Problem

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 April 2023**. The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A

Does there exist a partitioning X of $\mathbb{R}$ into infinite sets such that for every *choice map* $c: X \to \mathbb{R}$, i.e. a map c such that $c(S) \in S$ for all $S \in X$, the image of c is dense in $\mathbb{R}$?

Problem B

Show that for all $k \in \mathbb{Z}$ there exists an $x \in \mathbb{Q}$ for which there are at least two subsets $S \subseteq \mathbb{Z}_{\geq 1}$ such that $\sum_{s \in S} s^k = x$.

Problem C (proposed by Daan van Gent)

For a group G and $g \in G$ write $c(g) = \{hgh^{-1} \mid h \in G\}$ and $G^\circ = \{g \in G \mid \# c(g) < \infty\}$.

a. Show that G° is a normal subgroup of G and that $G^{\circ\circ} = G^\circ$.
b. Now define $G_\circ = G/G^\circ$. Show that there exists a group G for which the sequence $G, G_\circ, G_{\circ\circ}, \ldots$ does not stabilize, i.e. for none of the groups H in the sequence we have $H^\circ = 1$.

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