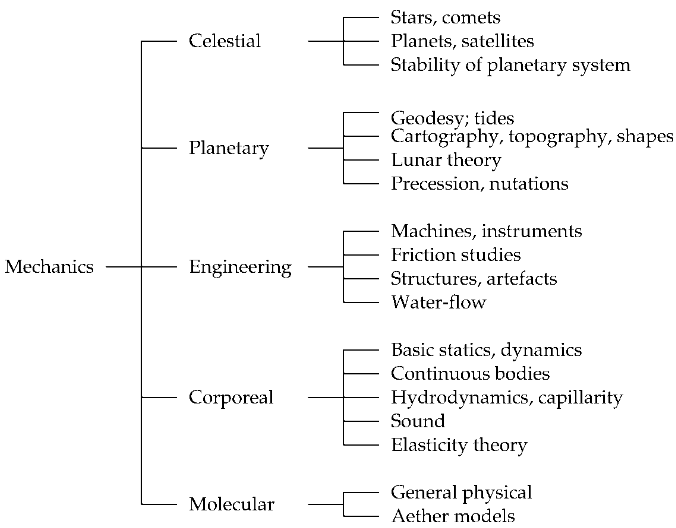


Figure 1

important results concerning series and (special) functions were found, often from solving these equations. Bernoulli was not a major figure here, though he made various useful studies of series, and his work in mechanics led him to consider the roots of equations and pioneer the study of some special functions.

By mid century another tradition in mechanics was beginning to emerge, initially with d'Alembert and then especially with Lagrange and his book of 1788: variational mechanics, run under principles such as d'Alembert's and least action, and later that of virtual velocities. They were chosen to form part of Lagrange's aim of algebraising as much mathematics as possible, with the calculus of variations given a prominent place; this tradition lay great (to its critics, excessive) emphasis on equilibrate situations in mechanics. I shall not detail it here, as Bernoulli used it rarely, although it came up in correspondence especially with Euler (Pulte 1990): conversely, the extent of its popularity was to reduce the reception of Bernoulli's contributions, for its stress upon algebra ran counter to the geometrical cast of most of Bernoulli's mechanics.

Most of the main issues and differences between these two and then three traditions lay in dynamics, but statics also had to be considered: indeed, a hope of the variational tradition was to reduce dynamical situations to statical ones. The basic principles of statics also needed study, as the young Daniel Bernoulli noticed.

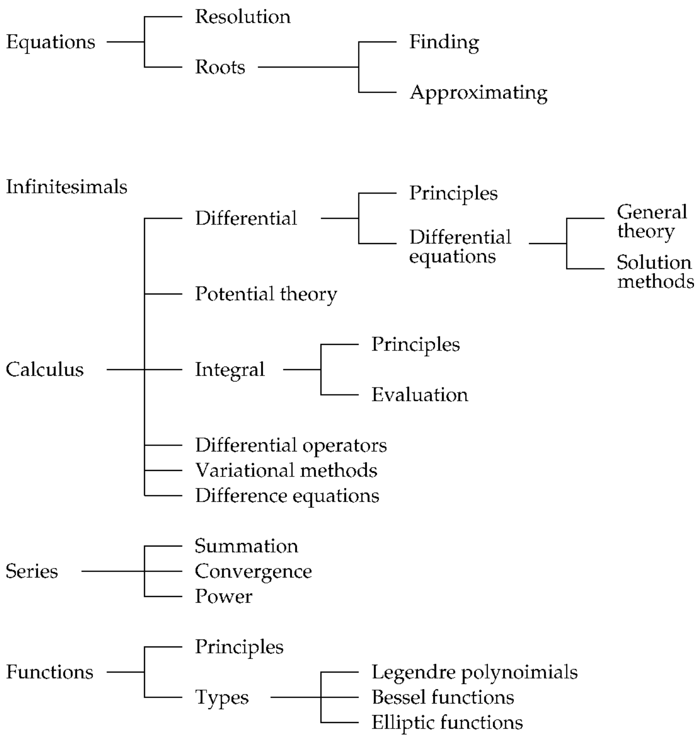


Figure 2

4 Mechanical principles with Bernoulli

In an early paper (Bernoulli 1728a) took up the epistemological status of the parallelogram law of two forces as deduced by Newton from his laws; is it empirically or necessarily true, and how derived? Aiming at necessity, he adopted seemingly general principles such as any force being replaceable by a parallel one of equal magnitude and sense, and built up the general parallelogram law from a sequence of special cases. Some further assumptions such as continuity slipped in en route; but the argument was clever. Strangely, nobody examined the case of composing two equal forces in parallel and with opposite senses until the beautiful analysis by the young Louis Poinsot (1777–1859) of the 'couple' (his word) (Poinsot 1803).

