Problemen

Problem Section

Problem A (folklore, communicated by Jaap Top)

Does there exist an integer $n > 1$ such that the *set* of leading digits of $2^n, 3^n, \ldots, 9^n$ is equal to $\{2, 3, \ldots, 9\}$?

Problem B (proposed by Bart de Smit and Hendrik Lenstra)

Rings are unital, and morphisms of rings send 1 to 1.
Let A and B be commutative rings. Suppose that there exists a ring C such that there are injective morphisms $A \to C$ and $B \to C$ of rings. Show that there exists a *commutative* such ring.

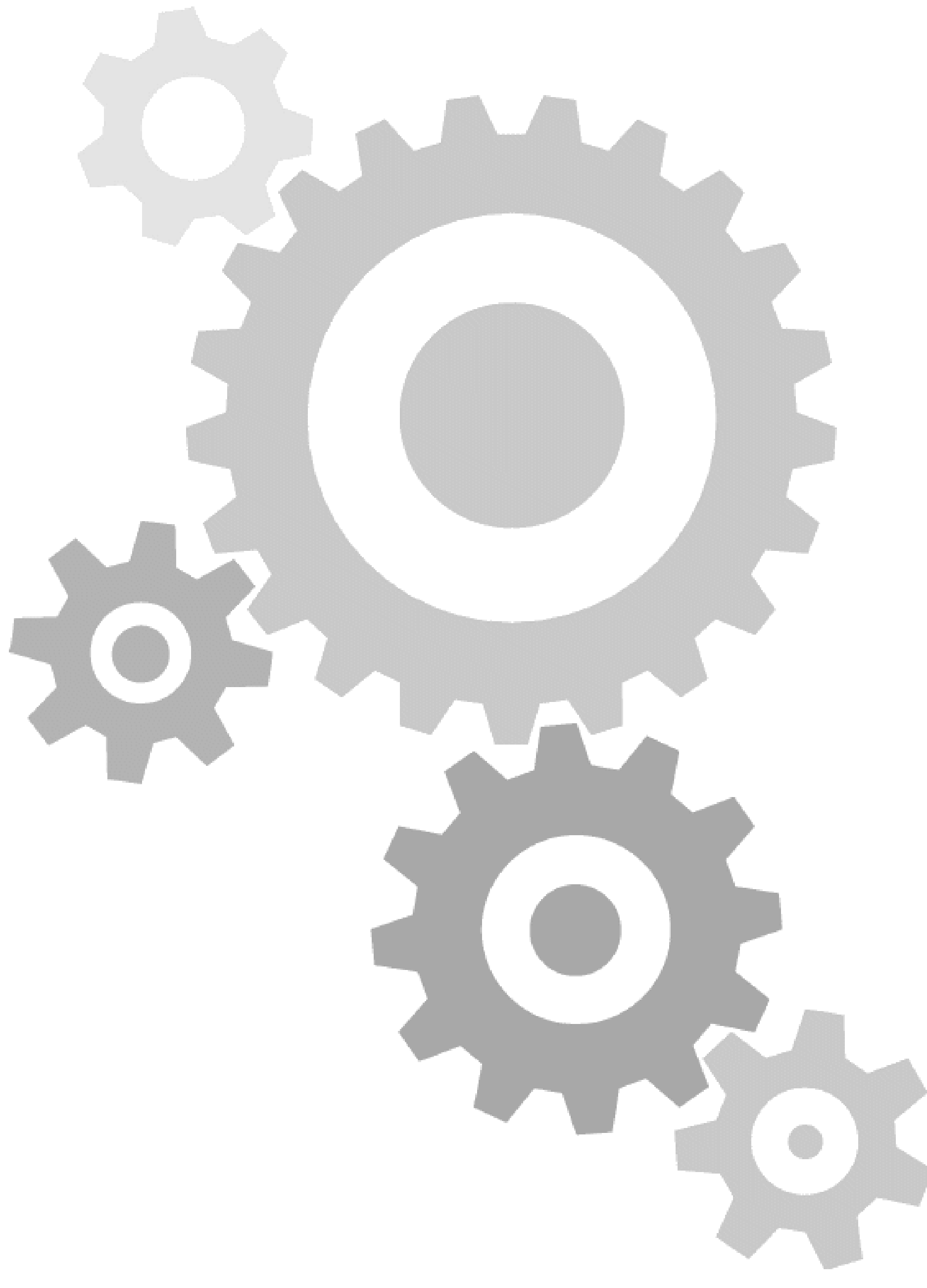
Problem C (proposed by Jinbi Jin)

Let $C(\mathbb{R}, \mathbb{R})$ denote the set of continuous maps from $\mathbb{R}$ to itself. A (not necessarily continuous) map $f\colon C(\mathbb{R}, \mathbb{R}) \to C(\mathbb{R}, \mathbb{R})$ is called *good* if it satisfies, for all $s, t \in C(\mathbb{R}, \mathbb{R})$, the identity

$$f(s \circ t) = f(s) f(t),$$

where the product on the right hand side is the point-wise multiplication of maps.

- Find a non-constant good map $f\colon C(\mathbb{R}, \mathbb{R}) \to C(\mathbb{R}, \mathbb{R})$.
- Show that $f(\exp) = 0$ for all non-constant good maps $f\colon C(\mathbb{R}, \mathbb{R}) \to C(\mathbb{R}, \mathbb{R})$. (Here, exp is given by $x \mapsto e^x$.)



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