Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 July 2021**. The solutions of the problems in this issue will appear in the next issue.

Problem A (proposed by Onno Berrevoets)

A hot frying pan contains 2^{2021} potato slices. Each time we toss the slices, each slice has a chance of 0.5 to land on its other side. These probabilities are individually independent. How often do we need to toss the slices so that with probability at least 0.5 all slices will have lain on both sides?

Problem B (folklore)

1. Five young ladies in a school walk in a line for ten days in succession. It is required to arrange them daily so every pair of ladies occupies all ten possible (pairs of) positions in this line. More precisely, it is required to find ten permutations $\pi_1,\dots,\pi_{10}$ of $\{1,2,3,4,5\}$ such that for all $i \neq j \in \{1,2,3,4,5\}$, and for all $i' \neq j' \in \{1,2,3,4,5\}$ there is a $k \in \{1,2,\dots,10\}$ such that $\{i',j'\} = \{\pi_k(i),\pi_k(j)\}$. Is this possible? If so, produce such permutations, and if not, prove so.
2. Replace 5 by n (with $n \in \{4,6,7,8,9\}$) and replace 10 by $\binom{n}{2}$ in the above problem. For which of these values of n does a set of permutations as in the previous part exist?

Problem C (proposed by Onno Berrevoets)

Let R be a commutative ring, and consider the set X of R-ideals J with $J^2 \neq J$. Suppose that I is a maximal element of X (with respect to inclusion). Prove that I is a maximal ideal of R.

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