Bernoulli's most substantial concern with Newtonian principles lay in the law of angular momentum; in (1746) he examined cases of motion of a sphere under various conditions in which angular momentum was conserved. Was this property true in general? He thought so, but an ingenious argument failed to establish it, and contemporary efforts by Euler led to the conclusion that the general law of conservation of angular momentum was in fact independent of Newton's laws (Truesdell 1968, ch. 5) — a fact which is still too little recognised.

Some years earlier (Bernoulli 1740) had made a foundational contribution of a different kind; in the course of analysing the oscillation of bodies connected by a thread he proposed a new principle based upon considering their free motion and then the effect of the constraints imposed by the connections. It is similar to the principle which d'Alembert was to propose soon afterwards and which still carries his name, in which such motions were taken as the combination of free and constrained sub-motions (Truesdell 1960, 159–163). In the later and clearer hand of Lagrange this principle was prominent in variational mechanics; Bernoulli did not follow that path, although optimal principles occurred on occasion (an example is given in §6) and he discussed them in correspondence with Euler.

For Bernoulli the preferred route drew upon vis viva, perhaps influenced by his father's advocacy, which itself was partly inspired by physiology (Maclean 1972). His own interest in this tradition, and also in physics, is evident in his medical studies, in which his doctorate of 1721 was awarded for a thesis on respiration (Huber 1958, 29–40). Another early paper (1728b) dealt with the effect of motion upon the internal structure of muscles, which again he treated in terms of a physical theory about the actions of the supposed lateral fibres in the muscle; he followed his father, whose own doctorate at the university had partly been awarded for a thesis of 1694 on this topic (Kardel 1997). Later, in a lecture (1737) given at the university to present two of his own doctoral students in medicine Daniel discussed the work-rate of the heart and explicitly mentioned the conservation of vis viva.

In various papers Bernoulli also used this principle within the main areas of mechanics. He adopted it in an analysis (1729b) of pendula, where he built upon findings by Huygens. Similarly, in an essay (1747) on the motion of a heavenly body and the three-body problem he worked out from the first integral of Newton's second law, which related the square of its velocity to, in effect, its level of potential under attraction. In a sequel study (1750) on the generality of the principle he included the case where the body was acted upon by more than one centre of force. This was a most unusual way of working within celestial mechanics; Newton had rarely invoked energy equations or relationships, although he had also found results about external attraction and equipotential surfaces.

As was indicated in §3, the issue of generality in energy mechanics importantly involved in the cases of impact between extended bodies, and Bernoulli thought seriously about the various types involved. Most of the earlier work on the law of collision, especially that of Huygens, had treated only spheres, and the (apparent) losses of momentum and energy after lateral impacts; Bernoulli desimplified the theory in (1744) by allowing for rotation as well as translation in impact between bodies of general or of other shapes, such as impact between straight bars and a bar hitting a sphere. In a fine late successor (1771) he analysed the motion produced in an elastic bar at rest when struck at its mid-point by assuming that the shape adopted was given by the least measure of vis viva ; this principle became known in this century as the 'Rayleigh-Ritz principle' when developed (independently) in more general contexts (Szabó 1977, 465–470).

5 Fluid mechanics with Bernoulli

This topic was one of Bernoulli's main areas of interest, including the role within it of energy mechanics; his major book dealt with it, and construed one of his main achievements. This was the Hydrodynamica, published in Strasbourg as (1738b) though apparently much worked out earlier during his period at Saint Petersburg; a paper (1729a) of that time and academy on the motion of water in channels contained some of the main notions. On this book, and the preposterous priority dispute which his father and colleague professor later launched against him, see especially (Truesdell 1954, xxiii–xxxviii), and (Szabó 1977, 157–199).

