J.P. Murre
Mathematisch Instituut, Universiteit Leiden
Postbus 9512, 2300 RA Leiden
murre@math.leidenuniv.nl

On the life and work of Hendrik Douwe Kloosterman

This is the slightly modified and updated version of a lecture delivered on May 7, 1986 at the occasion of the visit of the first 'Kloosterman professor', Professor M. Artin. On the Kloosterman Centennial Celebration at April 7. at the Universiteit Leiden a biographical brochure was presented based on this text.

Some dates

Hendrik Douwe Kloosterman was born on April 9, 1900 in Rottevalle (Friesland), he died May 6, 1968 in Leiden.

Kloosterman studied mathematics at the University of Leiden from 1918–1922, his teachers in mathematics were J.C. Kluyver and W. van der Woude. After his master degree in 1922 he continued his studies in Copenhagen with H. Bohr and in Oxford with G.H. Hardy. In 1924 he obtained his Ph.D. degree in Leiden, his supervisor was J.C. Kluyver. From 1924–25 he performed his military service. A Rockefeller-Scholarship allowed him to study from 1926–28 in Göttingen and in Hamburg. From 1928–30 he was assistant in Münster.

In 1930 Kloosterman became lecturer ('lector') at the University of Leiden, this position was converted into a full professorship in 1947. Kloosterman remained professor in Leiden until his death in 1968. During the academic year 55–56 he was visiting professor at the University of Michigan, Ann Arbor.

In 1950 he was elected as a member of the Koninklijke Nederlandse Akademie van Wetenschappen (Royal Dutch Academy of Sciences).

During his stay in Göttingen Kloosterman met Margarete Träger. They married in 1930 and she remained his faithful companion until his death in 1968.

Kloosterman's scientific work

Kloosterman's work is deep and difficult and, as we shall see presently, it continues to play an important role in modern developments. In a report like this we can only superficially touch upon it an we shall limit ourselves to a short discussion of two of the most important topics.

As a student Kloosterman was attracted by analysis and analytic number theory, undoubtly this was due to the influcence of his teacher Kluyver and later Hardy. In his thesis [B1] he studied the problem of representing a positive integer n as the sum of squares, or more generally, in solving in integers the equation $n = a_1x_1{}^2 + a_2x_2{}^2 + \ldots + a_sx_s{}^2$, where the a_i's themselves are also positive integers. This was a classical problem studied among others by Euler, Lagrange, Jacobi and Liouville. These authors studied particular cases and their methods were (with the exception of those of Jacobi) mostly 'elementary'. In his thesis Kloosterman used powerful analytic tools developed by Hardy and Littlewood, and for $s \geq 5$ he obtained interesting, mostly asymptotic, general formulae for the number $r(n)$ of solutions of the above equation. However, for $s = 4$ there was an essential difficulty and the Hardy-Littlewood method did not yield good results.

Soon after his thesis Kloosterman returned to this particular case. By a deep and difficult analysis he refined the Hardy-Littlewood method and solved the problem completely. He published his result in 1926 in Acta Mathematica [B2]. This paper made him world-renowned. There Kloosterman introduced and carefully analysed certain exponential sums, now called 'Kloosterman sums'. These sums were crucial not only for the problem at hand, but in fact they turned out to be fundamental for many problems in analytic number theory and in the theory of modular functions and they also play a role in parts of algebraic geometry.

As early as 1927, stimulated by Hecke, Kloosterman applied his methods to estimate the Fourier coefficients of certain modular forms [B3]. The best estimates for the Kloosterman sums were later given by Weil by using powerful tools from algebraic geometry. The fundamental nature of the Kloosterman sums is clear from the numerous papers, from 1926 up to the present day, in which they have been studied, used, and generalized. In order to give an idea we mention some authors and dates of such papers: Esterman (1930), Salié (1931), Davenport (1930), Weil (1948), Lomadze (1959), Mordell (1961, 1972), A.I. Vinogradov (1962), Linnik (1962), Selberg (1965), Deligne (1977), Serre (1977). Katz (1979, 1988), Hooley (1983), Michel (1985), Duke-Friedlander-Iwaniec (1997). For further details see also the nice paper [A3], as well as the section Number Theory, 11 L05 'Gauss and Kloosterman sums, generalizations' in Mathematical Reviews.

