Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 October 2022**. The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A

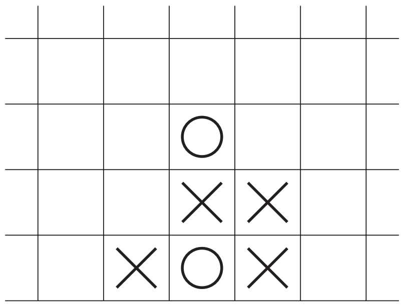
1. Let $n \in \mathbb{Z}_{\geq 1}$ and let $f : \mathbb{R}^n \to \mathbb{R}^n$ be continuous such that for all $x \in \mathbb{R}^n \setminus \{0\}$ we have $|f(x)| < |x|$. Write f^m for the mth iteration of f. Prove that
$$\lim_{m \to \infty} f^m(x) = 0.$$
2. Denote by ℓ^2 the Hilbert space of square-summable sequences of real numbers. Prove that there exists a continuous map $f : \ell^2 \to \ell^2$ such that for all $x \in \ell^2$ we have $|f(x)| < |x|$ and for some $a \in \ell^2$ we have that $\{f^m(a)\}_{m=1}^{\infty}$ does not converge.

Problem B

Prove that for every integer n there exists a finite group G such that n equals the number of normal subgroups minus the number of non-normal subgroups.

Problem C

Olivia and Xavier play the game *Connect Three* on an infinite half grid on a sheet of paper. The rules are as follows: Olivia and Xavier take alternating turns, starting with Olivia. In her turn, Olivia draws an ○ in a square with no empty squares below. In Xavier's turn, he twice draws an $\times$ in a square with no empty squares below. Olivia wins if she gets three ○'s in a row, either horizontally, vertically, or, diagonally. Can Xavier prevent Olivia from winning?



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