The name of the book was itself Bernoulli's innovation; it covered both the statics and dynamics of fluids, the latter then often called 'hydraulics'. He covered a wide range of phenomena, including oscillation of fluids within vessels, motion within machines, and the issue of jets from orifices and their impact upon surfaces. An important sect. 10 was devoted to properties of gases, especially air, analysed in a way strikingly combining energy with Newtonian force and somewhat anticipating later kinetic theories of gases (Pacey and Fisher 1967). As normal with him, he also reported on his own experiments. Perhaps for the special difficulties concerning viscosity, he did not consider blood flow.

Among principles, the 'conservation of vis viva' naturally took prime place, where the 'potential ascent' of a system of fluid particles against gravity was balanced by the 'actual descent' of their centre of gravity achieved after equilibrium has been achieved. While too few details were provided to cover the generality of mechanical phenomena, the emphasis on energy was clearly registered; in particular, Bernoulli produced an energy-oriented equation which asserted a relationship between the vis viva and the (in effect) wall pressure for the flow of fluid within a vessel (sect. 1). Rather more general versions of this equation are still named after him; his own use of it here was fairly limited, and he did not have a general conception of hydrostatic pressure. Further, from it he concluded that the loss of vis viva was measured in terms of the difference between the squares of the initial and final velocities in a situation, whereas Carnot and others were to realise later that the square of the difference of velocities was required.

The notion of ascent and descent recall Leibniz's concern with the concentric levels of energy within a system of vortices which a heavenly body traverses (§3). Bernoulli applied it also to a continuity principle designed specifically for (incompressible) fluids, which had been known before him but received a newly prominent place here and became known later in the century as 'the hypothesis of parallel slices'. It stated that a fluid body was presumed to achieve its ascents and descents by moving in slices of common velocity without interaction of other particles in the direction of the motion. (An analogy denied to Bernoulli is of a wrapped loaf of sliced bread moving en bloc without interpenetration of the slices.) The Leibnizian calculus played a significant role, for the slices were infinitesimally thin, and so expressible in terms of differentials; conversely, the utility of the calculus gave the hypothesis extra prestige.

While aware of the simplifications embodied in his principle, Bernoulli deployed it with great dexterity in various cases of motion. It became a strong favourite, especially among engineers; de Prony used it as the basis of much of treatment of incompressible fluids while also being aware of its limitations (see, for example, 1790, esp. pp. 330 (Hydrodynamica cited), 425–426). Later his former student Siméon Dénis Poisson (1781–1840) analysed it at length in his textbook on mechanics (1811, 444–460; 1833, 721–746).

Another well recognised achievement by Bernoulli in fluid mechanics concerned the motion of tides, where he and Euler were two of four victors of a Paris prize problem set in 1738 for 1740. Concentrating upon surface behaviour, his main insight was again based upon the notion of level in fluids (and also Newton's inverse-square law): he determined the height of the neap tide under the influence of one external large heavenly body B by balancing columns of elongated sea-water created by B pulling along the axis of attraction. From this analysis he corroborated Newton's value for the height of the solar tide, and also compared it with that of the lunar tide. In addition, he found the same ratio 501:500 as Newton for the ratio of the poles of the Earth considered as a spheroid of revolution, although his poles were elongated rather than flattened (Greenberg 1995, 400–412).

At various time Bernoulli studied questions in the technological side of fluid mechanics, some of them inspired by Paris prize problems. They included the maintenance of reliable water-clocks of ships at sea; the optimal shape of anchors; and the effects of wind upon their navigation of ships, especially rolling and pitching. These efforts gained little interest among naval engineers, a fate also to befall most of Euler's contributions. However, a late prize analysis (1769a) of the effect of winds upon the manoeuvre of ships included a 'new theory of the economy of forces and its effects' which pioneered some notions in ergonomics, and was soon to influence a major figure in that area, namely Charles Coulomb (1736–1806) on the action of windmills (Coulomb 1785, art. 16).

