Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 April 2022**. The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A (proposed by Hendrik Lenstra)

Let R be a ring. We say $x \in R$ is a *unit* if there exists some $y \in R$ such that $xy = yx = 1$ and write R^* for the set of units of R. Show that $1 < \#(R \setminus R^*) < \infty$ implies $1 < \#R < \infty$.

Problem B (proposed by Hendrik Lenstra)

Let G be a group. For $n \in \mathbb{Z}_{>0}$ write $G[n] = \{g \in G \mid g^n = 1\}$ and $G^n = \{g^n \mid g \in G\}$.

1. Suppose G is abelian and $m, n \in \mathbb{Z}_{>0}$. Show that $G[n] \subseteq G^m$ if and only if $G[m] \subseteq G^n$.
2. Show that there exist $m, n \in \mathbb{Z}_{>0}$ such that the above is false when we drop the assumption that G is abelian.

Problem C (proposed by Onno Berrevoets)

Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Suppose that $a < b < c$ are real numbers such that $f(a) = f(b) = f(c) = 0$. Prove that there exists $x \in (a, c)$ such that

$$f'(x) + f''(x) = f(x)^2 + 2f(x)f'(x).$$

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