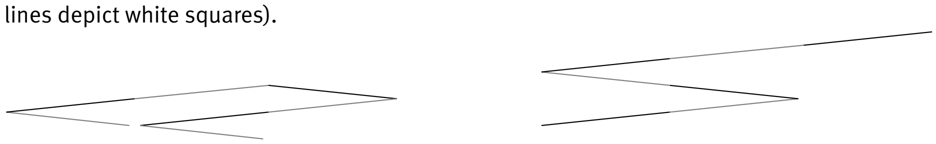
Problemen

Problem Section

Problem A (folklore)

Let n be a positive integer. Given a $1 \times n$-chessboard made out of paper, one is allowed to fold it along grid lines, and in such a way that the end result is a flat rectangle, say $1 \times m$. For example, the following figure shows side views of valid ways of folding a 1×7-chessboard (gray lines depict white squares).



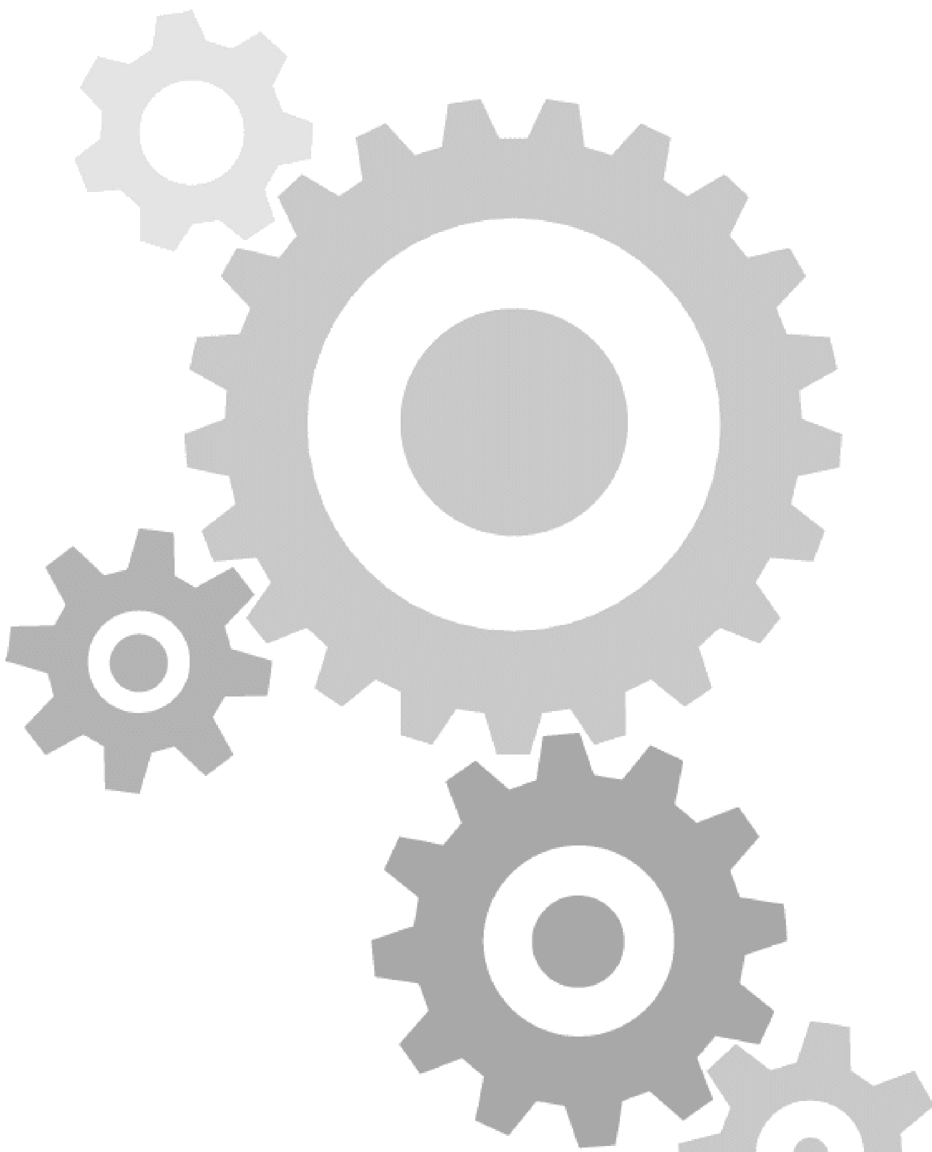
Let a_i for $i = 1, 2, \ldots, m$ be the number of black squares under the i-th square of the resulting rectangle, and consider the tuple $(a_1, a_2, \ldots, a_m)$. So in our examples, the respective corresponding tuples are $(1, 1, 1)$ and $(2, 1, 1)$.
Show that for any positive integer m the m-tuple $(a_1, a_2, \ldots, a_m)$ of non-negative integers can be obtained via the above process if and only if for all $i, j \in \{1, 2, \ldots, m\}$ such that $i + j$ is odd, we have $(a_i, a_j) \neq (0, 0)$.

Problem B (proposed by Jinbi Jin)

Let A be a commutative ring with unit, and let I be an ideal of A with $I \neq 0$ and $I^2 = 0$. Let B be the ring of which the elements are triples (a_1, a_2, a_3) where $a_1, a_2, a_3 \in A$ are such that $a_1 + I = a_2 + I = a_3 + I$, with coordinate-wise addition and multiplication. Show that there exist at least four distinct ring homomorphisms $B \to A$.

Problem C (proposed by Hendrik Lenstra)

Does there exist a non-trivial abelian group A that is isomorphic to its automorphism group?



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