Problem Section

Problem A

Reconstruction. Suppose you get to know the n midpoints of the n edges of a polygon. Can you determine the polygon?

Problem B

Large is odd. Let *G* be a graph with vertices *V* and edges *E*. A subset $U \subset V$ is called large if every vertex that is not in *U* has a neighbor in *U*. Prove that the number of large subsets is odd.

Problem C

Meanders. On our way to this problem, we first meander through a Martin Gardner eye test:



One of the two spirals in the illustration consists of a single piece of rope that has its ends joined. The other spiral consists of two separate pieces of rope, each with joined ends. Can you identify which is which using only your eyes?

Now we define a meander. Let *n* be an even number. Consider the integers 1 to *n* in the complex plane and connect them by semicircles centered around $\frac{n+1}{2}$ in the upper half plane. Let n = a + b for even numbers *a* and *b*. Connect the first *a* integers by semicircles around $\frac{a+1}{2}$ in the lower half plane. Similarly, connect the final *b* integers by semicircles around $a + \frac{b+1}{2}$ in the lower half plane. The resulting curve, or set of curves, is a meander. For which *a* and *b* is it connected?

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