H.D. Kloosterman, 1955

Next we want to focus on another important piece of work of Kloosterman. During his stay in Göttingen and Hamburg, influenced by E. Noether, E. Artin and E. Hecke, Kloosterman became more and more interested in algebraic methods. Both his algebraic and analytic skills are demonstrated in his beautiful papers *The behaviour of general theta functions under the modular group and the characters of binary modular subgroups* [B7] and *The characters of binary modulary congruence groups* [B8] which appeared, respectively, in 1946 in the Annals of Mathematics and in 1952 in the Proceedings of the International Congress of Mathematicians (the latter being the written version of his invited lecture at the ICM, 1950 in Cambridge, Mass.). In these papers Kloosterman uses theta functions to study representations of the factor group Γ mod $\Gamma(N)$, where Γ is the modular group and $\Gamma(N)$ the principal congruence subgroup modulo N. This work was continued by several of his students, and further by S. Tanaka in 1966, who related it to the representations of the sympletic group as studied by A. Weil (Acta Math. 111, 1964), and also by Nobs and Wolfert in 1974 and 1975.

The above report is short and incomplete. Still, we hope that it gives some idea of the profundity and richness of Kloosterman's work, which is still very much alive in modern mathematics!

Kloosterman Professors

1986 M. Artin, Massachusetts Institute of Technology, Mass., U.S.A.
1987 H. Brézis, Université Paris VI, Paris, France
1988 A. Salomaa, University of Turku, Turku, Finland
1989 S.M. Ross, University of California, Berkeley, Cal., U.S.A.
1990 M. Barth, University of Erlangen, Erlangen, Germany
1991 J.C. Butcher, University of Auckland, Auckland, New Zealand
1992 P.J. Bickel, University of California, Berkeley, Cal., U.S.A.
1993 H. Ehrig, Technical University Berlin, Berlin, Germany
1994 T. Oshima, University of Tokyo, Tokyo, Japan
1995 H.W. Lenstra, University of California, Berkeley, Cal., U.S.A.
1996 G. Cybenko, Thayer School of Engineering, Dartmouth College, Hannover, N.H., U.S.A.
1997 J.K. Hale, Georgia Institute of Technology, Atlanta, U.S.A.
1998 A.A. Borovkov, University of Novosibirsk, Novosibirsk, Russia
1999 A. Granville, University of Georgia, Athens, U.S.A.
2000 J. Mallet-Paret, Brown University, Providence, RI, U.S.A.

Added in 1997: In the summer of 1997 I asked P. Sarnak, professor at Princeton University and one of the experts on modern number theory about the influence of Kloosterman's work. Professor Sarnak sent me the following lines.

"I am happy to comment on Kloosterman and his influence on modern number theory. His paper in the twenties on quaternary quadratic forms is one of the most influential in analytic number theory. He introduces a refinement of the Hardy-Littlewood circle method which has been at the heart of much modern work (Linnik, Selberg, Iwaniec, Hooley, et cetera). It is such an important refinement that some even refer to the circle method as the Hardy-Littlewood-Kloosterman method. This refinement led him to the Kloosterman sums. Besides introducing these basic objects (they are the 'Bessel functions' of finite fields) he derived their basic properties. It was left to Hasse and Weil to give the well known and deep estimate for these sums. Kloosterman sums find their applications in many proofs of results in automorphic forms, L-functions, quadratic forms... He has certainly left his mark on modern number theory (arithmetical and analytic)."

