

Problemen

| Problem Section

Problem A

Let f be a function from the set of positive integers to itself such that, for every n , the number of positive integer divisors of n is equal to $f(f(n))$. For example, $f(f(6)) = 4$ and $f(f(25)) = 3$. Prove that if p is prime then $f(p)$ is also prime.

Problem B

Let n be a positive integer and $F \subseteq 2^{[n]}$ a family of subsets of $[n] = \{1, 2, \dots, n\}$ that is closed under taking intersections. Suppose that

1. For every $A \in F$ we have: $|A|$ is not divisible by 3.
2. For every pair $i, j \in [n]$ there is an $A \in F$ such that $i, j \in A$.

Show that n is not divisible by 3.

Problem C* (proposed by Hendrik Lenstra)

Cut three squares of equal size in exactly the same way into three pieces each in such a way that the resulting nine pieces can be rearranged to form a regular twelve-gon. Open question: Can you cut the three squares into *eight* pieces that form a regular twelve-gon?