Problemen

Problem Section

Problem A (proposed by Wouter Zomervrucht)

Let n be a positive integer. Let M be an $n \times n$-matrix with entries in $\{1, 2, \dots, n\}$. Let r be the complex eigenvalue with the largest absolute value. Show that $n \leq |r| \leq n^2$.

Problem B (proposed by Hans Zwart)

Let X be a unital $\mathbb{R}$-algebra with multiplicative unit 1, and let $\|\cdot\|$ be a *submultiplicative norm* on X, i.e. a map $\|\cdot\| \colon X \to \mathbb{R}$ satisfying the following properties:

- $\|1\| = 1$;
- if $x \in X$ satisfies $\|x\| = 0$, then $x = 0$;
- for all $a \in \mathbb{R}, x \in X$, we have $\|ax\| = |a|\,\|x\|$;
- for all $x, y \in X$, we have $\|x + y\| \leq \|x\| + \|y\|$ and $\|xy\| \leq \|x\|\,\|y\|$.

Let $C \colon \mathbb{R} \to X$ be a map such that $C(0) = 1$ and such that for all $s, t \in \mathbb{R}$, we have

$$2C(s)C(t) = C(s+t) + C(s-t).$$

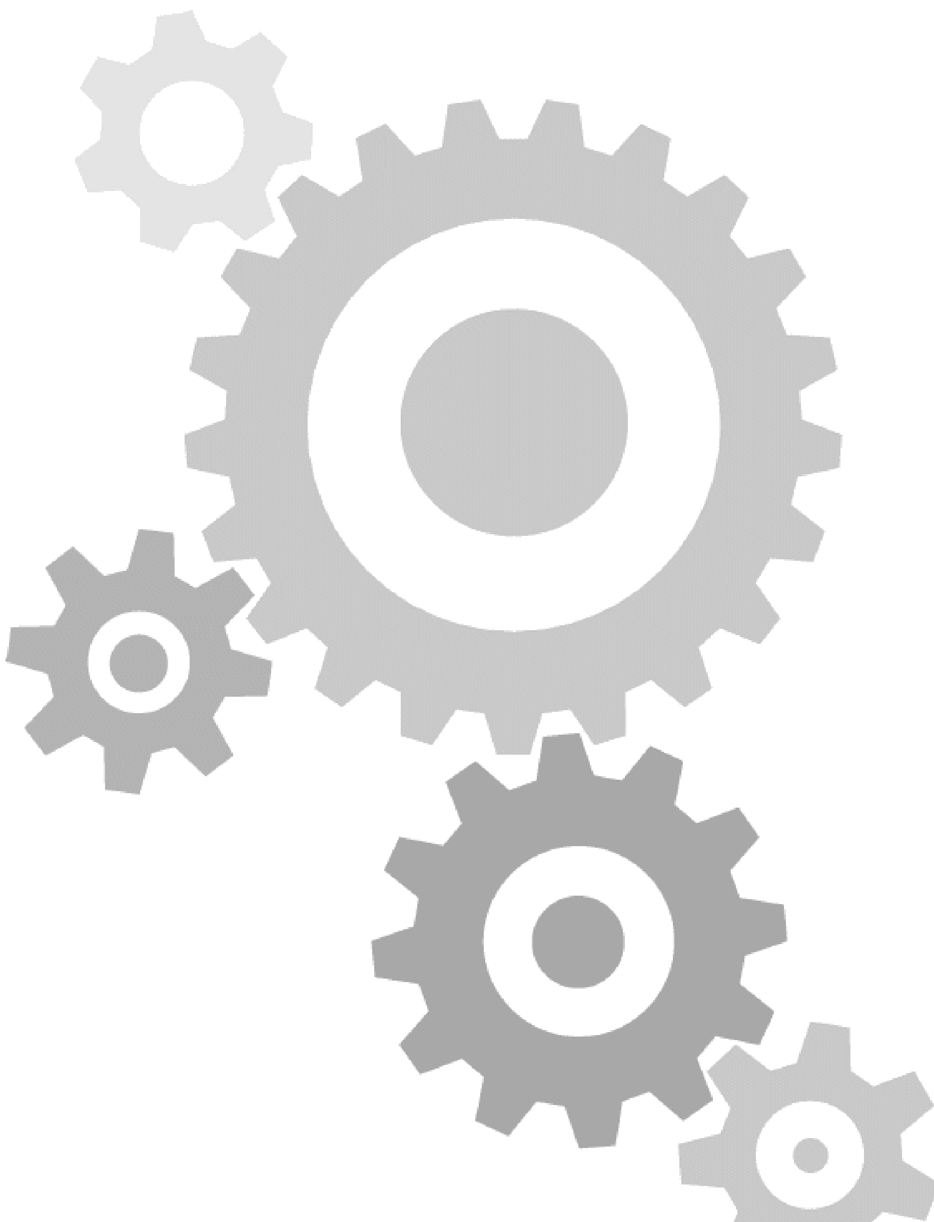
Suppose that

$$\sup_{s \in \mathbb{R}} \|C(s) - 1\| < \tfrac{3}{2}.$$

Show that $C = 1$.

Problem C (proposed by Hendrik Lenstra)

Let G be a finite group. Let n be the number of automorphisms σ of G such that for all $x \in G$, the element $\sigma(x)$ is conjugate to x. Show that every prime divisor of n divides the order of G.



Redactie:
Johan Bosman
Gabriele Dalla Torre
Christophe Debry
Jinbi Jin
Marco Streng
Wouter Zomervrucht

Problemenrubriek NAW
Mathematisch Instituut
Universiteit Leiden
Postbus 9512
2300 RA Leiden
problems@nieuwarchief.nl
www.nieuwarchief.nl/problems