Teaching and influence on students

Before the war, i.e. before 1940, the size of the mathematics staff at the Dutch universities was very small (maybe with the exception of the University of Amsterdam). In Leiden it consisted of two professors (W. van der Woude and J. Droste) and one lecturer (Kloosterman). During the period 30–41 (the University of Leiden was closed from 1941 until the middle of 1945) the principal task of Kloosterman was to give first and second year courses calculus and analysis for students in mathematics, physics, astronomy, chemistry, and geology. However, besides these elementary courses Kloosterman voluntarily lectured each year on an advanced subject, each year different. During that period he lectured on the theory of groups, on analytic and algebraic number theory, ideal theory, theory of Galois, on quadratic forms, on linear operators in Hilbert space, and on mathematical tools in quantum theory. After the war he continued this series with courses on representation theory, lattice theory, measure theory, modular functions, elliptic functions and on algebraic function fields.

These capita selecta were exciting lectures and modern for that period. They were carefully planned and prepared, masterly presented, starting elementarily but advancing steadily to a high level. Kloosterman was convinced of the unity of mathematics and in his lectures he stressed the connections between different areas. To the audience a beautiful panorama unfolded, completely natural and harmonic.

In fact, to attend a lecture of Kloosterman, elementary or advanced, was always a pleasure, both for mathematicians and non-mathematicians. Those of us present there will remember his very clear style, his skilful use of the blackboard (always filled at — but never before — the end of the lecture), his fine sense of humour and certainly also his choice of a 'completely arbitrary' number, which invariably was 37.

Under Kloosterman's supervision 17 students wrote their Ph.D. thesis. Several worked on subjects stemming from Kloosterman's own work, but he encouraged and stimulated also those going their own way. Several of these students later held professorships at the universities of Utrecht, Amsterdam and Leiden, and at the technical universities of Delft, Eindhoven and Twente.

H.D. Kloosterman (1900-1968)

After the war Kloosterman was the principal architect in building our Mathematical Institute in Leiden. Under his leadership several new chairs were created, first in pure mathematics, but later he also laid the foundations for the chairs in applied mathematics (and by this indirectly also for computer science).

Conclusion

We remember Kloosterman with admiration and gratitude. We are pleased and honoured to name this new special chair of our Department, the Kloosterman Chair.

References

A. On Kloosterman's life and work

[A1] F. van der Blij and N.G. de Bruijn, *In memoriam H.D. Kloosterman, 1900–1968*. Nieuw Archief voor Wiskunde, Serie 3, 16 (1968), 139–147.

[A2] N.G. de Bruijn, *Levensbericht van Hendrik Douwe Kloosterman*. Jaarboek Kon. Ned. Akademie van Wetenschappen 1967–1968, 326–330.

[A3] F. van der Blij, *Commentary on Kloosterman sums*. In: Two decades of mathematics in The Netherlands, 1920–1940 (A retrospection of the bicentennial of the Wiskundig Genootschap), Math. Centre, Amsterdam 1978, 63–82.

B. Some papers of Kloosterman
(For a complete list see [A1])

[B1] *Over het splitsen van geheele positieve getallen in een som van kwadraten*. Thesis Leiden University, 1924.

[B2] *On the representation of numbers in the form* $ax^2 + by^2 + cz^2 + dt^2$. Acta Mathematica 49 (1926), 407–464.

[B3] *Asymptotische Formeln für die Fourierkoeffizienten ganzer Modulformen*. Abh. Math. Sem. Hamburg 5 (1927), 337–352.

[B4] *Theorie der Eisensteinschen Reihen von mehreren Veränderlichen*. Abh. Math. Sem. Hamburg 6 (1928), 163–188.

[B5] *Thetareihen in total-reellen algebraischen Zahlkörpern*. Math. Ann. 103 (1930), 279–299.

[B6] *Simultane Darstellung zweier ganzen Zahlen als einer Summe von ganzen Zahlen und deren Quadratsumme*. Math. Ann. 118 (1942), 319–364.

[B7] *The behaviour of general theta functions under the modular group and the characters of binary modular congruence groups I, II*. Annals of Math. 47 (1946), 317–447.

[B8] *The characters of binary modulary congruence groups*. Proc. Intern. Congress of Mathematicians 1950, 275–280.