Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 October 2020**. The solutions of the problems in this issue will appear in the next issue.

Problem A (proposed by Onno Berrevoets)

Let $f:(-1,1)\to\mathbb{R}$ be a function of class C^∞, i.e., all higher derivatives of f exist on $(-1,1)$. Let $c\geq 0$ be a real number. Suppose that for all $x\in(-1,1)$ and all $n\in\mathbb{Z}_{\geq 0}$ we have $f^{(n)}(x)\geq -c$. Also assume that for all $x\in(-1,0]$ we have $f(x)=0$. Prove that f is the zero function.

Problem B (proposed by Onno Berrevoets)

Consider the map $f:\mathbb{Z}^2_{\geq 0}\to\mathbb{Z}^2_{\geq 0},\ \ (a,b)\mapsto(2\min\{a,b\},\max\{a,b\}-\min\{a,b\})$.
We call $(a,b)\in\mathbb{Z}^2_{\geq 0}$ *equipotent* if there exists $n\in\mathbb{Z}_{\geq 0}$ such that $f^n(a,b)=(x,x)$ for some $x\in\mathbb{Z}_{\geq 0}$ (where $f^n=f\circ\cdots\circ f$). Show that $(a,b)\in\mathbb{Z}^2_{\geq 1}$ is equipotent if and only if $\frac{a+b}{\gcd(a,b)}$ is a power of 2.

Problem C* (folklore)

Uncle Donald cuts a 3 kg piece of cheese in an arbitrary, finite number of pieces of arbitrary weights. He distributes them uniformly randomly among his nephews Hewey, Dewey, and Louie. Prove or disprove: the probability that two of the nephews each get strictly more than 1 kg is at most two thirds.

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