6 Elasticity and acoustics with Bernoulli

Partly in connection with friction, Bernoulli also wrote on the design and maintenance of compasses, which linked to another major interest in mechanics. One of the factors was the friction of the mounting (Bernoulli 1748, again for a Paris prize problem). Soon afterwards and especially late in life he tackled in general this exceptionally difficult area of corporeal mechanics, examining cases of both rolling and sliding friction; some cases in (1769b) were inspired by technology such as ploughing with minimal effort and pulling ropes and sledges along the ground. Despite much ingenuity, he found only special results. In his late years he seems to have contributed heavily to the researches in elasticity theory of his nephew John III Bernoulli (1744–1807).

Much greater success and indeed some fame had come to Bernoulli with his study of vibrating elastic strings (Truesdell 1960, pt. 3). Again thinking in terms of physics more than of mechanics, he carried out both analyses and experiments on the modes of vibration and oscillation in many physical situations, such as stretched horizontal strings, suspended linked bars or masses, bars or needles pinned or clamped at one or both ends in various ways, and bodies rocking on water (Cannon and Dostrovsky 1981, esp. chs. 9, 12–13). He concluded that such systems of bodies exhibited natural and/or forced modes of vibration. The paper (1738a) opened up this theory, with the consequent analysis including some properties of the (new) Bessel function (as it was called later). The successor paper (1740) was the venue for the d'Alembert principle mentioned in §4, and it was followed by an essay (1751) on forced oscillations.

None of this work gained the general recognition that it deserved. By contrast, in one of his most famous papers, (Bernoulli 1755) proposed that the vertical displacement y of a uniform elastic string vibrating in the horizontal plane between two fixed pivots of equal height and distant a apart be expressed in terms of the distance x from one pivot by an infinite trigonometric series

$$y = \alpha \sin \frac{\pi x}{a} + \beta \sin \frac{2\pi x}{a} + \gamma \sin \frac{3\pi x}{a} + \delta \sin \frac{4\pi x}{a} + \&c. \qquad (1)$$

The physical principle was that each sine term represented a basic vibration of the string (as claimed by Newton's follower Brook Taylor (1685–1731) in 1713), and that the full behaviour was to be understood as their superposition. While another important suggestion in the science of acoustics, the formulation was mathematically rather inadequate, lacking the cosine and time terms or means of calculating the constants α, β, γ, δ, ... Further, at that time it was seen as inferior to the functional solution to the underlying partial differential 'wave' equation which d'Alembert and Euler had debated in the previous decade:

$$y = f(x + Kt) + g(x - Kt), \qquad K \text{ a physical constant} \qquad (2)$$

for the functions f and g were explicitly exhibited and determined by the initial conditions of release of the string. The controversy over the solutions became a major topic for the rest of the century, especially from the mathematical point of view, because the wave equation was the first partial differential equation to be solved in detail; for example, both Lagrange and Laplace wrote on it early in their careers.

Thus Bernoulli's proposal (1) became well known. However, (2) was the favourite candidate until in the 1800s Joseph Fourier (1768–1830) rehabilitated (1) in full form including time terms (Grattan-Guinness 1990a, ch. 9), by both determining the coefficients and understanding the periodicity of the trigonometric terms relative to y (Bernoulli had thought that y be defined as zero outside the interval of specification). His own main principal later contribution during that interim was a study (1767) of the vibrations of a non-uniform string, where he found some special solutions (Truesdell 1960, 307–309).

Somewhat isolated from this string of papers though still closely guided by the physics of vibrations was Bernoulli's long analysis (1764) of the tones emitted from organ pipes (Truesdell 1955, *lv–lix*). This time layers of air were held to move simply harmonically within the pipe, of which the size, shape and openings determined the overtones and thus the quality of sound. Some mathematics on the vibrating string problem could have been brought into play, and in certain respects the paper is rather passé.

