| Problem Section

Problemen

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 October 2023**. The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A

Let $n > 0$ be an integer and let $\varphi : \mathbb{R}^n \to \mathbb{R}^n$ be an isometry, i.e., a map such that for all $x, y \in \mathbb{R}^n$ we have $|\varphi(x) - \varphi(y)| = |x - y|$. Let $X \subset \mathbb{R}^n$ be a set such that $\{\varphi(x) \mid x \in X\} \subseteq X$. Show that if X is closed and bounded, then $\{\varphi(x) \mid x \in X\} = X$, and show that we can drop neither of these two assumptions.

Problem B

Let X be a normally distributed random variable and let $t \in \mathbb{R}_{>0}$. Show that $x \mapsto \mathbb{P}(X \leq x + t \mid x \leq X)$ is a decreasing function.

Problem C (proposed by Hendrik Lenstra)

Let $p = 2n + 1$ be a odd prime and consider the finitely presented group G with generators $x_1, \ldots, x_n$ and for each $0 < i, j, k \leq n$ such that $ij = k$ or $ij = p - k$ the relation $x_i x_j = x_k$. Show that G is a cyclic group of order n.

Edition 2023-2 We received correct solutions from Rik Biel, Rik Bos, Pieter de Groen, Alexander van Hoorn, Nicky Hekster, Marnix Klooster, Timo van der Laan, Thijmen Krebs, Kees Roos and Andrés Ventas.