Problemen

| Problem Section

Problem A

Consider two identical bags of stones, all having integral weights. No two weights in a bag are the same. Suppose that the stones from these two bags are divided into proper subsets of equal cardinality and equal total weight. No two weights in a subset are the same. Is it then possible to readjust this division such that each subset contains stones from the same bag?

Problem B

Let $H_1, \ldots, H_k$ be k planes in $\mathbb{R}^3$. Prove that the unit ball contains an open ball of radius $\frac{1}{k+1}$ which does not intersect any of the planes.

Problem C (proposed by Hendrik Lenstra)

Suppose that a, b, n, m are integers, and that m, n are positive, such that $a^n \equiv b^n \equiv -1 \bmod m$. Prove there exists an integer c such that $ab \equiv c^2 \bmod m$. Also provide a fast method to compute such a c which works even if the prime factors of m are unknown.

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