7 Bernoulli and Euler

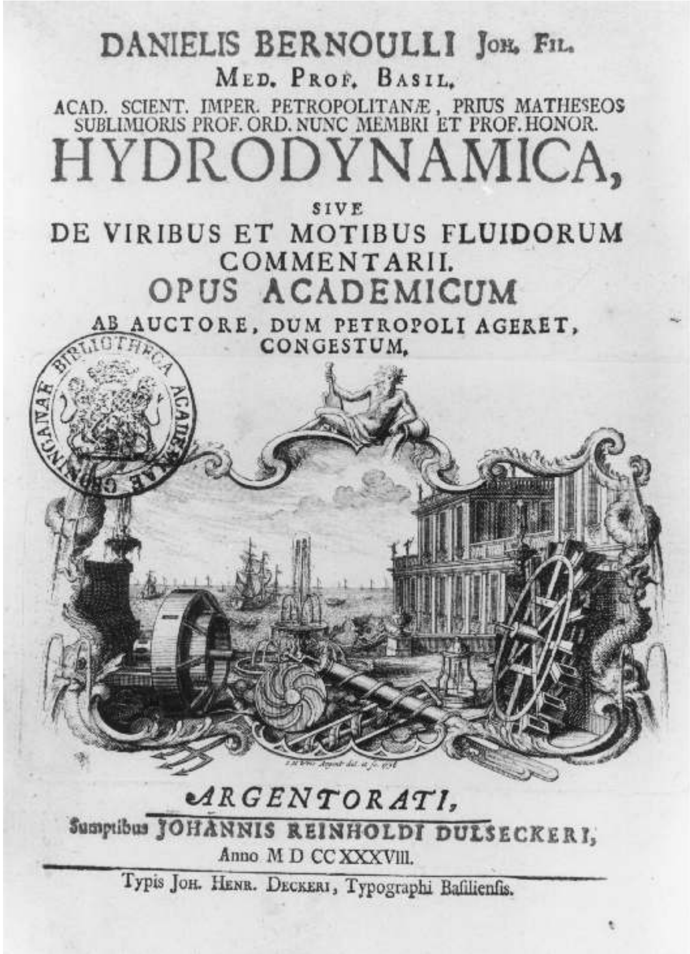
Of the three traditions which reigned or developed in Bernoulli's lifetime, energy mechanics was his favourite choice, seemingly for its kinship to physics and physiology. But some limitations of his mathematical insights here are well captured by (1), which only expressed some of the features of the superimposed simple vibrations of which he held the string to be capable. Elsewhere, while he often showed great ingenuity, his theories tended to be developed ad hoc, tied to the particular needs of the type of problems at hand. Moreover, they related to an unusually large extent at that time to physics, which then held a status much lower than that of mechanics (Kuhn 1976).

Bernoulli's choice of problems suggest a *lateral* thinker at work, taking up questions and making connections eschewed or not noticed by others. (For example, his interest in probability and especially statistics was very unusual at that time.) His training in both mathematics and medicine was itself a singularity, though profitable for the common links found in physics. The study of friction, and of water-clocks at sea,

Leonhard Euler

show a researcher not afraid to tackle really hard problems — but then inevitably not advancing far forward in solution. Again and sadly, his penchant for careful experimentation was then not common in mechanics. Finally, he worked little or not at all in several trendy areas of mathematics of his time. One was solutions to differential equations; although he used them in some contexts, he did not use partial ones such as the wave equation for the vibrating string problem. Another popular branch was celestial mechanics, especially perturbation theory after the 1740s, where he took a non-standard approach using energy (§4) rather than the normal method of proceeding directly from Newton's second law.

A comparison with Euler is clearly useful, and best effected in areas where they were both important. Bernoulli made significant contributions to elasticity theory and fluid mechanics, but he did not achieve the general equations of equilibrium and motion which Euler was to find in the 1740s and the 1750s. His treatment of tides (§5) was notable, but it did not match the insight in Euler's competition essay that the horizontal component of the force disturbing the motion of the sea is responsible for many of the subsequent actions (Aiton 1955). Bernoulli also did not match other of Euler's innovations elsewhere, such as using trigonometric series (not to be confused with (2)) to analyse perturbation theory in celestial mechanics (late 1740s), a fundamental change which obliterated Bernoulli's deployment of vis viva noted in §4; applying Newton's second law in any direction in a problem, and finding the general equations for a body rotating about a fixed point (both 1753); clarifying to a noteworthy extent the relationship between real and complex variables with his equations connecting the trigonometric and complex exponential functions (1748); and reformulating the Leibnizian calculus with the notion of the differential coefficient,



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HYDRODYNAMICA,
SIVE
DE VIRIBUS ET MOTIBUS FLUIDORUM COMMENTARII.
OPUS ACADEMICUM
AB AUCTORE, DUM PETROPOLI AGERET, CONGESTUM.
ARGENTORATI,
Sumptibus JOHANNIS REINHOLDI DULSECKERI,
Anno M D CCXXXVIII.
Typis JOH. HENR. DECKERI, Typographi Basiliensis.

