This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before 15 October 2022. The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A

1. Let \( n \in \mathbb{Z}_{\geq 1} \) and let \( f : \mathbb{R}^n \to \mathbb{R}^n \) be continuous such that for all \( x \in \mathbb{R}^n \setminus \{0\} \) we have \( |f(x)| < |x| \). Write \( f^m \) for the \( m \)th iteration of \( f \). Prove that

\[
\lim_{m \to \infty} f^m(x) = 0.
\]

2. Denote by \( \ell^2 \) the Hilbert space of square-summable sequences of real numbers. Prove that there exists a continuous map \( f : \ell^2 \to \ell^2 \) such that for all \( x \in \ell^2 \) we have \( |f(x)| < |x| \) and for some \( a \in \ell^2 \) we have that \( \{f^m(a)\}_{m=1}^{\infty} \) does not converge.

Problem B

Prove that for every integer \( n \) there exists a finite group \( G \) such that \( n \) equals the number of normal subgroups minus the number of non-normal subgroups.

Problem C

Olivia and Xavier play the game Connect Three on an infinite half grid on a sheet of paper. The rules are as follows: Olivia and Xavier take alternating turns, starting with Olivia. In her turn, Olivia draws a 5 in a square with no empty squares below. In Xavier’s turn, he twice draws an \( \times \) in a square with no empty squares below. Olivia wins if she gets three \( \circ \)’s in a row, either horizontally, vertically, or, diagonally. Can Xavier prevent Olivia from winning?