Problemen

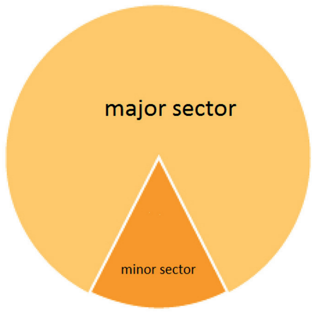
| Problem Section

Problem A

Show that there are no infinite antichains for the partial order $\leq$ on $\mathbb{N}^k$ defined by $(x_1, x_2, \ldots, x_k) \leq (y_1, y_2, \ldots, y_k)$ iff $x_i \leq y_i$ for all i, $1 \leq i \leq k$.

Problem B

A sector is a portion of a disk enclosed by two radii and an arc. For each pair of radii, there are two complementary sectors. If the two complementary sectors have unequal area, then we say that the larger sector is the major sector.



Let S be a subset of the plane. We say that $x \in S$ is virtually isolated if x is the only element of S in a major sector of a disk centered on x. Suppose that all elements of S are virtually isolated. Prove that S is countable.

Problem C (proposed by Hendrik Lenstra)

Let n be a natural number > 1. Suppose that for every prime $p < n$ we have that $p^n \equiv (p-1)^n + 1 \bmod n^2$. Prove that $n = 2$.

Consider the morphism $f: R \to S$ of rings given by $x_i \mapsto t_{i-1} t_i t_{i+1}$, $y_i \mapsto t_i^3$ and $z_i \mapsto t_i^2$. Does there exist a finite number of elements $r_1, \ldots, r_n \in R$ such that the kernel I of f is generated as an ideal in R by $\{\tau^i r_j : i \in \mathbb{Z}, j = 1, \ldots, n\}$?

Redactie:

Gabriele Dalla Torre
Christophe Debry
Jinbi Jin
Marco Streng
Wouter Zomervrucht

Problemenrubriek NAW
Mathematisch Instituut
Universiteit Leiden
Postbus 9512
2300 RA Leiden

problems@nieuwarchief.nl
www.nieuwarchief.nl/problems