The frontispiece of Bernoulli's Hydrodynamica (1738b)

the forerunner of the derivative (1755). Euler chose his problems more conservatively than offbeat Bernoulli, but he enriched them to the extent of making radical innovations and revolutions in the theories involved; the list just given is not exhaustive. Further, nobody matched him in the sheer quantity of output. Bernoulli also did not produce any textbooks in mathematics; his situation in Basel forced no such requirement. In addition but not demonstrated here, in contrast especially to Euler, his writing style and manner of presentation can be difficult to follow. For these reasons he has been rather eclipsed by Euler, and also by his father John and uncle James.

8 A decline and rise of fame?

This is not a new view of Bernoulli's overall contributions to mechanics; indeed, appraisal of his preferred methods and topics conforms to that conveyed in the editorial material in volumes of the edition, where many more details are given. My aim has been to provide a more general perspective, and to this end a note of some histories of mechanics would be useful.

During his lifetime Bernoulli became very well known as a scientist. The histories of mathematics produced soon after his death gave him his due: in particular, the last two volumes of the giant account by Jean-Etienne Montucla (1725–1799), as completed by Jerome Lalande (1732–1807), covered mechanics pretty comprehensively (Bogolubov and Djukobskaya 1976), and Bernoulli was quite well covered (Montucla 1802 passim). However, Sylvestre-François Lacroix (1765–1843), an author with formidable historical knowledge who helped Lalande with the parts of Montucla covering the calculus, cited Bernoulli only for the work on the vibrating string problem in his own huge treatise on the calculus (Lacroix 1800, 546; 1819, 735). By contrast, a fair proportion of Euler's contributions already published was gaining a good reception in the dominating French community of the time, even though several of the theories (like Bernoulli's) clashed with the dictats of Lagrange (Grattan-Guinness 1983).

But most of Bernoulli's contributions to mechanics seem to have become overlooked in the later histories dealing with the subject. To take two good examples, the history of engineering mechanics made by Moritz Rühlmann (1811–1896) included only a few sections of the Hydrodynamica and a mention of (1728) on the parallelogram law (Rühlmann 1881-1885, 159–167, 482). Similarly, the latter paper was the only Bernoulli item in the nice presentation by Emile Jouguet (1871–1943) of 'mechanics taught by its authors' (Jouguet 1909, 58–70, including part of the later treatment in (Poisson 1833)). In between these two books there appeared various editions and translations of Ernst Mach's history, much better known though in my view much inferior to them, where Bernoulli fared no better (Mach 1902).[4] More recent historical sources show similar levels of attention, up to (Dugas 1950); at most the statics paper, bits of the Hydrodynamica, and the studies of the vibrating string.

However, the efforts of Clifford Truesdell (1919–2000) from the 1950s gave Bernoulli much more exposure. With a typical mixture of insight and exaggeration he characterised Bernoulli as "one of the most mysterious figures in the history of science, mysterious for no other reason than that nobody troubles to study through his works, while historians continue to attribute to him fame or blame for things he did not do or say" (1968, 278).

Indeed, the historical literature on any aspect of Bernoulli's life or work remains modest. But much of this mystery is being replaced by understanding and recognition, through Truesdell's own efforts and consequent work; and also the edition in progress of Bernoulli's works, which itself builds upon the bibliography in the excellent encyclopaedia survey article (Straub 1970) on Bernoulli. For example, the index of a general encyclopaedia for the history and philosophy of mathematics which I edited a few years ago contained 22 sub-entries for him, on all topics (Grattan-Guinness 1994, 1728); for comparison, there are 15 sub-entries for James Bernoulli, 18 for John, 27 for d'Alembert, and 68 for Euler.

While not wishing to overturn the normal appraisal of Bernoulli compared to his elders and to Euler, I would claim that he has been rather underrated since his death. His choice of unusual problems and/or methods of solution may be one reason; the production of only one overall book-length statement in any branch of mathematics will have reduced impact; and the accidental co-existence of Euler must be significant. But as his publications and manuscripts become more available and read, a great lateral scientist of the 18th century will be revealed.

