Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 January 2020**. The solutions of the problems in this issue will appear in the next issue.

Problem A (proposed by Onno Berrevoets)

Let $F \in \mathbb{Z}[X]$ be a monic polynomial of degree 4. Let $A \subset \mathbb{Z}$ with $\#A \geq 5$ be such that for all $a \in A$ we have $2^{2019} \mid F(a)$. Prove that there exist distinct $a, a' \in A$ such that $a \equiv a' \bmod 2^{202}$.

Problem B (proposed by Jan Draisma)

Let $n \in \mathbb{Z}_{\geq 2}$ and let S be a non-empty subset of $\{1, 2, \ldots, n-1\}$.

1. Prove that there exists an n-th root of unity $z \in \mathbb{C}$ such that the real part of $\sum_{i \in S} z^i$ is smaller than or equal to $-\frac{1}{2}$.
2. Suppose that n is not a power of 2. Prove that the $-\frac{1}{2}$ in the previous part is optimal in the following sense: there exists a non-empty subset S of $\{1, 2, \ldots, n-1\}$ such that for all n-th roots of unity z, the real part of $\sum_{i \in S} z^i$ is at least $-\frac{1}{2}$.

Problem C (proposed by Onno Berrevoets)

Let $n \geq 1$ be an integer. Denote by S_n the permutation group on $\{1, \ldots, n\}$. An element $\sigma \in S_n$ is called *representable* if there exists a polynomial $f \in \mathbb{Z}[X]$ such that for all $x \in \{1, \ldots, n\}$ we have $\sigma(x) = f(x)$. What is the number of representable elements of S_n?

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