Problemen

| Problem Section

Problem A (folklore)

Consider a regular n-gon $P_1P_2\ldots P_n$, and draw $n-3$ diagonals such that there are no intersection points in the interior. The polygon is now divided into $n-2$ triangles. Let t_i be the number of such triangles that have a vertex at P_i. Show that

$$t_1 - \cfrac{1}{t_2 - \cfrac{1}{\ldots - \cfrac{1}{t_{n-1}}}} = 0.$$

Problem B (MSRI Emissary)

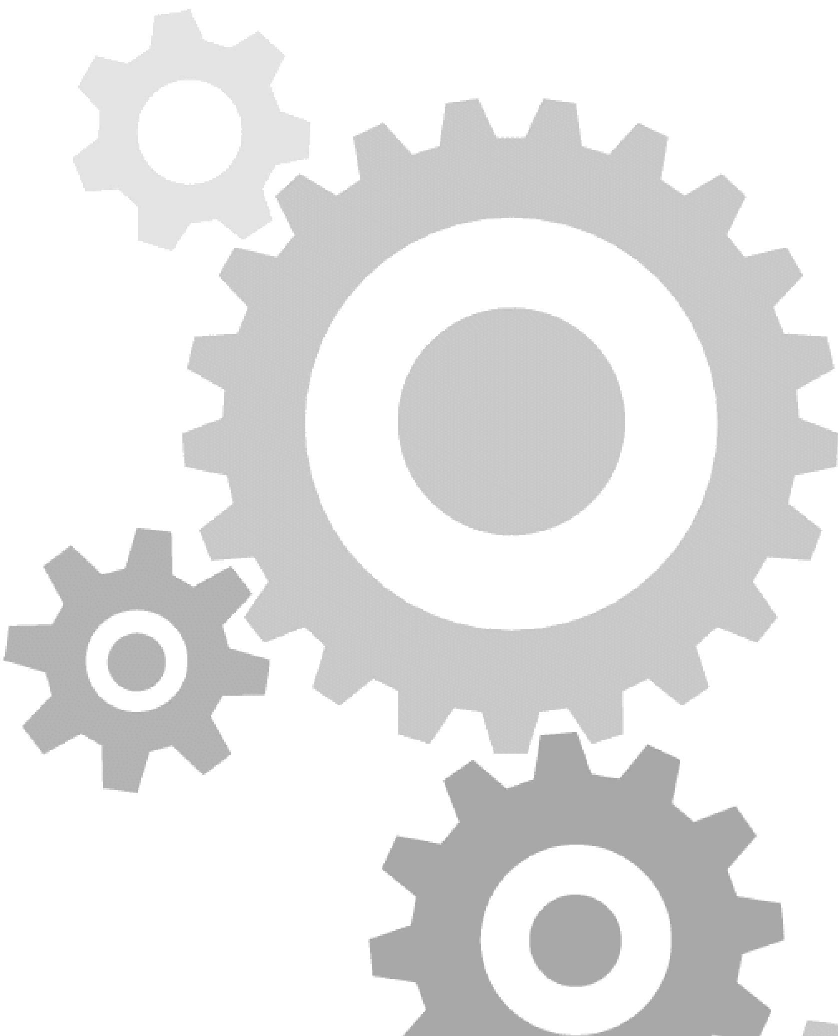
You are allowed to transform positive integers n in the following way. Write n in base 2. Write plus signs between the bits at will (at most one per position), and then perform the indicated additions of binary numbers. For example, $123_{10} = 1111011$ can get + signs after the second, third and fifth bits to become $11 + 1 + 10 + 11 = 9_{10}$; or it can get + signs between all the bits to become $1+1+1+1+0+1+1 = 6_{10}$; and so on.
Prove that it is possible to reduce arbitrary positive integers to 1 in a bounded number of steps. That is, there is a constant C such that for any n there is a sequence of at most C transformations that starts with n and ends at 1.

Problem C (proposed by Jinbi Jin)

Let R be a commutative ring with 1. Consider the set

$$S = \left\{(i,j) \in R^2 : i^2 = i, j^2 = j, ij = 0\right\}.$$

Show that the cardinality of S is a power of 3 if S is finite.



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