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4 Moritz Cantor's vast history, also published at that time, did not cover applied mathematics for the 18th century onwards.

Bibliography

Abbreviations

CP Commentarii Academiae Scientarum Imperialis Petropolitanae

NCP Novi CP

MAB Mémoires de l'Académie Royale des Sciences et Belles-Lettres de Berlin

MAP Mémoires de l'Académie Royale des Sciences [Paris]

RPP Receuil des pièces qui ont remporté les prix de l'AP

St. n Item n in H. Straub's list in each of volume of Bernoulli's works

Works. *Werke*, 8 vols., in progress, Basel (Birkhäuser), 1982–.

Bernoulli items

1728a. *Examen principiorum mechanicae...*, CP, 1, 126–142 = Works, 3, 119–135. St. 9.

1728b. *Tentamen novae de motu musculorum theoriae*, CP, 1 (1726/28), 297–313 = Works, 1, 92–106. St. 10.

1729a. *Theoria nova de motu aquarum per canales quoscunque fluentium*, CP, 2 (1727/29), 111–125. St. 12.

1729b. *De mutua relatione centri virium, centri oscillationis et centri gravitatis, demonstrationes geometricae*, CP, 2 (1727/29), 208–216 = Works, 3, 136–144. St. 13.

1737. *De vita*, manuscript; in Verh. Naturf. Gesell. Basel, 52 (1940-41), 219–234. St. 75. [Introd. and German trans. by O. Spiess and F. Verzár on pp. 189–218].

1738a. *Theoremata de oscillationibus corporum fili flexili connexorum et catenae verticaliter suspensae*, CP, 6 (1732-33/38), 108–122 = (Cannon and Dostrovsky 1981), 125–141 [with English trans. on pp. 156–167]. St. 23.

1738b. *Hydrodynamica, sive de viribus et motibus fluidorum commentarii*, Strasbourg (Dulsecker). St. 41.

1740. *Demonstrationes et theorematum suorum de oscillationibus corporum fili flexili connexorum et catenae verticaliter suspensae*, CP, 7 (1734-35/40), 162–173 = (Cannon and Dostrovsky 1981), 142–155 [with English trans. on pp. 168–176]. St. 25.

1744. *De variatione motuum a percussione excentrica*, 24) 6 CP, 9 (1737/44), 189–206 = Works, 3, 145–159. St. 27.

1746. *Nouveau problème de mécanique résolu*, MAB, 1 (1745/46), 54–70 = Works, 3, 179–196. St. 40.

1747. *Commentatatio de immutatione et extensione principii conservationis virium vivarum...*, CP, 10 (1738/47), 116–124 = Works, 3, 160–169. St. 29.

1748. *Mémoire sur la manière de construire les boussoles d'inclinaison...*, RPP, (1743/48), 3–61 = Works, 7, 67–112. St. 39.

1750. *Remarques sur le principe de la conservation des forces vives pris dans un sens général*, MAB, (1748/50), 356–364 = Works, 3, 197–206. St. 43.

1751. *Excerpta ex litteris. A Danieli Bernoulli ad Leonhardum Euler*, CP, 13 (1741-43/51), 3–15 = Works, 2, 81–93. St. 35.

1755. *Réflexions...sur les nouvelles vibrations des cordes...*, MAB, 9 (1753/55), 147–172. 1) 8St. 45. [On forced oscillations].

1764. *Recherches physiques, mécaniques et analytiques, sur le son et sur les tons de tuyaux d'orgues différement construits*, MAP, (1762/64), 431–485 = [Bologna] (Forni), 1983. St. 53.

1767. *Mémoire sur les vibrations des cordes d'une épaisseur inégale*, MAB, (1765/67), 281–306. St. 54.

1769a. *Recherches sur...l'action du vent sur les grands vaisseaux...*, RPP, 7, 99 pp. St. 47.

1769b. *Commentatio de utilissima ac accommodissima directione potentiarum frictionibus mechanicis adhibendarum*, NCP, 13 (1768/69), 242–256 = Works, 3, 207–218. St. 57.

1771. *Examen physico-mechanicum de motu mixto qui laminis elasticis a percussione simul imprimitur*, NCP, 15 (1770/71), 261–370. St. 61.

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