Problemen | Problem Section

Problem A (proposed by Mark Veraar)

Let $(X_n)_{n\geq 1}$ be a sequence of independent random variables with values in $[0,+\infty)$ satisfying

$$\mathbf{P}(X_i > t) = \frac{1}{1+t}$$

for all i and all $t \geq 0$. Let $(c_n)_{n\geq 1}$ be a sequence of positive real numbers. Show that the sequence $(c_n X_n)_{n\geq 1}$ is bounded with probability 1 if and only if the series $\sum_{n=1}^{\infty} c_n$ converges.

Problem B (proposed by Simone Di Marino)

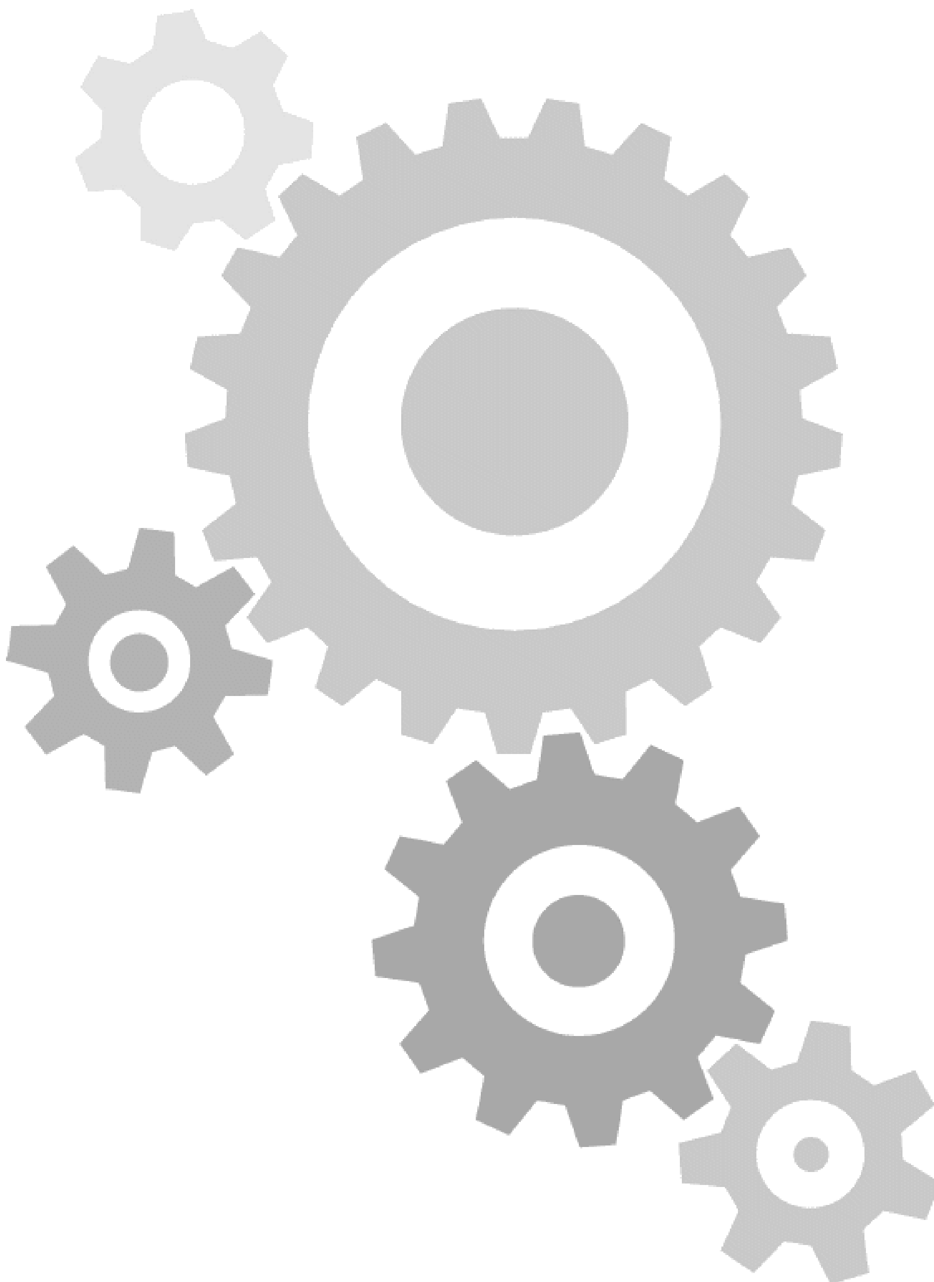
Determine all pairs (a, b) of positive integers such that there are only finitely many positive integers n for which n^2 divides $a^n + b^n$.

Problem C (proposed by Hendrik Lenstra)

Let $f \in \mathbb{Z}[X]$ be a monic polynomial, and let R be the ring $\mathbb{Z}[X]/(f)$. Let U be the set of all $u \in R$ satisfying $u^2 = 1$. Show that U has a ring structure with the following properties: the *zero element* is 1, the *identity element* is -1, the *sum* of two elements in U is their product in R, and the *product* $*$ in U is such that for all u, v, s, t in U the identity $u * v = s * t$ holds in U if and only if

$$(1-u)(1-v) = (1-s)(1-t)$$

holds in R.



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