Problemen

| Problem Section

Problem A (folklore)

Let $a, b > 1$ be integers such that $2a \leq b$. Does there exist a map

$$f : \{a, a+1, \cdots, b\} \to \{a, a+1, \cdots, b\}$$

without fixed points, such that for all $n \in \{a, a+1, \cdots, b\}$ we have $f^{f(n)}(n) = n$? Here, for a positive integer k, f^k denotes the k-fold composition

$$\underbrace{f \circ f \circ \cdots \circ f}_{k \text{ times}}$$

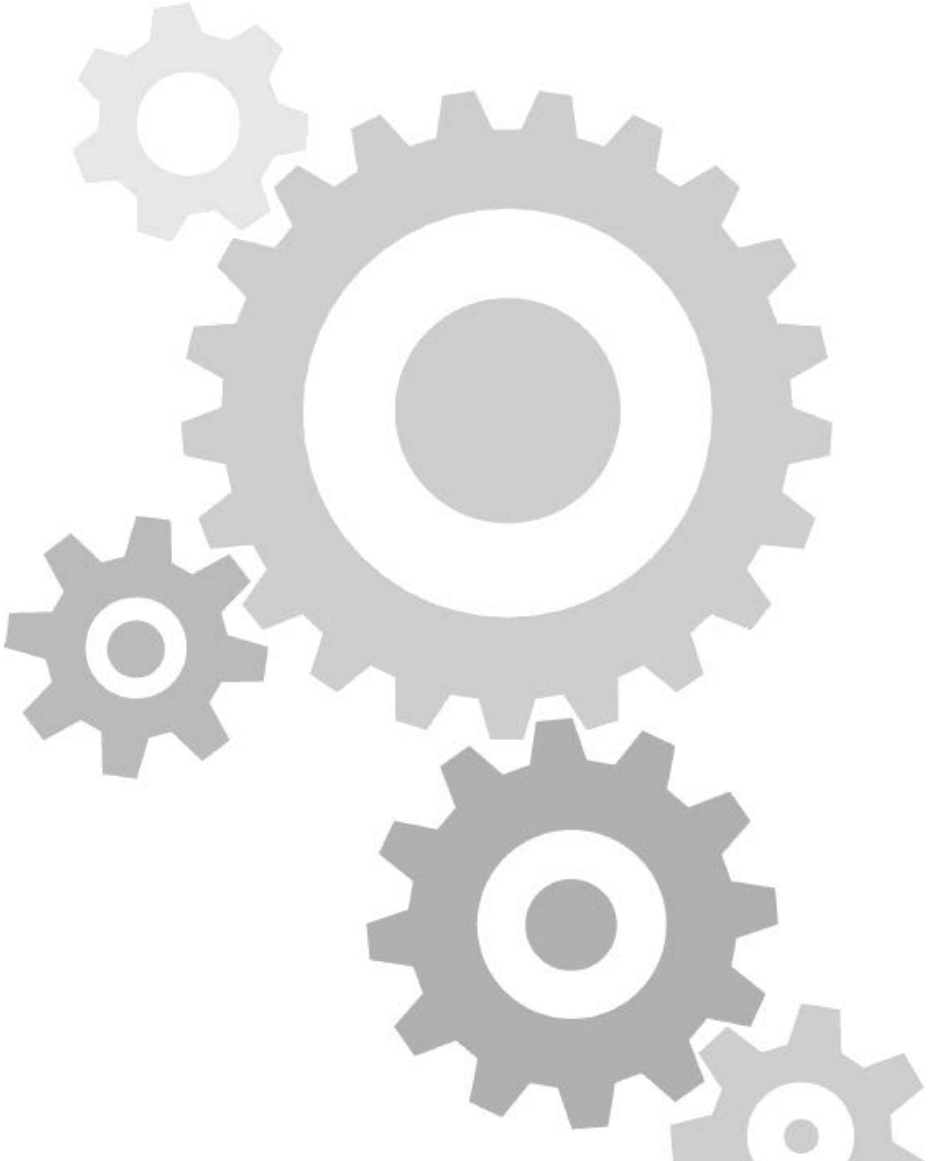
of f.

Problem B (folklore)

Let $n \geq 3$ be an integer. Two players play the following game. Starting with a sheet of paper with the numbers 1 and 2 on them, the players take turns writing down a new number from 1 to n that is the sum of two numbers already on the sheet. The player who writes down the number n wins.
For which n does the first player have a winning strategy?

Problem C (proposed by Hendrik Lenstra)

Determine all two-sided infinite sequences of positive integers in which each number is the Euler-phi of the next.



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