Problemen

Problem Section

Problem A (folklore)

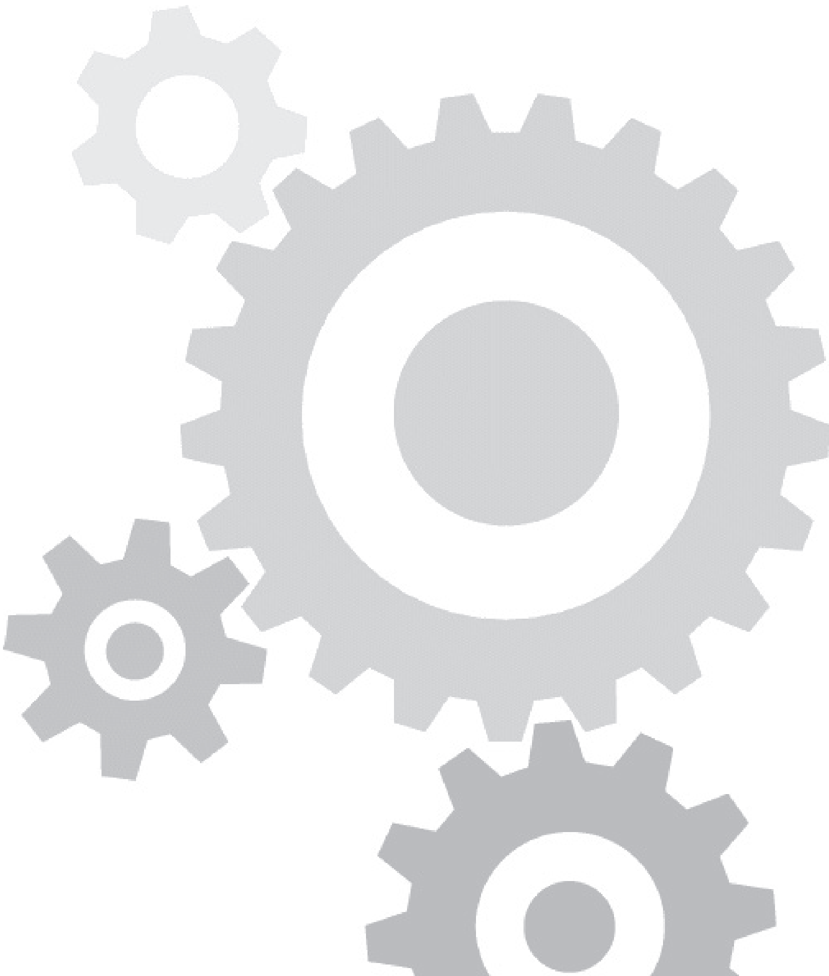
Show that a group G is torsion-free if and only if for all integers $n \geq 2$ and finite subsets $S, T \subseteq G$ with $\#S = \#T = n$ we have $\#\{st : s \in S, t \in T\} > n$.

Problem B (proposed by Hendrik Lenstra)

Show that for all groups G the commutator subgroup $[G,G] = \langle xyx^{-1}y^{-1} : x,y \in G \rangle$ of G has order at most 2 if and only if every conjugacy class in G has at most 2 elements.

Problem C (proposed by Carlo Pagano and Mima Stanojkovski)

A subgroup H of a group G is said to be *solitary* if no other subgroup of G is isomorphic to H. A group G is said to be *totally solitary* if all of its subgroups are solitary. Show that a group G is totally solitary if and only if it is isomorphic to a subgroup of $\mathbb{Q}/\mathbb{Z}$.



Redactie:

Gabriele Dalla Torre
Christophe Debry
Jinbi Jin
Marco Streng
Wouter Zomervrucht

Problemenrubriek NAW
Mathematisch Instituut
Universiteit Leiden
Postbus 9512
2300 RA Leiden

problems@nieuwarchief.nl
www.nieuwarchief.nl/problems