

Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 April 2024**. The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A

Prove that for all three line segments of length 1 in a closed disc of radius 1 there are two of those line segments with distance at most 1.

Problem B (proposed by Hendrik Lenstra)

Let A be an abelian group and write $\text{End}(A)$ for the ring of group homomorphisms from A to A . Show that A is free as $\text{End}(A)$ -module if and only if A admits a commutative ring structure so that the Cayley map $A \rightarrow \text{End}(A)$ given by $x \mapsto (y \mapsto xy)$ is an A -module isomorphism. Show that for all subrings of \mathbb{Q} and rings $\mathbb{Z}/p\mathbb{Z}$ with p a prime the Cayley map is an isomorphism, and give an example of an uncountable ring with this property.

Problem C (proposed by Albert Visser)

A *Gollum ring* is a ring that is isomorphic to the subring $\mathbb{Z} + 2R$ of a commutative ring R . Show that there is a sentence in first-order logic in the language of rings that is true in all Gollum rings, but not true in all commutative rings.

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