Problemen

| Problem Section

Problem A (proposed by Arthur Bik)

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \in \mathrm{SO}(3)$$

be a matrix not equal to the identity matrix. Prove: if the vector

$$\begin{pmatrix} (a_{23}+a_{32})^{-1} \\ (a_{13}+a_{31})^{-1} \\ (a_{12}+a_{21})^{-1} \end{pmatrix}$$

exists, then A is a rotation using this vector as axis.

Problem B (proposed by Onno Berrevoets)

1. Let $k \in \mathbb{Z}_{>0}$ and let $\mathcal{X} \subset 2^{\mathbb{Z}}$ be a subset such that for all distinct $A,B \in \mathcal{X}$ we have $\#(A \cap B) \leq k$. Prove that $\mathcal{X}$ is countable.
2. Does there exist an uncountable set $\mathcal{X} \subset 2^{\mathbb{Z}}$ such that for all distinct $A,B \in \mathcal{X}$ we have $\#(A \cap B) < \infty$?

Problem C (proposed by Onno Berrevoets)

Let $A : \mathbb{R}^2 \to \mathbb{R}$ be a continuous function such that for every $x,y,z \in \mathbb{R}$ we have

1. $A(x,y) = A(y,x)$,
2. $x \leq y \Rightarrow A(x,y) \in [x,y]$,
3. $A(A(x,y),z) = A(x,A(y,z))$,
4. A is not the $\max$ and not the $\min$ function.

Prove that there exists an $a \in \mathbb{R}$ such that for all $x \in \mathbb{R}$ we have $A(x,a) = a$.

Redactie: Onno Berrevoets, Rob Eggermont en Daan van Gent

problems@nieuwarchief.nl

www.nieuwarchief.nl/problems